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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY
OF HARVARD COLLEGE.

THE CHANGEABLE HYDROLYTIC EQUILIBRIUM OF DISSOLVED CHROMIC SULPHATE.

By Theodore William Richards and Frederic Bonnet, Jr.

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THE CHANGEABLE HYDROLYTIC EQUILIBRIUM OF DISSOLVED CHROMIC SULPHATE.*

BY THEODORE WILLIAM RICHARDS AND FREDERIC BONNET, JR.

Presented May 13, 1903. Received May 9, 1903.

INTRODUCTION.

MANY eminent chemists have attempted to solve definitely the riddle involved in the dual nature of chromium solutions; but some doubt still remains concerning the precise interpretation of the facts. A complete review of earlier work would require far too much space at present; instead of such a review, there is given at the end of this paper a chronological bibliography of the subject, which will be referred to throughout the discussion by simply naming the appropriate date. The most satisfactory interpretation, based chiefly upon the work of Krüger (1844), Siewert (1863), Recoura (1887, 1890, 1895, 1899), Doyer van Cleef (1881), and Whitney (1896, 1899), seems to point to the following situation. The violet solution may be inferred to contain the chromic salt in a state comparable to that of other normal salts, while the green solution modified by heating appears to be hydrolyzed into acid and basic substances.† The hydrolysis seems in many cases to proceed to such an extent that about a gram atom of ionized hydrogen is developed for every two of chromium present, but the results are too conflicting to prove whether this is the indication of a nearly completed reaction or merely an accidental coincidence caused by an equilibrium which happens to yield an approximately commensurate mixture.

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* A part of this paper formed a portion of a thesis presented by F. Bonnet, Jr., for the degree of Ph.D. at Harvard University.

† Recoura has pointed out sharply the difference between the solutions thus modified and those obtained from salts which have been heated in the dry state. He names the two respectively "modified" and "green" chromic salts. The present paper has to do only with the former.

The present investigation was begun with the hope of discovering more about the nature of the complex substances produced, and thus explaining the mechanism of the change. The mode of attack consisted, first, in the more persistent and thorough application of the devices of others, and secondly of the introduction of other methods not used before on this problem. Among the former dialysis, migration experiments, electrical conductivity, and the distribution between two liquids were applied; among the latter, solid solution, catalytic sugar inversion, and the depression of the freezing point. While not a final unravelling of the tangle, the research has succeeded in narrowing the possibilities of interpretation.

The Preparation of Chromic Sulphate.

The first problem to be solved was the preparation of chromic sulphate in a pure state on a scale large enough to supply plenty of material for the research. The sulphate was chosen for the research because when once converted into the green mixture, it regains its violet condition far more slowly than almost any other salt. Thus more time is available in which to study the green solution. This very fact, however, introduces a serious difficulty into the original preparation of the violet material, which is much hydrolyzed by a very moderate rise in temperature.

After sundry preliminary experiments which need not be recounted, it was found most convenient to prepare the violet salt according to the method of Traube (1848) with the additional precaution of careful cooling. When alcohol is run into a diluted mixture of sulphuric and chromic acids much heat is evolved, and the violet salt is accordingly hydrolyzed. By immersing the vessel containing the reacting solution in a freezing mixture and by adding the alcohol very slowly with continued agitation, the temperature may be kept below 10°, and less green chromic sulphate is formed. Upon adding subsequently an excess of alcohol the violet salt is precipitated in beautiful pale violet pearly flakes, which may be further purified by solution in water and reprecipitation by alcohol. We used an initial solution of chromic and sulphuric acids containing about 200 grams and 300 grams respectively of these substances dissolved in 600 grams of water. The yield of crystallized salt was about twenty per cent of the theoretical yield.

The salt thus obtained was analyzed for chromium by ignition and for sulphuric acid by precipitation, taking care to use conditions which prevent the occlusion of chromium in the baric sulphate.* For every double gram atom of chromium there were found in two analyses 2.98 and 2.99 gram molecules of the sulphuric group, showing that the salt was normal. It was found by difference to contain 42.81 per cent or 16.3 molecules of water.

DIALYSIS.

Van Cleef in 1881 used dialysis as a means of partially separating the acid and the basic substances, and his method seemed worthy of a further trial. Accordingly we made a number of experiments in this direction. To our disappointment we found the results difficult to interpret quantitatively, partly because dialysis is a slow process, and during its dilatory progress the green solution has time to revert partially into the violet one. The experiments are, however, worthy of brief chronicling, because their qualitative interpretation is clear.

In the first place violet solutions were subjected to dialysis at temperatures from 16° to 20°, the results showing that the diffusing solution is essentially like that which remains behind. Parchment paper, carefully tied and with all accidental apertures sealed with collodion, was used as the diaphragm ; and the level of the liquids was always adjusted in such a way as to prevent filtration.

For analysis, two precisely similar portions of a given solution were measured by means of a pipette. One of these portions was evaporated and ignited in a platinum crucible, and the other was diluted, and treated with slight excesses of ammonic hydroxide, baric chloride, and hydrochloric acid in succession. The mixture was heated after the addition of the ammonia, and was digested for some time on the steam bath before filtration. This method, an application of that devised by Küster and Thiel † for exclusion of iron from baric sulphate, is successfully applicable to the case of chromium also. Here are the results : —

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* A gram of the salt yielded in one case 0.2226, and in another 0.2229 grams of chromic oxide ; again in one case 1.017 and in another 1.021 grams of baric sulphate. The baric sulphate was precipitated from a very dilute cold solution containing hydrochloric acid. The mixture was subsequently heated before filtration. The baric sulphate formed in this way is colorless and contains only traces of chromium. This matter will be discussed at length in another paper.

† Zeit. anorg. Chem., **19**, 97 (1899). This matter will be discussed in detail in another paper.

DIALYSIS OF THE VIOLET SOLUTION.

No. of Exp.	Solution Analyzed.	Cr_2O_3.	$BaSO_4$.	Average Atomic Ratio Cr : SO_4.
1	Dialyte	0.0455	0.2066	2 : 2.95
2		0.0450	0.2043	
3		0.0508	0.2276	
4		0.0504	0.2285	
5	Residue	0.1120	0.5180	2 : 2.99
6		0.1119	0.5118	
7		0.1713	0.7847	
8		0.1717	0.7815	

Thus the "dialyte," or the portion which passed through the diaphragm, seemed to be slightly more basic than that which remained behind — but the difference is scarcely greater than the possible errors of analysis. The diffusion is obviously concerned chiefly with the normal salt, $Cr_2(SO_4)_3$.

Very different results were obtained when the green solution was subjected to dialysis. Below are given the corresponding analyses of material which had been hydrolyzed by heating to the boiling point: —

DIALYSIS OF THE GREEN SOLUTION.

No. of Exp.	Solution Analyzed.	Cr_2O_3.	$BaSO_4$.	Average Atomic Ratio Cr : SO_4.
9	Dialyte	0.0470	0.2951	2 : 4.09
10		0.0471	0.2947	
11	Residue	0.1174	0.5071	2 : 2.82
12		0.1172	0.5062	

Here the difference between the analyses of the "dialyte" and residue is vastly greater than any possible analytical error, and in the opposite direction from the slight one observed in the case of the violet salt. The portion which diffused through the diaphragm contained much more of the sulphuric group in proportion than that which remained behind. From the analysis, the deficiency of acid in the residue seems to be less than the excess of acid in the dialyte merely because less than quarter of the material was allowed to pass through the diaphragm. The total deficiency, on the one hand, of course exactly equalled the total excess on the other. These facts agree essentially with the results of van Cleef.

In support of these data it is worthy of note that the green boiled solution is strongly acid to methyl orange.

Clearly the results of dialysis, complicated as they are by the continual reversion of green to violet, are too involved to furnish more than qualitative evidence that hydrolysis has really taken place in the green solution. The qualitative evidence is nevertheless indubitable.

Catalytic Action.

Long suggested in 1897 that the green chromium solution probably inverts cane sugar, while the violet does not. Modern hypothesis contends, as is well known, that ionized hydrogen is the cause of this kind of catalytic action; hence the speed of the inversion might afford a measure of the extent to which the chromic solution has been hydrolyzed.

The strength of color of the solutions makes the use of the polarimeter impossible, hence the ingenious method used by Kahlenberg and Davis * was employed for determining the extent of the inversion. This method depends upon the fact that invert sugar has about twice as great a depressing effect on the freezing point of water as the cane sugar from which it is made. The catalyzer is assumed to exert a constant effect on the freezing point. Thus the progress of the inversion may be followed by taking successive measurements of the freezing point of the solution.

Our sugar solution contained about 250 grams in the litre. It was carefully sterilized by intermittent heating on three successive days; and the permanent apparatus for delivering measured quantities of the solution was also sterilized. If suitable precautions of this kind are taken, the solution will of course preserve its strength for months. A

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* Jour. Am. Chem. Soc., **21**, p. 1 (1899).

fourth-molar solution of crystallized chromic sulphate (170 grams in a litre) was made; and a portion of this was boiled in a Jena flask with a reflux condenser and rapidly cooled. Thus were obtained a violet and a green solution containing the same concentration of chromium, the latter being always used very promptly after its preparation in order to forestall as much as possible the reversion to the violet condition.

The freezing point determinations were made in large test-tubes in a Beckmann apparatus. Each tube contained 20.00 cubic centimeters of the sugar solution, and 5.00 cubic centimeters of the chromic solution or of another solution whose catalytic action was to be tested. In order to determine the freezing point of the cane sugar solution another tubeful was prepared in which these last 5.00 cubic centimeters were of pure water.

The freezing points of these solutions were determined in the first place immediately after mixing. They were then placed in a water tank of fairly constant temperature (not far from 17°); and from time to time further determinations of the freezing point were made.

Oddly enough the green and violet solutions caused in the first place the same depression of the freezing point of the sugar solution. It is not wholly safe to conclude from this, however, that the average number of molecules in a given bulk of the two solutions would be identical if the sugar were absent; for it is not impossible that a portion of the complex substances existing in the green solution may combine with sucrose, as we later found them to affect gelatine.

In a short time the catalytic action of the two solutions was seen to be very different. The green solution inverted the sugar rapidly, while the violet solution had only a slight action. This is indicated by the following table of data and results, most of which explains itself. The sixth column labelled "acid" represents the freezing points of a 0.468 normal solution of hydrochloric acid. Each of the temperatures represents an average of at least four readings of the thermometer. The changes in the zero point of the thermometer were registered by freezing-point determinations of pure water; these were essentially parallel with the changes in the pure sugar solution and are not recorded in the table on the opposite page.

It is evident from this table that the green solution possessed at least twelve times as great a catalyzing power as the violet, while the acid catalyzed at first about 3.5 times as fast as the green. As the sucrose diminishes in concentration the speed of the reaction diminishes from this cause, and as time goes on, the green slowly reverts to the violet,

PRELIMINARY SERIES OF CATALYTIC EXPERIMENTS.

Fresh Violet and Boiled Green Solution at 17°±.

Time.	Data : Observed Freezing Points.				Results : Lowering of Freezing Point with Time.		
Hours.	Sugar.	Violet.	Green.	Acid.	Violet.	Green.	Acid.
	°	°	°	°	°	°	°
. .	4.060	3.760	3.760	3.655	. . .	. . .	. . .
18	4.071	. . .	3.672	3.326	. . .	0.099	0.340
46	4.086	3.770	3.617	3.127	0.016	0.169	0.464
68	4.086	3.765	3.537	3.007	0.021	0.249	0.674
138	4.071	3.735	3.339	2.690	0.036	0.432	0.976

hence the first few hours of the experiment give the best clue to the composition of the solution.

The inference to be drawn from this result is, then, that the fourth molar green solution contains about as much ironized hydrogen as a $\frac{0.468}{3.5} = 0.133$ normal solution of hydrochloric acid, or every ion of hydrogen corresponds to 3.7 atoms of chromium. This is a much smaller proportion of ionized hydrogen than that found by Recoura and Whitney (1 : 2); but it must be remembered that both of these experimenters made their determinations by neutralization, a proceeding which might well displace an existing equilibrium and make the amount of ionized hydrogen seem too large.

In another series of experiments the conditions were varied. The chromic solution was hydrolyzed for many hours at 50°, instead of by boiling. A temperature of 30° was chosen for the inversion in order that its speed might be greater than before; this temperature was maintained by means of an Ostwald thermostat. The acid solution which served as a measure of the ionized hydrogen present was much diluted for this comparison, because it had been shown before to be stronger even than the acid in the chromic solution which had been boiled. By two or three preliminary tests it was found that an acid having about the same inverting power as the green solution prepared at 50° was very nearly tenth normal. Below are given the data and results of the series, arranged in the same way as before. The hydrochloric acid whose results are recorded in the fifth and last columns was 0.105 normal.

Further Catalytic Experiments.

Fresh Violet and Green Solutions at 30°.

Time.	Data: Observed Freezing Points.				Results: Lowering of Freezing Point with Time.		
Hours.	Sugar.	Violet.	Green.	Acid.	Violet.	Green.	Acid.
. .	3.835°	3.542°	3.537°	3.922°	. . .°	. . .°	. . .°
6	3.846	3.528	3.390	. . .	0.011	0.154	. . .
20	[3.835]	. . .	. . .	3.487	. . .	. . .	0.435
23	3.860	3.470	3.130	. . .	0.083	0.428	. . .
29	[3.850]	3.435	3.040	. . .	0.108	0.508	. . .
47	3.183*	2.640	2.133	. . .	0.236	0.748	. . .
53	[3.183]	2.593	2.010	. . .	0.283	0.871	. . .
68	[3.835]	. . .	. . .	3.032	. . .	. . .	0.890

These results contain much of interest; their outcome is best seen in the accompanying diagram. (Dotted curves H, B, and A.) The typical logarithmic curve whose inclination depends solely upon a constant concentration of catalyzer and the changing concentration of the sucrose is shown by the line A, representing the performance of the acid recorded in the last column above. Any departure from this curve must indicate some other superposed reaction.

Neither of the curves of the chromium solutions (B and H) shows precise similarity to the typical curve, but that of the green solution (B) is of the same general character. Especially at first its irregularity is slight; here it acts as if the concentration of its ionized hydrogen were perhaps 0.095 normal.

On the other hand the curve of the violet solution (H) indicates clearly an important superposed reaction. At first, in confirmation of the previous results, it seems to contain about a tenth as much acid as the green salt; but the rate of inversion steadily *increases* with time, instead of slightly decreasing. This anomaly is only to be explained by supposing the formation of more green substance and acid, — that is, by the hydrolysis of the violet salt at 30°. This conclusion was verified by the

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* Thermometer reset.

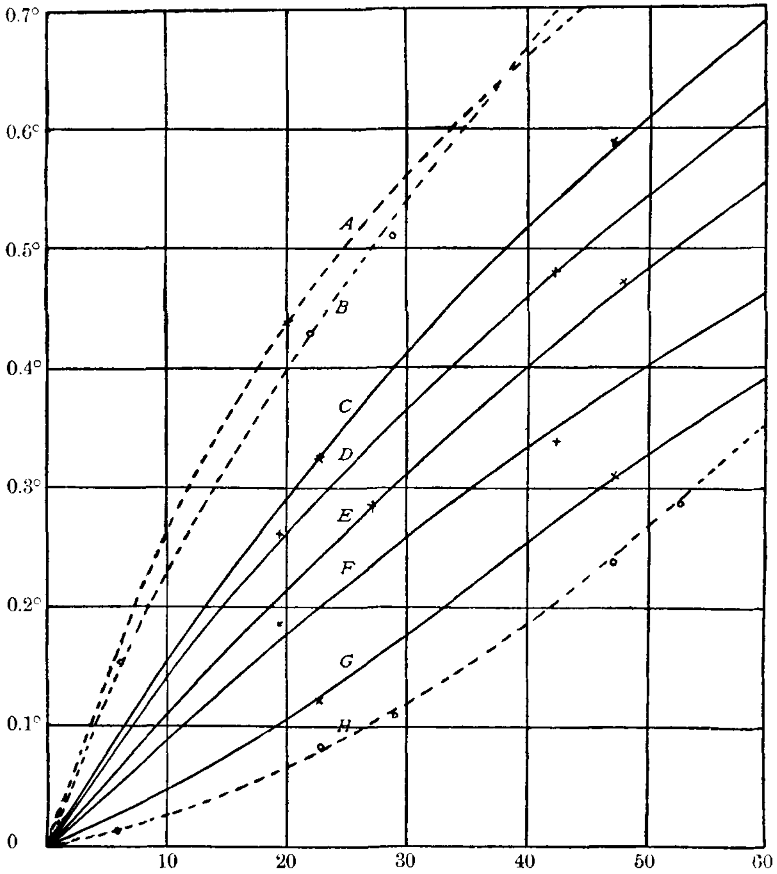

FIGURE 1. — THE RESULTS OF CATALYSIS.

Hours are plotted abscissally, and changes in the freezing point ordinately. The curve *A* corresponds to pure acid, *B* to freshly boiled green solution, *H* to freshly made violet solution, and *E* to both of these solutions after having been kept for a month at 30°.

color of the solution, which at the end of five days had become decidedly green.

This series of experiments thus showed the following facts: —

First, the green salt hydrolyzed at 50° contained slightly less acid than that hydrolyzed at 100°, the respective acidities being perhaps 0.10 and 0.13 normal in a $\frac{1}{4}$ molar solution.

Secondly, at 30°, in the presence of sucrose and glucose, the violet solution is converted at least in part into the green.

From these experiments it seemed probable, since the effect of the sugar could hardly be great, that the relation between the green and the violet solution is a changeable one, depending upon the temperature, and attaining at each temperature a definite equilibrium. It seemed highly desirable to test this conclusion by suitable experiments.

Accordingly a fourth-molar violet and a fourth-molar boiled green solution, both free from sugar, were placed in a thermostat at 30°. After five days, 5 c.c. of each solution was removed by means of a pipette, mixed with two portions of 20.00 c.c. each of a sugar solution, and followed by means of the freezing point. Unfortunately the supply tube containing the green solution was then broken, so that a new one was placed in the thermostat. At the end of twelve days for the violet and seven days for the new green solution, new portions were mixed with sugar; again eight days afterward this process was repeated, and yet again after nine days more. There are given below the figures of the first two and the last two of these freezing point determinations; the others confirm the verdict of these and are omitted to economize space. For the same reason only the additional depressions of the freezing point are given; — these were found in the manner already described. The results are plotted on the diagram (curves C, D, E, F, G), which gives also in dotted lines those of the previous series.

EQUILIBRIUM OF GREEN AND VIOLET SOLUTIONS.

Depression of Freezing Points of Solutions Maintained at 30°.

Time of Inversion.	G. Violet Hydrolyzed 5 Days at 30°.	C. Green Dehydrolyzed 5 Days at 30°.	Time of Inversion.	F. Violet Hydrolyzed 12 Days at 30°.	D. Green Dehydrolyzed 7 Days at 30°.	Time of Inversion.	Violet Hydrolyzed 20 Days at 30°.	Green Dehydrolyzed 15 Days at 30°.
22.7	0.120	0.323	19.5	0.190	0.263	19	0.205	0.213
47.2	0.302	0.587	42.5	0.338	0.473	66	0.473	0.538
70.7	0.433	0.768	66.5	0.497	0.672	E.	29 Days.	24 Days.
94.7	0.550	0.880	91	0.635	0.817	27.2	0.285	0.286
165	0.827	1.100	116.7	0.720	0.900	48	0.470	0.468
184	0.877	1.120	186.7	0.900	1.030	96	0.735	0.740
207	0.923	1.140	212	0.937	1.060	150	0.900	0.890

Thus in about a month the violet and green solutions, kept in a thermostat at 30°, had become identical in their inverting power and probably in every other respect. (Curve F.) The concentration of acid corresponding to this equilibrium is not far from 0.043 normal, judging from the inclinations of the curves at the origin.

Thus the conclusion which prompted the last experiments is satisfactorily confirmed. Evidently the hydrolytic equilibrium between the green and the violet salts varies continuously with the temperature. At 18°, in a fourth-molar solution of salt, the violet salt is chiefly in evidence, the inverting power corresponding to only a small fraction of the green solution. At this temperature even the slight hydrolysis which occurs must take place very slowly, and months would be needed to reach the true equilibrium. This conclusion is in accord with the spectroscopic results of Erhard (1875) and Vogel (1878), and explains the fact noted by Lecoq de Boisbaudran (1874) that a violet chromic solution increases slightly in volume during long standing at 20°. At 30° the acid resulting from the hydrolysis attains in equilibrium a concentration of about 0.04 normal; at 50° of about 0.10 normal, and at 100° of at least 0.13 normal. The last two figures are probably too low because the green solution must obviously begin to "dehydrolyze" as soon as it cools.

In the light of these facts it is clearly impossible from any direct analysis of the solution to determine the composition of the compounds resulting from the hydrolysis. Evidently, too, the process of neutralization by alkali adopted by several experimenters will tend to hasten the incomplete hydrolysis by removing the ionized hydrogen; but at the ordinary room temperature it is unlikely that any definite equilibrium will be reached in a short time. Hence the composition assigned to the basic compound by Recoura and Whitney is by no means certain; it might have been varied by time and by change of temperature. This indeed is indicated by the disagreeing formula of Siewert.

It seemed possible, however, that if the acid were in any manner removed, without introducing any other substance, the basic substance might finally attain a definite composition. Two methods suggested themselves for this purpose; first, the repeated "shaking out" of the acid by another solvent, after repeated warming; and secondly, the long continued agitation of the green solution with chromic hydroxide. Descriptions of experiments of these kinds are recorded below.

THE EXTRACTION OF THE GREEN SOLUTION WITH ALCOHOL AND ETHER.

No. of times boiled and extracted.	Weight of Cr_2O_3.	Weight of $BaSO_4$.	Atomic Ratio $Cr_2 : (SO_4)_x$. $x =$
1	0.1408 0.1407	0.5448 0.5435	2.52
1	0.1034	0.4000	2.52
2	0.3981	1.4992	2.46
3	0.0724	0.2676	2.41
5	0.1571	0.1571	2.25
6	0.0474	0.1498	2.06
9	0.0669	0.1553	[1.52]
14	0.0322 0.0321	0.0978 0.0970	1.98
12	0.0760 0.0760	0.2272 0.2261	1.94

DISTRIBUTION BETWEEN TWO SOLVENTS.

In 1863 Siewert endeavoured to distinguish between the acid and basic salt by distributing the two substances between two solvents, in which it was reasonable to suppose that the two would have different solubilities. He treated an aqueous solution with a mixture of ether and alcohol for this purpose, and found that the viscous aqueous solution remaining was basic, containing only about 2.4 SO_4 instead of 3 SO_4 for each Cr_2. In the light of the facts described above, it seemed to us possible that this basic substance was mixed with some unchanged chromic sulphate; because there is no proof that the hydrolysis was complete in the presence of the acid, and several reasons for believing that it was not complete. This inference could be tested by boiling the green viscous residue with water after its acid had been removed. If the compound had been hydrolyzed as much as possible, further extraction with alcohol and ether could have no further effect; but if the hydrolysis were incomplete, the second boiling would produce more acid capable of extraction with alcohol and ether.

In our experiments a platinum vessel was used for heating the liquid, and the process of boiling and extraction was repeated fourteen times on one sample and twelve times on another. From time to time portions of the green viscous substance were dissolved in water, and two equal portions of the solution were taken for analysis. The chromic oxide was found by evaporation and ignition, the sulphuric acid by precipitation with successive additions of ammonia, baric chloride, and hydrochloric acid in the manner already described. Thus the "atomic ratios" between chromium and sulphuric acid were found. Above are the data and results. The first and the last two analyses were made in duplicate; the others were each single comparisons.

Thus the solution became more and more basic as the treatment proceeded. After the eighth extraction a precipitate appeared on each boiling; this was not filtered off before the ninth analysis, but subsequently it was removed before each analysis. Before the twelfth boiling in each case a constant ratio was reached, which was not altered by further treatment. This corresponds closely to $Cr_2 : (SO_4)_2$; and at first it was supposed that the formula of the compound might be simply $CrSO_4OH$. In order to test this assumption, it was thought desirable to discover if the same equilibrium could be attained in another way, the second method mentioned at the end of the last section.

THE TREATMENT OF THE GREEN SOLUTION WITH CHROMIC HYDROXIDE.

In order to attain a maximum of dissolved chromic oxide, the pure green solution used above was digested on the steam bath in a platinum dish with pure chromic hydroxide for several hours. A portion was then rapidly filtered, and analyzed, while the remainder was heated for nine hours more. The analyses of the two samples are given on the next page.

The relation of the chromium to the sulphuric acid in this solution, which had evidently attained a constant composition, was thus found to be essentially like that obtained by extraction with alcohol and ether. The two comparable average results are respectively 1.97 and 1.96, agreeing essentially with the formula $CrOHSO_4$.

Two circumstances imply that this is not the basic limit of chromic sulphate in cold solutions, however. One of these is the fact that upon extracting such a solution with ether, a clear solution was left which de-

MAXIMUM BASIC CONDITION OF HOT CHROMIC SULPHATE SOLUTION.

Time of Digestion.	Weight of Cr_2O_3.	Weight of $BaSO_4$.	Atomic Ratio $Cr_2 : (SO_4)_x$. $x =$
4 hours	0.1081 0.1080	0.3267 0.3260	1.971
13 hours	0.0732 0.0733	0.2205 0.2214	1.968

posited chromic hydroxide on boiling; and the other is the fact that this boiled solution was found to be strongly acid to methyl orange.

Thus the solution boiled with chromic hydroxide must contain ionized hydrogen, and hence the basic substance present must be even more basic than $CrOHSO_4$, the agreement with this formula being accidental.

In accordance with this inference, it was found that dilute caustic alkali could be added to the solution without producing a permanent precipitate. The same result — the removal of the ionized hydrogen — may be better accomplished by shaking the solution at ordinary temperatures with chromic hydroxide. Even the violet solution may be quickly converted into the green in this way, as Siewert has shown. The hydroxide was prepared by cold precipitation with a slight excess of ammonia, and was washed until free from sulphuric acid. The violet solution was treated with an excess of this precipitate, and the mixture was shaken in a tightly stoppered bottle in an automatic shaking machine for several days, portions being removed from time to time for analysis. Opposite are given the results of this work. The solution was fourth-molar and the temperature about 20°.

The constant results indicate saturation. Thus a decrease in acid from 1.97 to 1.59 — or perhaps better an increase in base from 2.00 to 2.48 — occurs on cooling in the presence of excess of hydroxide. This ratio corresponds almost exactly to $Cr_5 : (SO_4)_4$, a relation which demands the complex formula $Cr_5(SO_4)_4(OH)_7$. The possibility still existed that some ionized hydrogen was present even at 20°, because so much more ionized hydrogen was needed at 100° in order to preserve the equilibrium.

In order to test this point the rate of inversion of sugar at 30° by this

MAXIMUM BASIC CONDITION OF COLD CHROMIC SULPHATE SOLUTIONS.

Time of Shaking.	Weight of Cr_2O_3.	Weight of $BaSO_4$.	Atomic Ratio $Cr_2 : (SO_4)_x$ $x =$
2 days	0.1299	0.3162	1.588
3 days	0.1128	0.2757	1.593
	0.1127	0.2750	
4 days	0.1120	0.2740	1.595
	0.1120	0.2736	

saturated green solution was tested; and it was found that in thirty-six hours the additional depression of the freezing point of sugar was 0.017°, while in eight days it was 0.125°. This rate of reaction corresponds to a concentration of acid of only about $\frac{1}{400}$ normal, even supposing that the basic salt itself has no catalytic effect. Conclusions drawn from such an observation must of necessity be doubtful.

It seems probable, nevertheless, that there may exist in solution a chromium salt at least as basic as 5 : 4 — a substance far more basic than that found by Recoura and Whitney, whose ratio was 5 : 6.25. This probability of course by no means proves that there are not several stages of hydrolysis, each product being indistinguishably green. Moreover, the equilibrium in a solution at one concentration may be displaced at another. These considerations disclose an almost endless opportunity for investigation. Since, however, the time at our disposal was not unlimited, another direction was now pursued, in the hope that it might lead more directly to the desired goal.

THE OCCLUSION OF CHROMIC SALTS IN BARIC SULPHATE.

It is well known that iron is carried down with baric sulphate, when the iron is present in the ferric condition. This circumstance has formed the subject of numerous investigations.* In a recent paper,† it has been pointed out that this *occlusion* is probably due to the distribution of a definite molecular species between the solvent and the precipitate at the moment of formation of the latter. Various data concerning different precipitates seem to support this conclusion so strongly that it may be used as a working hypothesis.*

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* Among the most important of these are Jannasch and Richards, Jour. prakt. Chem. [2], **39**, 321 (1889); Schneider, Zeit. phys. Chem., **10**, 425 (1892); Küster and Thiel, Zeit. anorg. Chem., **19** (1899).

† Richards, Proc. Am. Acad., **35**, 377 (1900); Zeit. anorg. Chem., **23**, 383 (1900).

Thus the composition of the occluded matter may afford a clue as to the nature of some of the molecular species present in the solution. The only difficulty in the interpretation of the results is due to the possibility that more than one molecular species may thus distribute itself between the solution and the precipitate.

In the cases cited above, which furnished the basis for these conclusions, it was shown † that as a rule electrically neutral substances rather than ionized ones seemed to be thus distributed. This is indeed a reasonable conclusion; for the ionizing power of cool solids such as baric sulphate is probably much less than that of water.

In view of these inferences, it seemed possible that the careful analysis of baric sulphate precipitated in the presence of much green chromic salt might furnish valuable evidence as to the nature of the complex substances present. Accordingly precipitates were made under varying circumstances, and analyzed with great care. The first experiments showed that green chromic salt is occluded by baric sulphate to an extent fully as great as ferric iron, while the violet salt is scarcely occluded at all. Since the baric sulphate, after all the sulphuric acid had been precipitated from the solution, weighed less than the amount corresponding to the sulphuric acid known to be present, it was clear that the chromium must be present in the precipitate as sulphate. The case seemed, in short, to be precisely similar to that of iron, carefully studied by Jannasch and Richards in the paper already cited.

Our present object was to carry further the method of study followed in that paper, and to determine more precisely the nature of the occluded compound. A first step in this direction was to discover whether or not green chromic chloride might be occluded as well as sulphate; for the occlusion of chromic chloride would seriously interfere with the possibility of satisfactory interpretation.

For this purpose it was not enough to determine the chlorine in the precipitated baric sulphate, since baric chloride is known to be occluded by this salt.† It seemed possible, however, that since the complex green sulphate is so tardy in the attainment of its equilibrium (a fact indicated by the preceding work), it might also not form at once from the complex green chloride. In this case it might be possible to precipitate baric sulphate from a solution of chromic chloride before the green sulphate had been allowed time to form.

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* See for example Richards, McCaffrey, and Bisbee, These Proceedings, **36**, 377 (1901).

† Richards, *loc cit.*

The answering of this question was experimentally easy. Pure solutions of sodic sulphate, chromic chloride, and baric chloride were prepared of convenient concentrations, which corresponded to 1.558 gram of baric sulphate from the volumes used.

In a first experiment, the chromic chloride and sodic sulphate were heated separately to the boiling point, quickly and thoroughly mixed, and at once precipitated by baric chloride. After the precipitation the mixture was diluted four-fold from its small initial volume to about two-tenths of a litre, and was digested for several hours on the steam bath in order to facilitate filtering. The precipitate was weighed and its percentage of chromium found by fusion with alkaline carbonate and nitrate, reduction with alcohol, and precipitation with ammonia.

In this way, the baric sulphate was found to weigh 1.562 gram and to contain 0.0100 gram of chromic oxide. An experiment similar in every way, except that an hour elapsed between the mixing of chromic chloride with sodic sulphate and the precipitation, yielded only 1.472 grams of precipitate, containing 0.0582 gram of chromic oxide. Evidently far more chromic salt was occluded when time for the formation of the complex sulphate was allowed.

As would be expected, this difference became still more evident when the mixture and precipitation was carried on at 20°. A precipitate which had been formed as soon as possible after the mixing of the chromic and sulphate solutions appeared perfectly white, and contained only 5.5 milligram of chromic oxide; while fifteen minutes, an hour, and three hours' delay in precipitation led to greener and greener precipitates containing respectively 9.4, 13.6, and 14.7 milligrams of chromic oxide respectively.

These experiments were all verified by repetition, so that the following conclusions seem to be indubitable: —

(1) The chief occluded body is neither the simple chromic ion nor a complex chloride, but rather a sulphate.

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† Richards and Parker, These Proceedings, **31**, 67 (1894); Zeit. anorg. Chem., **8**, 413 (1895).

(2) That time is required for the formation of this complex sulphate, especially at low temperatures.

These reasonable conclusions are entirely in accord with the outcome of our previous work. It became now possible to determine by analysis the basic nature of the occluded body.

In order to attain this end, the occlusion was made as marked as possible by allowing plenty of time for the formation of the complex sulphate. Four portions of several grams each of baric sulphate thus badly contaminated with chromium were precipitated from a solution, which had been made strongly acid with hydrochloric acid in order to prevent excessive hydrolysis. Each portion of precipitate was analyzed twice, making eight analyses in all. Every analysis indicated a deficiency of sulphuric acid in proportion to barium and chromium (a deficiency at least as great as that demanded by Recoura's formula), although the extent of the deficiency was not exactly the same in every case. The variations are inessential, and the average result will serve for the present purpose. The precipitate was found to contain 57.24 per cent of barium, 0.39 per cent of chromium, and 40.84 per cent of the sulphuric radical. This quantity of barium should correspond to 40.01 per cent of the sulphuric radical; an amount which leaves 0.83 per cent to be associated with the chromium. This is a considerable deficiency, since 0.39 parts of chromium corresponds to 1.16 parts of the sulphuric radical in the normal sulphates.

Yet a greater amount of occlusion might obviously be obtained by causing the formation of more of the complex sulphate by neutralization of the acid before precipitation, but such a basic solution cannot be heated without the obvious deposition of chromic hydroxide, as has been already stated. Hence experiments were made with a cold green solution, saturated with chromic hydroxide. From the clear solution baric sulphate was precipitated, and allowed to stand for a long time. The baric sulphate thus formed was washed with cold water to prevent the accidental hydrolysis of the adhering filtrate. In four successive analyses of two separate specimens of gently dried baric sulphate thus made, the following percentages were found: barium, 55.46, 55.50; 55.14, 55.29; chromium, 0.64, 0.64; 0.65, 0.80; sulphuric radical, 39.67, 40.07; 39.53, 39.50. The averages of all these values are 55.35, 0.68, and 39.69. This quantity of barium corresponds to exactly one per cent less of sulphuric acid than the figure actually found, hence this 1.00 part may be supposed to be associated with 0.68 part of chromium. But this quantity of chromium should be associated with nearly twice as much sulphuric acid

(1.88 parts) if the salt were normal. The discrepancy indicated only 1.6 atoms of the sulphur for every 2 atoms of chromium, a proportion as nearly as possible identical with that obtained by the analysis of the pure ultra-basic solution.

Any conclusions based upon these figures must be somewhat insecure, because the data are only the somewhat variable differences between large quantities, and because traces of other groups besides the complex sulphate are probably occluded by the baric sulphate. Nevertheless it seems safe to say that the chief occluded body causing the green color of the impure precipitate is basic, and that nearly half of the sulphuric acid which should be combined with the chromium may be replaced by hydroxyl. There is nothing in the results to prove that there are not several soluble green basic salts, of different degrees of hydrolytic decomposition; but even the least basic of these, obtained in a strongly acid solution, seems to be more basic than that represented by the formula of Recoura.

Having thus determined in various ways the existence of a basic complex, it seems highly desirable to discover whether or not an acid complex containing chromium also exists, according to the possible equation,

$$xCr_2(SO_4)_3 + H_2O = Cr_{2x-1}OH(SO_4)_{3x-2} + HCr(SO_4)_2.$$

This matter forms the subject of the following section.

MIGRATION EXPERIMENTS.

Whitney showed in 1899 that the method of Lodge* for observing electrolytic migration afforded proof of the presence of ionized hydrogen in the green solution. We were desirous to carry this matter further, in order to discover whether or not there was any observable migration of a green complex anion, concerning whose possible existence Whitney was not anxious. In a number of experiments according to Lodge's method we obtained inconclusive results, which were found to be due to the fact that the green solution coagulates the gelatine and leaves free passages for convection in the expelled solution. Gelatine thus coagulated by the green chromium solution cannot be melted by warming, hence it is evident that a compound must be formed. On this account we rejected gelatine, and continued the migration experiments with the help of an apparatus similar to Hittorf's.

The diagram illustrates this arrangement. The glass cups or cells were provided below with carefully tied diaphragms of parchment paper made tight at the sides with collodion. The middle cup contained a green chromium solution which had been freed from acid by shaking with chromic hydroxide, while the others contained cadmic sulphate. Cadmium electrodes were used to prevent the formation of ionized hydrogen or hydroxyl.

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* O. Lodge, B. A. Reports, 395 (1886). See also Whetham, Phil. Mag. [5], **38**, 392 (1894); Orme Masson, Zeit. phys. Chem., **29**, 501 (1899).

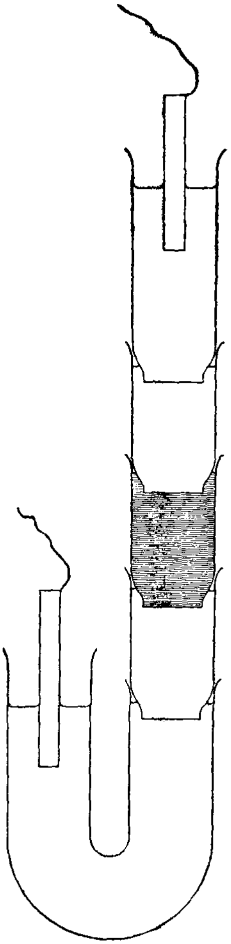

FIGURE 2.

Because gravity caused a slight filtration, only the presence of chromium in the cup above the green solution could be taken as evidence of certain electrolytic migration. Hence the positive current was run first downwards in order to determine if there was any anodic migration of chromium, and then in subsequent experiments upwards in order to determine if there was any cathodic migration. In order to determine the chromium, the cadmium in the cell was first removed as sulphide. No trace of chromium was found in the anode solution after 340 coulombs of electricity had been passed through the apparatus, as indicated by the deposition of 0.1119 gram of copper in a serially connected coulometer.

On the other hand, in an experiment in which the cathodic migration was tested, 183 coulombs carried 36.6 mg. of chromium, corresponding to 19.3 grams of chromium for each Faraday's equivalent of 96,580 coulombs.* The cation was green. The quantitative result is surprisingly large. Since the atomic weight of chromium is only 52, it is clear that each atom of chromium cannot be associated with over two charges, and probably with not more than a single charge. Assuming from the absence of chromium in the anodic fluid that the sulphuric ion alone migrates in that direction, and that the mobility of this anion is 70,† the mobility of the chromium group must be 41 on the assumption that each chromium has a single charge and 243 on the assumption that each has a double charge. The latter of these figures is improbably large, while the former is plausible, approximating the values for zinc and magnesium. Hence it seems probable that the number of charges does not greatly exceed the number of atoms of chromium. It is not impossible that the cation is $CrO^{\cdot}$ or $Cr(OH)_2^{\cdot}$, which has been shown by Siewert (1863) and Whitney (1896) to be the most probable cation of boiled chromic chloride and nitrate. On the other hand of course there is nothing to show that the ion is not a complex one, with a valence about equal to the number of chromium atoms, or at least a mixture of complexes with an average valence no greater than this. This conclusion would be consistent with that reached in other ways; for if not all the normal valence of chromium is available, it is obvious that some of it must be held by undissociable hydroxyl, or hydroxyl and sulphuric acid, in a basic complex.

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* The weights of chromic oxide found were 53.5 and 48.2 mg. respectively.

† *Kohlrausch and Holborn, Leitvermögen der Electrolyte*, p. 200.

The absence of chromium from the portion migrating towards the anode overthrows the hypothetical equation involving an ionized complex acid, which otherwise seemed plausible.*

MOLECULAR WEIGHTS.

The next step, suggested by the foregoing paragraph, is an investigation of the average molecular weights of the dissolved substances. In mixtures so complicated the interpretation cannot be certain, but may nevertheless afford some clue to the situation. It has already been pointed out that the depression of the freezing point is the same for a green solution hydrolyzed only by heating and a violet solution of equal concentration, when sugar is present, — namely about 0.30° with a twentieth molar solution. Since a molar solution should give a depression of 1.85° according to Raoult's law, there may be supposed to be $\frac{20 \times 0.30}{1.85} = 3.2$ active particles for each molecule of $Cr_2(SO_4)_3$ at this dilution, or the average molecular weight is 122. As has been already suggested, this conclusion is somewhat in doubt, because of the presence of sugar, which may have an unknown specific action. Since much ionized hydrogen is formed during the hydrolysis, it is obvious that the remainder of chromium is probably condensed into a more complex compound when the hydrolysis takes place — otherwise the number of active particles would increase on hydrolysis instead of remaining constant. This interpretation is supported by the freezing points of the more basic solutions. When the solution is saturated at 100° with chromic hydroxide, a mixture containing the same amount of chromium per litre as before causes a depression of the freezing point of only 0.080°, and a parallel mixture saturated at 20° with chromic hydroxide causes a depression of only 0.071°, instead of 0.30° found at first.*

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* Krüger (1838) and Siewert (1845); also Richards, Zeit. anorg. Chem., **23**, 383.

This decrease in the depression of the freezing point indicates a decrease in the number of dissolved particles, the final value corresponding to only 1.5 particles for every four chromium atoms. These results are only preliminary ones; they will be repeated and amplified in the near future. As far as they have any significance, they seem to indicate that one of the green bodies really possesses a high molecular weight, as Recoura and Whitney believed, although at least one of these bodies is more basic than these investigators assumed.

Chemical Retardation of the Hydrolysis.

The preceding experimentation is unanimous in indicating that at least one soluble green basic substance is found by the hydrolysis of violet chromic salts, and that the extent, and of course also the speed, of this hydrolysis is dependent upon the temperature.

If this is true, the neutralization of the acid formed by hydrolysis should greatly favor the formation of the basic substance, while on the other hand the addition of more acid should retard this formation. The former of these consequences has already been shown to hold true — it introduces an accidental circumstance into the work of both Whitney and Recoura. The latter of the consequences remains to be proved.

With this object freshly prepared solutions of the violet and merely boiled green solution, of one-fortieth molar strength, were each divided between four test-tubes, every tube containing ten cubic centimeters. To one test-tube of each color was added a gram of ammonic chloride, to another of each color a gram of sodic sulphate, and to a third of each color a gram of sulphuric acid, while the fourth of each color was left unmixed. Each was slightly diluted until all were equal in volume. All were allowed to stand together in the laboratory at temperatures varying from 20° to 27°. At the end of a week the green and violet tubes were distinctly less different than at first, and in each series the test-tube containing excess of acid was unquestionably the most violet. The sodic sulphate seemed very slightly, and the ammonic chloride somewhat more prominently, to assist the hydrolytic effect, as judged by the greenness of the solutions — but these changes were subordinate in magnitude, and inessential.

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* The actual depressions measured were respectively 0.053° and 0.108° for solutions yielding 0.0503 and 0.1156 Cr_2O_3 per 10 c.c. respectively.

Qualitative though they were, these experiments furnished final evidence that the effect is hydrolytic.

In this paper no attempt has been made to study the colloidal acid sulphates of Recoura and Whitney, made by heating the dry salts to a higher temperature. These introduce a new complication into the theoretical interpretation of the behavior of chromium; but they have been shown by Whitney to be unstable in solution, and hence probably do not enter into the equilibrium under consideration.

SUMMARY.

Our results, which amplify rather than contradict the best of the preceding work, may be summarized as follows: —

1. It is shown that the immediate yield of violet chromic sulphate from chromic anhydride is much increased if the reducing reaction is allowed to take place at low temperatures.

2. Violet chromic sulphate was found to dialyze in normal atomic proportions, while green chromic sulphate allows an excess of sulphuric acid to pass through the diaphragm. This result agrees with that of van Cleef, and indicates hydrolysis as the cause of the color-change.

3. With the help of the inversion of sugar it is shown that the extent and speed of the hydrolysis is dependent upon the temperature.

4. The acid formed by hydrolysis may be removed by successive shaking with a mixture of ether and alcohol, or by digestion with chromic hydroxide. This accelerates the hydrolysis.

5. Excess of acid accelerates the reversion to violet, and retards the formation of the green salt.

6. The formation of the green sulphate compound requires time, either from the double decomposition of the green chloride, or from hydrolytic action.

7. At 100°, a constant result is reached when the solution saturated with chromic hydroxide attains the atomic ratio $Cr : (SO_4)$; at 20°, the ratio is $Cr_5 : (SO_4)_4$.

8. The green substance occluded by baric sulphate from green chromic solutions also seems to be highly basic.

9. Migration experiments indicate no anion containing chromium in a green solution.

10. The molecular weight of one of the green substances is probably large.

11. All these experiments agree in pointing toward the existence of a green basic salt in which at least 47 per cent of the acid radical is replaced by hydroxyl. It is possible that there may be several compounds of different degrees of hydrolysis. The acid set free seems to be sulphuric acid, and not a complex acid. It is unsafe at present to apply any definite formula to the basic salt. Further investigation will be instituted in order to discover if possible its structure, as well as to determine whether or not a colloidal substance also exists in the hydrolyzed solution.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY,
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ON THE PROLONGATION OF SPECTRAL LINES.

BY THEODORE LYMAN.

WITH A PLATE.

INVESTIGATIONS ON LIGHT AND HEAT MADE OR PUBLISHED, WHOLLY OR IN PART, WITH APPROPRIATIONS FROM THE RUMFORD FUND.

ON THE PROLONGATION OF SPECTRAL LINES.

BY THEODORE LYMAN.

Received May 27, 1903.

INVESTIGATORS who have worked with concave diffraction gratings cannot have failed to observe the faint but sharp prolongations of strong lines which occur in the spectra produced by these instruments.

The cause of the difference in length of certain lines when a prism spectroscope is used is well known. Sir Norman Lockyer long ago made use of the phenomenon of "long and short lines" in his study of the chemistry of the Sun. He pointed out that when the vertical slit of the collimator is illuminated by the image of a light source formed by a lens, some of the lines in the resulting spectrum are longer than others. This of course arises from the fact that the portion of the source which illuminates the centre of the slit possesses some vibration frequencies which are wanting in those portions which come to focus at the top and bottom of the slit. When a concave grating is used, the astigmatism renders this phenomenon less striking.

The prolongations of strong lines which are referred to above present a different appearance, however, from those obtained with a prism spectroscope. They are, in fact, due to a different cause. As the author had never seen any explanation of the matter, it seemed that an investigation on the subject might prove of interest.

In making some adjustment of a concave grating of twenty-one foot radius the principal image of the slit came into view. It was at once noticed that this image was prolonged into two narrow streamers, one vertical and the other horizontal, each quite distinct and sharp. This suggested that the vertical continuation of the slit image and the vertical continuation of strong spectral lines were due to the same cause. Observations were accordingly made upon the principal image of the source. In order, however, to simplify the diffraction phenomena as much as possible, a circular opening of about 0.01 cm. diameter was substituted for the slit. In order to reduce spherical aberration to a minimum the light fell upon the grating at nearly normal incidence. In the final observations the polished surface was always covered. The first photograph was taken with the grating in its normal position and with all the ruled surface exposed. The result is an image of the pin hole accompanied by two sharp streamers, one vertical the other horizontal. The horizontal streamer is the longer and stronger of the two. The effect is shown in the Plate, Figure 1.

If the ruled surface of the grating be protected by a screen having a *rectangular aperture whose diagonal is less than the breadth of the ruled* surface, the orientation of this aperture throws light upon the nature of the phenomenon. When the longest side of the opening is parallel to the longest side of the ruled surface, the image of the point source presents exactly the same appearance as when the whole grating surface is exposed. When, however, the aperture is set askew so that its sides make angles of 45° with the sides of the ruled surface, the effect on the image is striking. The result is shown in Figure 2. The vertical streamer has revolved through 45° and the horizontal streamer has been broken into two parts. One of these parts has turned through 45° while the other and longer part has remained fixed. The streamers which accompany the image seem then to consist of two components, one a cross which turns as the sides which bound the ruled surface turned, the other a horizontal line or streamer which remains stationary. The idea that the movable cross is due to diffraction through a rectangular opening at once suggests itself. This theory may be further tested by covering the grating surface by a screen with a circular hole. The image of the source so obtained showed no vertical streamer, the horizontal line was still present, though less intense. This result indicates that the explanation of the origin of the movable cross is the correct one.

When the point source was replaced by a slit, the effects were of an exactly similar character.

It is easy to apply the results of these experiments to the vertical continuation of strong spectral lines. In order to demonstrate that the vertical streamers are due to the rectangular shape of the ruled surface, we have only to cover the surface with a screen whose rectangular aperture is set askew with respect to the direction of the lines of the grating. We obtain the effect shown in Figure 3. Here the continuations of the strong lines have turned and broadened, though the lines themselves remain straight and sharp. The diffraction pattern from the rectangular grating aperture is no longer a cross with sharp vertical and horizontal arms. The cross has turned, and, since the aperture is no longer symmetrical with respect to the slit, the arms have broadened.

The horizontal band which does not turn as the aperture is rotated may be due to either of two causes, or to a combination of them. The subordinate or secondary maxima which accompany every line in the spectrum furnish the first reason, irregularities in the grating ruling furnish the second. The author has shown that these irregularities may not only furnish a background, but, under favorable circumstances, may even produce sharp reproductions of real lines. The background or nearly continuous band can be noticed with almost every grating, and can be best observed in that portion of the extreme ultra-violet where no real lines are obtained. Its intensity varies greatly with different gratings. In investigations where long exposures are necessary it often proves very inconvenient, for faint real lines are much obscured by its presence.

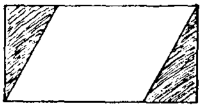
Figure A.

In many cases the horizontal band due to diffraction through a rectangular opening is much stronger and more troublesome than the band due to irregular ruling. In this case there is a remedy at hand. The ends of the ruled space may be covered with slanting pieces of blank paper and the rectangular ruled space thus converted into a parallelogram. The effect of this arrangement is to revolve the horizontal streamer due to the shape of the opening, the vertical streamer remaining fixed, with the result that the background of the spectrum is very materially cleared. The author can recommend this device to all those who investigate faint spectral lines, and to whom a clear field is a necessity.

Jefferson Physical Laboratory,
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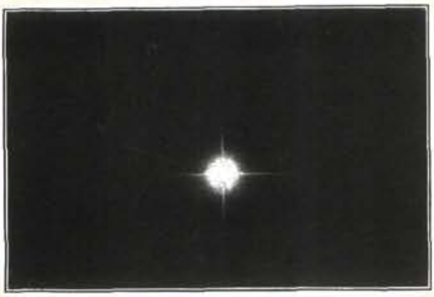

FIG. 1.

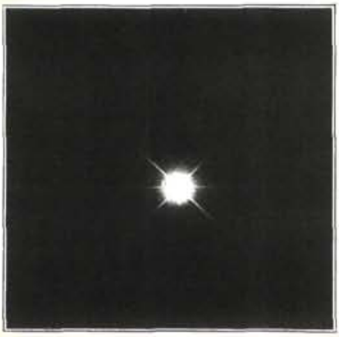

FIG. 2.

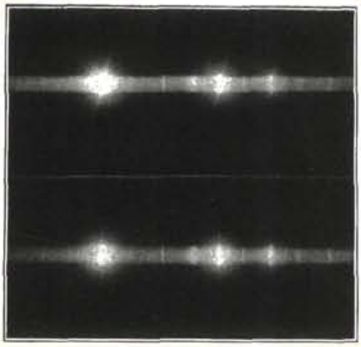

FIG. 3.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY,
HARVARD COLLEGE.

AN EXPLANATION OF THE FALSE SPECTRA FROM DIFFRACTION GRATINGS.

By Theodore Lyman.

With a Plate.

Investigations on Light and Heat made or published, wholly or in part, with Appropriations from the Rumford Fund.

AN EXPLANATION OF THE FALSE SPECTRA FROM DIFFRACTION GRATINGS.

By Theodore Lyman.

Presented May 13, 1903. Received May 27, 1903.

In a previous paper * the author has shown that the principal spectra produced by concave diffraction gratings are complicated by the presence of false spectra of lower order than the first, and that these false spectra are common and often of excellent definition. It is the purpose of this paper to show that these false spectra are not of the nature of "ghosts," and that, while the theories developed to explain the latter cannot be made to fit them, a theory proposed by Professor Carl Runge, after examining one of the plates which formed the basis of the earlier paper, furnishes an explanation of the phenomena.

The "ghost," so called, is a faint reproduction of a real line, and in general occurs within a few Ångström units of its parent. The false spectra not only occur in regions many hundred units from that occupied by the principal line, but also differ fundamentally from "ghosts" in other respects.

Mr. C. S. Pierce † has made a careful mathematical study of the subject, and his paper also contains experimental data on the position of "ghosts," showing a good agreement between the theory and the observed facts. His treatment, however, deals only with "ghosts" occurring very near the parent line. Rowland ‡ has given a theory, the formulae of which may be extended to the case of false lines occurring at a considerable distance from the parent. The author, however, has never been able to fit the positions or intensities of any of his observed false spectra into these formulae in the form given by Rowland. In the case of most gratings this does not seem surprising, since the false spectra are very numerous, of small intensity, and with wave-lengths which bear no simple

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* These Proceedings, **36**, No. 14.

† American Journal of Math., **2**, 330 (1879).

‡ Rowland's Physical Papers, p. 525.

relation to one another. The error in ruling which produces them must be of a very complex nature, and we cannot expect the phenomenon to be readily brought under mathematical analysis. In some cases, however, an exact treatment should be possible, for with certain gratings some few of the false spectra are very much stronger than the others. Here it is fair to suppose that we have a comparatively simple error combined with a complicated one. Under the circumstances it seemed profitable to attempt to modify the ordinary treatment as given by Rowland.

A grating of N lines may, if $N = n\,m$, be considered as a grating of m groups, each consisting of n lines. The spectrum of the first order may be considered as the spectrum of the nth order of a grating of m groups. Each group is in itself a grating of n lines. The intensity I_n, any point of a spectrum of a grating of n lines, is known to be proportional to $A^2\left(\frac{\sin n\,a}{\sin a}\right)^2$, where $a = e\,(\sin i - \sin r)\,\frac{\pi}{\lambda}$. A^2 is the intensity of one line, which may be different in different directions, and e is the grating space or distance between two consecutive lines.

If we consider the grating of m groups, the intensity at any point of its spectrum will be proportional to $I_n\left(\frac{\sin m\,a'}{\sin a'}\right)^2$, where $a' = n\,a$. The 1st, 2d, 3d. . . . . . $n - 1$ order will vanish for any value of A, because I_n vanishes for $a = \frac{\pi}{n}, \frac{2\,\pi}{n} \dots\dots \frac{(n-1)\,\pi}{n}$.

Now let us assume that in the ruling of n consecutive lines there is an irregularity, such that the grating of n lines which we take to be repeated m times is of itself an imperfect grating. Then I_n will not vanish at all places where $a = \frac{\pi}{n}, \frac{2\,\pi}{n} \dots\dots \frac{(n-1)\,\pi}{n}$ and some of the first $n - 1$ orders of the grating of m groups will be visible. So far this is the ordinary theory of false lines.

Further, let us assume that the grating of n lines has a periodic error. To make the matter definite, let us take this error as occurring every third line. That is to say, every third line is slightly out of place, while the other lines remain in their correct position. Then it is probable that I_n will have some appreciable value at $a = \frac{\pi}{3}$ and at $a = \frac{2\,\pi}{3}$. If we take the error to be somewhat irregular, the intensity I_n will spread to both sides of those positions where $a = \frac{\pi}{3}$ and $a = \frac{2\,\pi}{3}$. The intensity of the light given by the whole spectrum for the wave-length λ will be appreciable for those values $a = \frac{\pi}{n}, \frac{2\pi}{n}, \ldots\ldots \frac{(n-1)\pi}{n}$ which come near to the values $a = \frac{\pi}{3}$, $a = \frac{2\pi}{3}$. Here n may be taken at pleasure. For the sake of a definite case, let $n = 70$. Then I_n will have an appreciable value at $a = \frac{23}{70}\pi, \frac{24}{70}\pi, \frac{46}{70}\pi, \frac{47}{70}\pi$; for $\frac{23}{70}$ and $\frac{24}{70}$ are the nearest values to $\frac{1}{3}$: $\frac{46}{70}$ and $\frac{47}{70}$ are the nearest values to $\frac{2}{3}$. Moreover, since $\frac{1}{3}$ of 70 is 23.33, the 23d spectrum will be nearer this position than the 24th, and therefore the stronger of the two. Similarly, as $\frac{2}{3}$ of 70 is 46.66, the 47th spectrum will be stronger than the 46th. Therefore, taking the number 70, the two spectra which are the stronger will be the lower of the first pair and the higher of the second.

If, then, we let $n = 70$, and consider the error to occur every third line in the grating, for each line in the spectrum there will be four repetitions between its normal position and the direct image of the slit. These repetitions correspond to the 23d, 24th, 46th, and 47th order of the grating of m groups, the 70th order corresponding to the normal position.

This simple treatment of false spectra was suggested to the author by Professor Runge. The values used are those which seem most nearly to fit the case of the false lines obtained by the author from the grating called No. I, and illustrated in a former article.

Before proceeding further to the numerical verification of this theory, it may serve to illuminate the matter if we place in contrast with it the results which may be expected from the theory of Rowland.[*] The theory of Rowland is divisible into two parts, one dealing with the production of ordinary ghosts, the other part dealing with the production of lines at a considerable distance from the parent. It is this second part which could alone be expected to fit the case in hand. It does not do so, however, for it demands lines whose apparent wave-lengths bear simple ratios to the parent line. Professor Runge's theory shows the possibility of the total absence of lines at these positions indicated by Rowland, and it shows the probability of the formation of lines on either side of these positions at different distances and of an indicated relative intensity. Thus, in the present case, the theory of Rowland might be made to call for lines at positions corresponding to $\frac{2}{3}$ and $\frac{1}{3}$ the wave-length of the parent line. The lines actually observed do not fulfil this condition, but occur in flanking positions. For example, in the spark spectrum of magnesium, which is illustrated in Plate 1, the line of wave-length 2790.8,* which forms the most refrangible member of a characteristic group, is seen to be reproduced four times. Two of these false spectra are in the region between wave-lengths 1800 and 1900 and two in the region between 900 and 1000. It is to be noted that of the two groups near 1800 the less refrangible is the stronger, while of the groups near 900 the more refrangible is the stronger. This is in accord with the theory, for if the two strong reproductions are considered to be the 47th and 23d spectra of the grating of *m* groups, the two weak reproductions are the 46th and 24th spectra, while the real line 2790.8 corresponds to the 70th spectra.

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[*] Rowland's Physical Papers, p. 535.

When, however, the wave-lengths as measured in the previous paper were compared with those calculated from the fractions $\frac{47}{70} \times 2790.8$, $\frac{46}{70} \times 2790.8$, $\frac{24}{70} \times 2790.8$, $\frac{23}{70} \times 2790.8$, the observed facts did not seem to agree with the theory as accurately as might be expected. It seemed worth while to remeasure the wave-lengths of the false spectra, in order to determine if the fault lay in the theory or in the observed values. The author was encouraged in this step by the interest which Professor Runge took in the matter. In fact the method employed in re-measurement was suggested by him.

The arrangement of the apparatus was as follows: —

Two slits, *A* and *B*, were placed upon the circle whose diameter was the radius of the concave grating. The grating itself was kept fixed in position and the normal to its surface fell midway between the two slits. By this method the image of *A* was formed at *B*. An arm carrying the plate-holder *C* was pivoted at the centre of the circle. The result of this plan was that the first spectrum obtained when *A* was used as source, was shifted with respect to that obtained with *B* as source by a definite amount. The heights of the two slits were so arranged that the *A* spectrum fell directly over the *B* spectrum upon the photographic plate. The method of procedure was to illuminate *A* with the light from a magnesium spark and to place the plate-holder *C* in such a position that a photograph of the false lines under investigation was obtained. Next

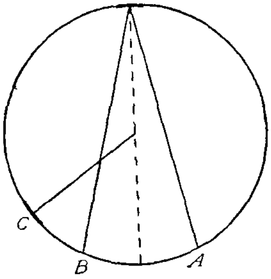

FIGURE A.

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* Exner and Haschek, K. Akad. der Wiss. in Wien, **106**, Abth. II (1897).

B was illuminated by light from a spark between iron terminals, and thus the standard iron spectrum was photographed under the false magnesium lines. To determine in Ångström units the amount by which the one spectrum was shifted with respect to the other, both A and B were illuminated with light from the iron spark and the relative position of two known lines was measured. As an example of the method, the false line, which by the theory should be the 47th order of the grating of m groups, was compared with the line in the iron spectrum 3541.2 * and found to have the relative position 3541.0. Next, the line 2737.0 * in spectrum A was compared with the line 4404.9 * in spectrum B. The difference between them was .5 Ångström units. Thus the shift of spectrum A with respect to B was $[(4404.9 - .5) - 2737] = 1667.4$, and the apparent wave-length of the false line was $3541.0 - 1667.4 = 1873.6$. In this way the apparent wave-lengths of the four false lines were determined. In order to check the accuracy of the method, slit B was moved toward slit A and the grating was re-adjusted. In this way the shift of one spectrum with respect to the other was altered and the unknown spectra were compared with a new position of the iron spectrum. The following table gives some idea of the accuracy of the method and of the agreement between the observed and the calculated values.

Calculated Wave-length.	Measured Apparent Wave-length.	
	First Observation.	*Second Observation.*
917.0	917.0	916.8
956.8	956.7	956.4
1834.0	1834.1	1833.9
1873.8	1873.7	1873.7

The method gives an average error of 0.17 of an Ångström unit. The average difference between the observed and the calculated values is .14 units. Thus the difference between theory and practice is within the errors in observation.

It might seem that values different from $n = 70$ and a period of 3 would satisfy as well the conditions when substituted in the equations.

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* Exner and Haschek, K. Akad. der Wiss. in Wien, **106**, Abth. II. (1897). Compare also Kayser and Runge Arc Spectrum.

This is not the case, as may be shown by trial. If there are to be but four reproductions and if the relative dispersions and intensities of these reproductions are to be explained, the values 70 and 3 give the result best in agreement with experiment.

Up to this point it has been the object of the discussion to contrast, in the light of these experiments, the theory of Professor Runge with that part of the theory as given by Rowland which would seem most obviously applicable to the case. This part of the theory was called "One line in *m* displaced.*" The other part of the analysis which deals with "ghosts" came under the title "Periodic Error."† A moment's consideration will show that the lines under discussion are not of this latter class.

In the case of Simple Periodic Error the position of any groove in the grating ruling is given by the equation, $y = a_0 n + a_1 \sin(e n)$. Thus the *n*th groove from a fixed line of reference is out of its true position by a term which varies as a sine function with period e; the maximum value of the displacement is of course a_1. Thus no groove in the grating surface is exactly in its proper place unless $\sin(e n) = 0$. The system of ghosts to which this form of error gives rise is characterized by the following properties.‡ Ghosts of any order must occur in pairs. Of a pair one lies to the right of the parent line, the other to the left. Ghosts of the second order must lie exactly twice as far from the real line as ghosts of the first order; ghosts of the third order three times as far. The distance of a ghost of a given order from its parent line is a constant independent of the order of the spectrum in which the parent line is measured. These three properties are not in any way possessed by the lines under discussion. This, together with the fact that no numerical application of the theory of ghosts to the case in hand seems possible, excludes it from further consideration.

In short, then, Professor Runge's explanation of the false spectra seems to fit the facts most accurately. It is perfectly possible to extend the theory to even the more complex cases where there are more than four reproductions of every real line. The period of the error and the value of *n* may be taken at pleasure, so that the treatment can be made to fit a great variety of cases. In practice, however, the errors of ruling in those gratings which give a great number of faint false spectra are too complex to make calculation profitable.

It may be of interest to remark that false spectra are not confined to concave diffraction gratings, but are to be found in the spectra produced by plane reflection gratings as well. The author has examined two instruments ruled upon speculum metal by Rowland's engine. In the experiment the light was collimated and brought to a focus by quartz lenses. In order to reduce the phenomena to the simplest possible form, a line in the visible blue spectrum of magnesium was separated out by a prism spectroscope and thrown upon the slit of the collimator. The camera was focussed on the image of this line in the first spectrum. The first grating was ruled in 1883 and had 14,438 lines to the inch. The image of the line used appeared in its proper place in the first spectrum — = 4481 Ångström units — and was not accompanied by any ghosts of the common kind. That is to say, there was no doubling of the line, nor were there any faint reproductions very near it. In the region near wave-length 3000, however, four sharp lines occurred; and again near wave-length 1550 four more reproductions were present. Thus this grating produces eight false spectra of a lower order than the first, corresponding to the real line 4481. Besides these eight, numerous faint reproductions may be detected, but they are of extremely feeble intensity.

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* Rowland's Physical Papers, p. 535. † Ibid., p. 536. ‡ Ibid., p. 519.

The second grating examined was ruled in 1887 and had also 14,438 lines to the inch. The spectrum obtained with it, however, was very different from that given by the first instrument. The first spectrum of the line 4481 was, as before, sharp and without ghosts, but the eight distinct false spectra were replaced by at least seventy reproductions of very feeble, but nearly equal, intensity. These extended between positions corresponding to wave-lengths 2000 and 900.

The results obtained with these two plane gratings are exactly similar in character to those obtained with the concave gratings called No. I and No. II and recorded in a previous paper. These gratings seem to belong to two types, the one in which the false spectra are all of nearly equal intensity and feeble, the other in which some few of the false spectra are many times more intense than the others. In the one case the grating gives a background of faint lines; in the other sharp, strong false spectra are present.

The author wishes to call attention to the plate which accompanies this article. It is from a concave grating of 6-foot radius and shows the false lines whose positions have been discussed in this paper. The plate is taken directly from a negative by photographic process. The two groups at positions corresponding to wave-lengths 956 and 1834 are very faint in this reproduction. Their positions are indicated, however. The character and dispersions of the two stronger groups is well shown. All the lines in this plate are false.

The pairs of sharp lines in the immediate neighborhood of the characteristic groups to which the treatment has been confined are reproductions of the real lines at 2936.8 and 2928.9. They form the 47th, 46th, 24th, and 23d spectra of the system in which the real lines are the 70th spectra. The single line near the middle of the plate is the 23d false spectrum of the real line at 4481.3. The lines not far from it are the 23d false spectrum corresponding to the real lines 3838.5, 3832.5, and 3829.5. The 24th spectra of all these lines are too feeble to reproduce. Their 46th and 47th spectra occur among real lines in the region between 3050 and 2500. The apparent wave-lengths of these false lines are marked upon the plate; the decimal place is omitted. The following table gives the measured apparent wave-lengths of all the lines appearing upon the plate, their calculated wave-lengths, their order as false spectra, and the wave-lengths of the real lines of which they are the reproductions.

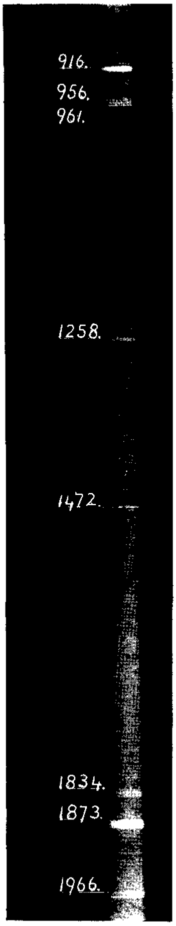
Apparent Wave-lengths.		Real Wave-lengths of Parent Lines.	Order as False Spectra.
Calculated.	Measured.		
917.0	917.0	2790.8	23d
956.8	956.7	2790.8	24th
962.3	961.8	2928.9	23d
1258.3	1258.4	3829.5	23d
1472.4	1472.8	4481.3	23d
1834.0	1834.1	2790.8	46th
1873.8	1873.7	2790.8	47th
1966.5	1966.5	2928.9	47th

It is well to remember that this grating is not unique, but that most reflection gratings produce false spectra. It is in the extreme ultra-violet that these false lines are most easily seen, and it is in this region that they may be most readily taken for real lines. The lines may serve to show by their strength and sharpness the danger which they offer in spectroscopic work.

In conclusion, it may be well to repeat the chief result of this paper. False spectra differ fundamentally in character from the commonly observed "ghost." The former seem due to a so-called periodic error in the grating ruling, an error which operates to displace every groove in the grating surface by an amount depending on a sine function of the position of the groove. It seems probable that the false spectra are due to an error of another type. Here the error operates to displace one, out of a given number of grooves, slightly, leaving the remainder in their proper positions. In order to make theory and the observed facts agree, this error must be considered somewhat irregular over the surface of the grating. While the theories proposed by C. S. Pierce and Rowland account in every way for the phenomenon of ghosts, they do not either qualitatively or quantitatively account for these false spectra, whereas the theory proposed by Runge, and given above, explains the phenomena qualitatively and very nearly quantitatively. That is to say, it explains the production of lines far from the parent line, lying entirely on one side of it; it explains their relative intensity, and it explains very nearly indeed their exact position. The maximum departure between the positions of these false spectra demanded by the theory and observed in the plates is not more than .4 Ångström units. This is slightly greater than the maximum difference found between sets of observations; but it is to be remembered that the measurements depended upon the wave-lengths of the parent group and upon the comparison iron lines, both of which are borrowed data.

JEFFERSON LABORATORY, HARVARD
UNIVERSITY, 1903.

LYMAN.—FALSE SPECTRA.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Proceedings of the American Academy of Arts and Sciences.

Vol. XXXIX. No. 4. — August, 1903.

THE ANOMALOUS DISPERSION, ABSORPTION, AND SURFACE COLOR OF NITROSO-DIMETHYL-ANILINE.

By R. W. Wood.

With Three Plates.

Investigations on Light and Heat made or published, wholly or in part, with Appropriations from the Rumford Fund.

THE ANOMALOUS DISPERSION, ABSORPTION, AND SURFACE COLOR OF NITROSO-DIMETHYL-ANILINE.

By R. W. Wood.

Presented May 13, 1903. Received May 30, 1903.

The very high dispersion of the aniline dyes and other absorbing media is due, as is well known, to the fact that the absorption band lies within the visible spectrum. The absorbing power of these substances for wave-lengths far removed from the centre of the absorption band is so great that only prisms of very small angle can be used, which puts a limit on the length of the spectrum which can be obtained with them.

In the case of the so-called transparent substances, the absorption band lies so far down in the ultra-violet, that the steepness of the dispersion curve in the visible spectrum is not comparable with that of the substances which are said to exhibit anomalous dispersion. If, however, we push the curve down to the absorption band, employing some photographic method, we find that it may be even steeper than the curve for cyanine in the red and orange.

I have found that nitroso-dimethyl-aniline is of peculiar interest, in that it fills in the gap existing between the aniline dyes and ordinary transparent substances. It has a band of metallic absorption in the violet, and is at the same time fairly transparent to the red, yellow, and green. The substance melts at 85° C. and can be formed into prisms between small strips of thin plate glass. The strips should be about two centimeters long and are best fastened together with one of the small clamps used with rubber tubing. It is best to melt the material on the end of one of the strips, the other being warmed over the same flame, and then clamp the two together with a piece of a match between the ends, to give the required prismatic form. A candle flame viewed through the prism is spread out into a most remarkable spectrum fifteen or twenty times as long as one given by a glass prism of the same angle. It is instructive to have a prism of the same angle made of Canada balsam or some such substance pressed out between two similar glass strips.

In addition to its remarkable dispersion, the nitroso, as I shall call it for short, exhibits a most beautiful violet surface color, which can best be exhibited by employing a small cell heated by steam, such as I shall describe later on.

The substance possesses in addition several other interesting features. It has, for example, in addition to its sharp and narrow band of metallic absorption in the violet, a weaker pair of bands near the end of the ultra-violet which flatten out the dispersion curve, but do not bend it into oppositely directed branches as the stronger band does. Moreover, the substance can be vaporized without decomposition, which makes it possible to compare its optical properties in the three states, — solid, liquid, and gaseous.

I shall take up in order the dispersion in the visible spectrum, the ultra-violet dispersion, the reflecting power in different parts of the spectrum, the principal azimuths and incidences for different wave-lengths, the angles of maximum polarization, and the changes in the position of the absorption band which accompany a change of state. The various results will finally be discussed in their bearing on the electro-magnetic theory of dispersion and absorption.

Dispersion in the Visible Spectrum.

As the refractive index of the nitroso changes very rapidly with the temperature, it was necessary in making the determinations of the dispersion to keep the temperature of the melted substance constant. The point chosen was the solidifying point, as it simplified the experimental conditions. The prism was constructed of a pair of interferometer plates accurately plane-parallel, and was mounted on the table of a spectrometer in a small clamp-frame made especially for it. A current of hot air was directed against the prism by means of a bent glass tube, under one end of which a small gas flame was burning. The slit of the spectrometer was illuminated with approximately monochromatic light furnished by a monochromatic illuminator built by Fuess. This extremely useful instrument is not as well known as it deserves to be, and a few words regarding it may not be out of place. It is essentially a small spectroscope with collimator and telescope at right angles. The two prisms, which are enclosed in the body of the instrument, are turned by means of a micrometer screw, from the reading of which the wave-length can at once be determined from the calibration curve of the instrument. The eye-piece can be removed and a draw tube carrying an adjustable slit inserted in its place. The spectrum can be made to pass across this slit by turning the micrometer screw, and by noting the readings when known lines in the spectrum fall on the slit, the instrument can be calibrated. This is easily and quickly accomplished by means of an ingeniously arranged microscope which can be thrown into and out of position as desired. A small lens carried in a tube in front of the slit focuses the monochromatic light which issues from it on the slit of the spectrometer.

The dispersion of the nitroso was measured in the following manner. The prism having been set at minimum deviation, and the usual adjustments made, the deviated image of the slit, illuminated in monochromatic light, was brought into the field of the telescope. The gas flame was then moved out of position and the current of hot air stopped. As the fluid nitroso cooled the deviation increased, the reading being taken just at the point of solidification, when the image disappeared gradually owing to the crystallization of the medium. The warm air was then turned on again and a second reading taken in the same way. A number of prisms were used, the angles varying from one to ten degrees, those of small angle being necessary when working with the bluish green, owing to the absorption. In this way the dispersion was measured between the extreme red and wave-length .00050, below which point the prisms refused to transmit sufficient light to make readings possible. The values obtained in this way are given in the following table.

Prism Angle 8° 7′				Prism Angle 53′	
λ	n	λ	n	λ	n
508	2.025	636	1.647	497	2.140
516	1.985	647	1.758	500	2.114
525	1.945	659	1.750	506	2.074
536	1.909	669	1.743	513	2.020
546	1.879			577	1.826
557	1.857			647	1.754
569	1.834			669	1.743
584	1.815			696	1.723
602	1.796			713	1.718
611	1.783			730	1.713
620	1.778			749	1.709
627	1.769			763	1.697

The results are shown graphically in Plate 1, together with the dispersion curve for bisulphide of carbon for the same region of the spectrum. The remarkable dispersive power of the nitroso is at once apparent when we compare the two curves, and contrast it with that of the bisulphide, which has the highest dispersive power of any substance in common use.

Over the region of the spectrum given above, the nitroso can be considered as a transparent substance, and the dispersion formula for transparent substances can be applied to it. From three values of n and the corresponding values of λ the value of λ', the centre of the absorption band, can be calculated.

In the case of selenium, which has a dispersion curve resembling that of the nitroso, the absorption appears to increase steadily from the yellow down to the extreme ultra-violet, making it impossible to determine experimentally the centre of the absorption band which is chiefly responsible for the dispersion. Applying the formula for transparent substances to the values found for selenium, I found that if we assume a single absorption band, its centre must be at wave-length .00056. To account for the continued absorption as we pass down the spectrum, we have only to assume that there is a series of bands of which the calculated one is the first member. I expected that the nitroso would behave in a similar manner, but found on examining its transmission that the absorption, which commenced at $\lambda = .0005$, ended quite abruptly at about .00037, the substance transmitting the ultra-violet almost down to the last cadmium lines. This property of the substance enabled me to prepare screens transparent only to ultra-violet light, which I have described in a previous paper.*

The transparency of the nitroso on the ultra-violet side of the absorption band is, however, very much less than on the green side, as I soon found in endeavoring to measure the dispersion by crossing a prism of the substance formed between quartz plates, with the prism of a quartz spectrograph. No trace of the spectrum on the more refrangible side of the absorption band appeared on the photographic plate. After a number of failures, which obviously resulted from the insufficient transparency of the nitroso prism, combined with the necessarily short exposure, I abandoned this method, and made some rough determinations of the ultra-violet dispersion, by observations on the angle of maximum polarization for these wave-lengths. The results, while not very accurate, gave unmistakable evidence of anomalous dispersion, the refractive index for wave-lengths immediately adjacent to the absorption band ranging from 1.1 to about 1.5. I was subsequently able to use the method of crossed prisms, by employing very acute prisms, kept warm by means of an electrically heated platinum wire, which made long exposures possible.

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* Phil. Mag. and Astrophysical J., 1903.

I shall first, however, consider the polarization phenomena accompanying the reflection of light from the surface of the fluid nitroso. The substance was kept in a liquid condition by means of a small cell heated by a current of steam. Figure A.

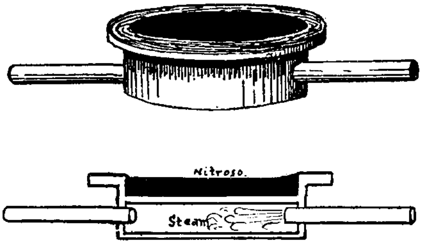

FIGURE A.

On examining the light reflected at a fairly large angle with a Nicol prism, it will be found to vary from light blue to deep violet and purple, as the angle of incidence is increased, the Nicol being held in such a position as to refuse transmission to the light polarized by reflection. If a spectroscope is placed behind the Nicol, a dark band will be seen crossing the spectrum, which shifts its position as the incidence angle varies. The centre of this dark band is evidently the wave-length, for which the angle of incidence happens to be the angle of maximum polarization, or in other words the refractive index of the substance for this wave-length is the tangent of the angle of incidence. In the case of glass and substances of low dispersion, the different colors are polarized at nearly the same angle, i.e., very little color effect is observed when the reflected light is examined with the Nicol. In these cases the dark band to which I have referred is so broad as to occupy practically the entire visible spectrum. In the case of a substance with as high a dispersion as that of the nitroso, the angle of maximum polarization is quite different for the different colors, so that the dispersion may be determined by observing the position of the centre of the dark band and the angle of incidence. It was found that values agreeing very closely with those obtained with the prisms could be obtained by this method in the yellow, green, and greenish-blue portions of the spectrum. On attempting to drive the band through the orange and into the red, it broadened so much that it was quite impossible to locate its centre with any precision. This was of course due to the fact that the dispersion in the red and orange is not sufficient to make the method very accurate in this part of the spectrum. The dispersion of selenium was also determined in this manner, and found to agree very well with the results obtained with prisms. To apply the method to the ultra-violet the observations were made photographically with a small quartz spectrograph made by Fuess. This instrument was furnished with a Rochon prism mounted immediately behind the quartz collimation lens. This prism as furnished by the maker of the instrument refused to transmit the ultra-violet, and I found that the two halves had been cemented together with balsam, which I replaced with glycerine. Some preliminary experiments were made with selenium mirrors, as they were easier to handle than the fluid cell. The light from a cadmium spark was reflected into the slit of the instrument at various angles of incidence, and dark bands were found running across the two polarized spectra furnished by the Rochon prism. I spent some time endeavoring to extract data from these bands, and finally came to the conclusion that either they were not due to the selective polarization by reflection, or else that the dispersion curve as determined for selenium by means of the interferometer was in error. On experimenting further I found that these bands were due solely to the rotatory dispersion of the light in the quartz collimating lens, some colors being rotated through such an angle as to be quenched in one spectrum by the Rochon, and others in such a degree as to be absent in the other. I mention this defect in the instrument as it may be of interest to others working along similar lines. The proper design of the instrument should have called for a collimating lens made of two thin lenses, one of right-handed, the other of left-handed quartz. I remedied the defect in my instrument by placing a flat plate of L. H. quartz immediately behind the lens. This plate had a thickness equal to the thickness of the lens at its centre, and abolished the bands entirely, when the lens was diaphragmed down to a small area at the centre.

I found, however, in working with the horizontal cell of fluid nitroso that better results were obtained by using a Nicol prism in front of the slit of the spectrograph, than with the Rochon prism. The Nicol was made transparent to ultra-violet light by separating the two halves, cleaning off the balsam, and subtituting glycerine.

Evidence of the very low value of the refractive index on the ultra-violet side of the absorption band was obtained before any polarization experiments were tried. The light of a cadmium spark was reflected from the pool of liquid nitroso at nearly normal incidence, and then thrown into the spectrograph by means of a quartz total reflecting prism. A series of spectra was taken, with times of exposure varying from 2 seconds to 3 minutes. By comparison of the different spectra it was possible to form a rough estimate of the reflecting power in different parts of the spectrum. A similar series was made with a flat reflector of magnalium, which is remarkably constant in reflecting power throughout the entire spectrum. It was found that the nitroso, in the region of its absorption band, reflected almost as strongly as magnalium, while just on the ultra-violet side of the band there was a gap in the spectrum, due to the very low reflecting power which the substance has for these waves. The reflection coefficient at this point was estimated at about 2 per cent only, while a short distance further up the spectrum the coefficient is at least 80 per cent.* This indicates that the refractive index has a very low value at the point of minimum reflection. The following table gives an idea of the reflecting power of the nitroso in the various regions of the spectrum as compared with the magnalium. The values are of course only approximate, being deduced by picking out two spectra (one from nitroso the other from magnalium) which showed the same intensity at the given wave-length and then comparing the times of exposure. The reflecting power is expressed as a fractional part of the reflecting power of magnalium.

λ	Ref. Power	λ	Ref. Power
217	$\frac{1}{2}$	380	$\frac{1}{2}$
240	$\frac{1}{2}$	399	1
290	$\frac{1}{3}$	467	1
330	$\frac{1}{6}$	508	$\frac{1}{2}$
343	$\frac{1}{12}$	535	$\frac{1}{4}$
350	$\frac{1}{40}$	569	$\frac{1}{4}$

When the incidence angle is increased and the reflected light examined with a Nicol prism, a most beautiful surface color appeared, ranging from a brilliant blue, through violet, to a reddish purple. This is very easy to understand. The violet light within the region occupied by the absorption band is metallically reflected at all angles of incidence, consequently it is always present in excess in the reflected light, and is never cut off by the Nicol. As the angle of incidence is increased, the polarizing angle for red light is first reached, and the red end of the spectrum disappears, being cut off by the Nicol, the reflected light appearing blue. As the angle increases, more of the spectrum is removed by the Nicol, and the color changes to deep violet, the dark band advancing down the spectrum. Finally by further increase the red appears again in full intensity, being reflected unpolarized, and mixing with the metallically reflected violet gives the brilliant reddish purple. Very similar appearances can be observed with selenium, though the colors are not as saturated and not nearly so brilliant.

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* This value is much too high. It was based on the supposition that the magnalium plate reflected 80 per cent, which was subsequently found to be incorrect, the metal containing too little magnesium and having a rather low reflecting power.

Having roughed out, so to speak, the dispersion in the ultra-violet by photographing the spectrum of the reflected light at various angles of incidence through a Nicol prism, an attempt was next made to secure more reliable data by means of the method of crossed prisms. A prism of small angle, held in a clamp, was mounted with its refracting edge in a horizontal position, immediately behind the prism of the quartz spectrograph. A fine platinum wire immediately below the edge of the prism, when heated by a current of suitable strength, kept the nitroso in a fluid condition. A system of small screens was arranged so that the light could either be made to pass through the nitroso prism, or through a small clear space in the quartz plates immediately above it, thus photographing the deviated and undeviated spectra one above the other. The current strength was so adjusted as to keep the temperature of the prism as nearly as possible at the melting point of the nitroso. It was possible in this way to give exposures of an hour or more, and obtain a photographic record of the dispersion curve from the orange down to the extreme end of the ultra-violet. The cadmium spark was employed as a source of light, and the slit of the spectrograph was diaphragmed down to a length of less than a millimeter in order to obtain very narrow spectra. As the conditions necessary for success were determined by repeated experiments, prisms of larger angle were used, and the two spectra, which overlapped in the first experiments in the region where the refractive index had a low value, were completely separated. I found that a great improvement resulted from carefully grinding the edge of the quartz plate, which rests against the other plate, perfectly straight with fine emery. It was not until this expedient was adopted that the larger angles became possible. The deviated spectrum is quite sharp except close to the edge of the absorption band, where absorption produces a broadening of the image by reducing the effective width of the beam of light, as is always the case with strongly absorbing prisms. It is even possible to follow the general trend of the dispersion curve right through the band of metallic absorption, though the broadening resulting from diffraction is, in this region, so great that accurate measurements were impossible. The continuity of the curve is shown, however, better than I have ever seen it in any photograph. To deduce numerical values from the photographs, the distances between the spectra were measured with a dividing engine at the principal cadmium lines. From these distances the actual angular deviations were calculated, making proper allowance for the fact that the plate stood at an angle, — i. e., the focal length of the lens of the spectrograph was very different in the different regions of the spectrum. The angle of the prism was calculated from the deviations in the yellow and green, for which region the refractive index had been already determined with considerable accuracy. In spite of the rather crude method, the results obtained with the different prisms agreed surprisingly well.

It will be seen from the photographs, some of which are reproduced in Figure 1, Plate 3, that the deviated and undeviated spectra come together at the ultra-violet edge of the principal absorption band, indicating a refractive index not much above unity.

The prism angles in these experiments varied from 20′ to 2°.

It was found possible to get a fairly accurate determination of the refractive index for wave-length 48 from these photographs, the mean of several determinations giving 2.28 as the value. The values found on the ultra-violet side of the band are given in the form of a curve in Plate 2. This portion of the curve is especially interesting, as the effect of the ultra-violet absorption bands is most strikingly shown. The weak band at wave-length 25 causes a dip in the curve, which has a maximum at 285, after which it again rises rapidly as it approaches the heavier absorption band situated not far from 20.

On this same Figure I have given the dispersion in the visible spectrum, together with the value $n = 1.85$ for wave-length, $\lambda = 43$, the centre of the absorption band, which was determined from the constants of elliptical polarization. The trend of the curve within the absorption band I have sketched in freehand, as well as the general form and position of the absorption bands.

I have already shown that the reflecting power of the nitroso for the wave-lengths immediately adjoining the absorption band on the ultra-violet side, is exceedingly small, which is not surprising when we consider that the refractive index for this region is not very different from air. If now we consider a quartz-nitroso surface, instead of an air-nitroso surface, we should expect fairly strong reflection at this point, the nitroso acting as the rarer medium, while further down the spectrum, at about wave-length 29, where the indices of the quartz and nitroso are very nearly the same, we should expect scarcely any reflection at all.

This was found to be the case. An acute prism of quartz was ground and polished, and one face of it brought in contact with a small drop of nitroso fused on a strip of blackened brass. One end of the strip was heated in a flame, the nitroso remaining fluid as a result of the heat conduction along the strip. The object of using the blackened brass was to get rid of reflection from the back surface of the nitroso. The light reflected from the front surface of the quartz was thrown to one side, owing to the inclination, so that everything was eliminated except the reflection from the quartz-nitroso surface. The spectrum of this selectively reflected light is shown in Figure 2, Plate 3. The minimum at wave-length 36 will be seen to have disappeared, and a new one will be found at 29 just about where we should expect it. This minimum would doubtless be more pronounced were it not for the fact that the absorption coefficient has at this point a not inconsiderable value, which will cause the bounding surface in question to have a higher reflecting power than if both media were perfectly transparent.

Absorption of Solid, Liquid, and Gaseous Nitroso.

No determinations of the dispersion of the solid nitroso have been made, owing to the difficulty of getting suitable prisms: when the fluid prisms cool off the nitroso crystallizes in aborescent forms, and no longer transmits regularly. If the prism is held close to the eye, and a lamp flame viewed through it, enormously deviated spectra are seen, due to the formation of crystals having a much larger angle than the prism. The substance in the solid condition is double refracting, the two spectra which every prism furnishes being extinguished in turn by a revolving Nicol prism. The absorption band of the solid nitroso was studied by photography. A drop of fluid nitroso was pressed in a clamp between two hot plates of quartz, which were allowed to cool under pressure. In this way a very thin film of the solid substance was obtained. On photographing a spectrum through this screen it was found that the principal absorption band had broken up into two, placed symmetrically with respect to the band shown by the fluid. One of these doubtless belongs to the ordinary, the other to the extraordinary ray. The centres of the bands are at wave-lengths 36 and 46, while the centre of the single band possessed by the liquid is at $\lambda = 43$.

In Figure 3, Plate 3 we have two photographs of this double band.

The absorption of the liquid nitroso does not differ very materially from that of its solution in glycerine, which I have described in a previous paper on screens transparent only to ultra-violet light. The absorption spectrum was photographed by pressing out a drop of the fluid substance between a quartz lens and plate, the whole being kept warm by a current of hot air. In this way a film of variable thickness was obtained, which when brought close to the slit of the quartz spectrograph, enabled a record to be made of the relative intensities, positions, and general form of the absorption bands. One of these photographs is shown in Figure 4, Plate 3.

The nitroso begins to vaporize at a temperature only a little above its melting point, and the absorption of the vapor is interesting, for unlike most absorbing vapors and gases, this substance shows a broad absorption band, similar to the liquid, only shifted well down into the ultra-violet. The nitroso was vaporized in a glass tube, the ends of which were closed with quartz plates. The tube was previously exhausted, and was heated by a water bath through which it passed. The absorption spectrum was photographed at different temperatures, ranging from 85°, the melting point, to 100°. The absorption does not begin as a narrow line, as is usually the case with gases, but with a broad band, which increases in intensity as the density of the vapor increases. The centre of this band is at wave-length 34, while the centre of the band in the case of the liquid is at 43.

I made numerous experiments to see whether the density of the vapor could be sufficiently increased to cause a shift in the position of the band towards the red, all of which failed, owing to the fact that the nitroso decomposes at temperatures above 150. The substance was heated in strong sealed bulbs in an air bath, but decomposition always resulted before a density sufficient to shift the absorption band in a measurable degree had been obtained.

On the other hand the position of the absorption band can be shifted by increasing the density of the medium in which the nitroso vapor is present. If a solution of nitroso in ether is heated above its critical temperature in sealed glass tubes, the centre of the absorption band can be given almost any position between that of the vapor band and the solution band, by varying the amount of ether in the tube, or in other words by varying the density of the vapor.

I made a number of attempts to prove that the double overlapping band shown by the solid was connected with the double refraction of the nitroso crystals, by placing a Nicol before the slit of the spectrograph and photographing the absorption spectrum of a thin crystalline film of the substance between two quartz plates. A number of spectra were taken with the Nicol in different positions, a comparison of which gave unmistakable evidence that one band belonged to the ordinary, the other to the extraordinary ray. In no case did either of the bands disappear entirely, owing to the fact that they overlap, which makes it impossible to have complete transparency with the Nicol in any position. No measurements have as yet been made of the values of the extinction coefficient in different parts of the spectrum, owing to the difficulty of preparing a film thin enough to transmit light within the absorption band. Though it is possible to get a wedge-shaped film between a lens and a flat plate which transmits all wave-lengths to a certain degree in its thinest portions, a film of this nature is quite unsuitable for quantitative measurements of the absorption. Probably by working with solutions in glycerine of different concentrations and thicknesses a fairly correct idea of the absorption curve could be obtained by calculation, though this method would be open to some objections. Until the substance has been investigated in the infra-red and until the dielectric constant has been determined, the dispersion formula cannot be applied to it to the best advantage.

At the present time facilities for investigating these two points are not at my disposal, but I expect in the near future to investigate them. In the meantime it is instructive to apply the formula to the results which have already been obtained.

The Dispersion Formula.

For parts of the spectrum in which the extinction coefficient has a small value the dispersion is represented by the formula

$$n^2 = 1 + \sum \frac{m_\kappa \lambda^2}{\lambda^2 - \lambda_\kappa^2},$$

in which λ_κ are the wave-lengths of the centres of the absorption bands (nearly) and m_κ constants for these bands, the summation being taken for all of the absorption bands, whether near or far removed from the portion of the spectrum under consideration. If the bands are far removed from this region the fraction $\frac{\lambda^2}{\lambda^2 - \lambda_\kappa^2}$ is practically equal to unity and the constants m', m'', etc., have merely to be added or subtracted according to whether they lie on the more refrangible or less refrangible side of the spectral region under investigation, i. e., according as the sign of $\frac{\lambda^2}{\lambda^2 - \lambda_\kappa^2}$ is positive or negative.

For a medium having but a single absorption band the formula takes the form

$$n^2 = 1 + \frac{m' \lambda^2}{\lambda^2 - \lambda'^2}.$$

This formula can, I think, be applied to sodium vapor, judging from results recently obtained. Most other substances have, however, a second band further along in the ultra-violet, though for the region investigated experimentally this is so far removed that m'' can be considered constant. The formula then takes the form

$$n^2 = m + \frac{m' \lambda^2}{\lambda^2 - \lambda'^2},$$

in which $m = m'' + 1$. This formula has been found to represent the dispersion of most transparent substances.

I have applied this formula to the region of the spectrum for which the nitroso is most transparent, using the data obtained with the spectrometer and prisms of comparatively large angle.

The position of the centre of the absorption band can be calculated from three observed values of the refractive index, n_1, n_2, n_3, for wave-lengths λ_1, λ_2, λ_3, from the formula,

$$\lambda'^2 = \frac{\lambda_3^2 (\lambda_1^2 - \lambda_2^2) - \lambda_2^2 (\lambda_1^2 - \lambda_3^2)\, C}{(\lambda_1^2 - \lambda_2^2) - (\lambda_1^2 - \lambda_3^2)\, C}, \text{ in which } C = \frac{n_2^2 - n_1^2}{n_3^2 - n_1^2}.$$

This formula gives $\lambda' = 43.1$ for the centre of the absorption band, a value agreeing closely with the observed value. Further,

$$m' = \frac{(n_3^2 - n_1^2)(\lambda_1^2 - \lambda'^2)(\lambda_3^2 - \lambda'^2)}{\lambda'^2 (\lambda_1^2 - \lambda_3^2)}$$

$$m = n_2^2 - \frac{m' \lambda_2^2}{\lambda_2^2 - \lambda'^2}.$$

The values found for m' vary slightly with the region of the spectrum in which the value of n and λ are chosen. This indicates the presence of infra-red absorption bands. The mean value found was

$$m' = 0.53$$
$$m = 2.13$$

If it had turned out that m equalled one, the inference would have been that the absorption band at $\lambda = 43$ was the only one present. The large value of $m - 1$ points to one or more other bands in the ultra-violet. As I have already shown, a strong absorption-band exists a little below $\lambda = 20$ and a weak one at $\lambda = 25$. Though the formula as it stands represents the dispersion in the red, yellow, and green fairly well, it breaks down if we try to apply it to the values found in the ultra-violet by the photographic application of the method of crossed prisms. This is due to the fact that we are getting into the region in which $\frac{\lambda^2}{\lambda^2 - \lambda''^2}$ is no longer approximately equal to one.

This quantity increases in magnitude as λ decreases, and the values of n will, consequently, be higher than those calculated on the assumption that the quantity is equal to unity. To meet this contingency we must use the formula (neglecting the weak band at $\lambda = 25$),

$$n^2 = 1 + \frac{m' \lambda^2}{\lambda^2 - \lambda'^2} + \frac{m'' \lambda^2}{\lambda^2 - \lambda''^2}.$$

Since $m = 2.13 = m'' + 1$, we can take $m'' = 1.13$.

The value of the refractive index n, for $\lambda = 34$ calculated from the original formula, is $n = 1.1$, the value observed is 1.3; in other words the value is raised by the influence of the remote ultra-violet band. Using this observed value of n, it is possible to calculate the wave-length λ'' of the ultra-violet band, assuming as above that $m'' = 1.13$.

This was found to be $\lambda'' = 18$, which looks reasonable. Of course it is impossible to determine the centre of the band experimentally, since the nitroso cuts off everything below 20. Whether or not a return of transparency would be found further along by employing fluorite plates and a vacuum spectrograph, it is impossible to say. The calculated value of λ'' is about what we should expect it to be, judging by an inspection of the photographs of the absorption in this region.

We are now in a position to calculate other values of n in the ultra-violet and compare them with the observed. For $\lambda = 36$ (the wave-length for which the lowest value of n was found experimentally), n calculated by first formula was $n = .92$, by second formula $n = 1.08$, observed $n = 1.05$. For $\lambda = 31$, by second formula $n = 1.42$, observed $n = 1.43$. Obviously we cannot apply the formula to that portion of the curve between wave-lengths 23 and 29 until we know the value of the extinction coefficient within this region. I feel certain that there are absorption bands in the infra-red, not only on account of the indications which the dispersion formula gives, but because when working with prisms of large angle I found that a fairly strong absorption occurred in the extreme visible red. It was not sufficiently intense to interfere with measurements, and whether it was due to the presence of a weak, diffused band, similar to the one at $\lambda = 25$, or to a strong band of metallic absorption, I was unable to say, the observations being confined to its extreme edge. I expect to investigate this point, as soon as our apparatus for the study of the infra-red region of the spectrum is in working order. I hope to be able, in the near future, to make a determination of the dielectric constant of the substance. With these points thoroughly investigated, a more rigorous application of the dispersion formula to the results will be possible. I feel less hopeful of getting satisfactory data regarding the absorption, but shall at all events make the attempt.

THE DISPERSION OF TOLUINE.

In examining the ultra-violet absorption of some organic solvents, I found that toluine has a fairly strong and narrow absorption band at wave-length .00027, in addition to the band of metallic absorption below .00020, which is the one which chiefly influences the dispersion of the substance. It occurred to me that it would be interesting to determine the dispersion of the substance through this region, and see to what extent the curve was modified by the weaker band. So far as I was able to find, the ultra-violet dispersion of toluine has never been determined, which is not very surprising, since only very acute prisms transmit anything below the band above mentioned. The same method was employed as in the case of nitroso-dimethyl-aniline, namely the crossed prism method adapted to the quartz spectrograph.

The dispersion was first measured in the visible spectrum with a hollow prism of 60° angle. Small prisms were then made of quartz plates with the toluine between them, the angles varying from two to six degrees. The angles were determined from the photographs of the deviated spectra obtained with the quartz spectrograph, using the data obtained with the spectrometer in the blue region of the spectrum. Figure 6, Plate 3, shows one of the deviated spectra, the ultra-violet being to the left, and illustrates well the general form of the dispersion curve in this region. Figures 7 and 8 show the deviated and undeviated spectra, the one above the other, obtained with prisms of two different angles. It is at once apparent that the absorption band at wave-length .000271 modifies the curve to no small degree. On the original negatives a very slight trace of oppositely curved branches, such as are always present at the edges of strong absorption bands, is discernible. The band is quite narrow and fairly strong, yet it does not seem to modify the curve to any great degree on the red side, — at least the curve has nothing like the steepness which it has in the remote ultra-violet. On the other side of the band, however, the curve is seen to be depressed to a considerable distance. The influence of the band seems, therefore, to be unsymmetrical. This is to be ascribed, I believe, to the fact that the absorption on the ultra-violet side of this band is quite strong, as will be seen from Figure 8.

The plates were measured in the same manner as the nitroso plates and a table of the refractive indices is given below.

λ.	n.	λ.	n.
2266	1.885	2681	1.640
2288	1.8503·	2750	1.628
2314	1.821	2980	1.595
2322	1.808	3250	1.570
2330	1.807	3400	1.554
2372	1.767	3659	1.542
2470	1.709	3995	1.526
2527	1.1679	4799	1.507
2570	1.649		

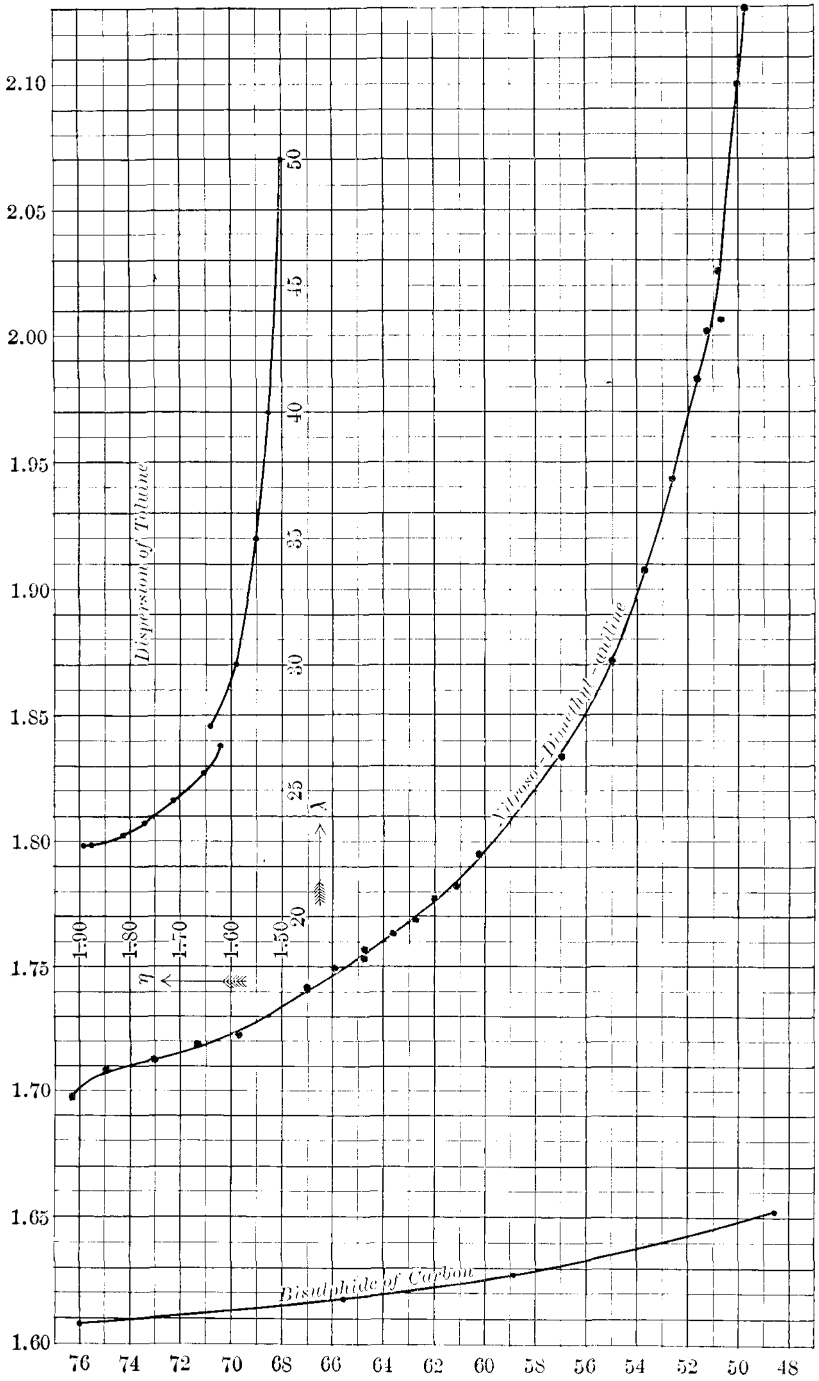
The curve is shown graphically on Plate 1.

Applying the dispersion formula to the results we find, neglecting the band at .00027 for the centre of the band of metallic absorption in the ultra-violet, the value .000182, which is not inconsistent with the photographic records. For the other constants we find $m = 1.37$ and $m' = .77$.

The sum of these two constants should give us the dielectric constant of the substance if no other bands of absorption are present. The sum $m + m' = 2.14$, while the dielectric constant is 2.36 (determination by Palaz), from which it seems probable that there is an absorption band in the infra-red.

The formula expresses the dispersion fairly well except for a narrow range immediately on the more refrangible side of the band at wavelength .00027. Adding another term to the formula will not help matters much in this region, in my opinion, owing to the indefinite nature of the absorption.

The substance seems to be worthy of a more complete investigation.


2.10
2.05
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1.95
1.90
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1.80
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76
74
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50
48
Dispersion of Toluine
50
45
40
35
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λ
1·90
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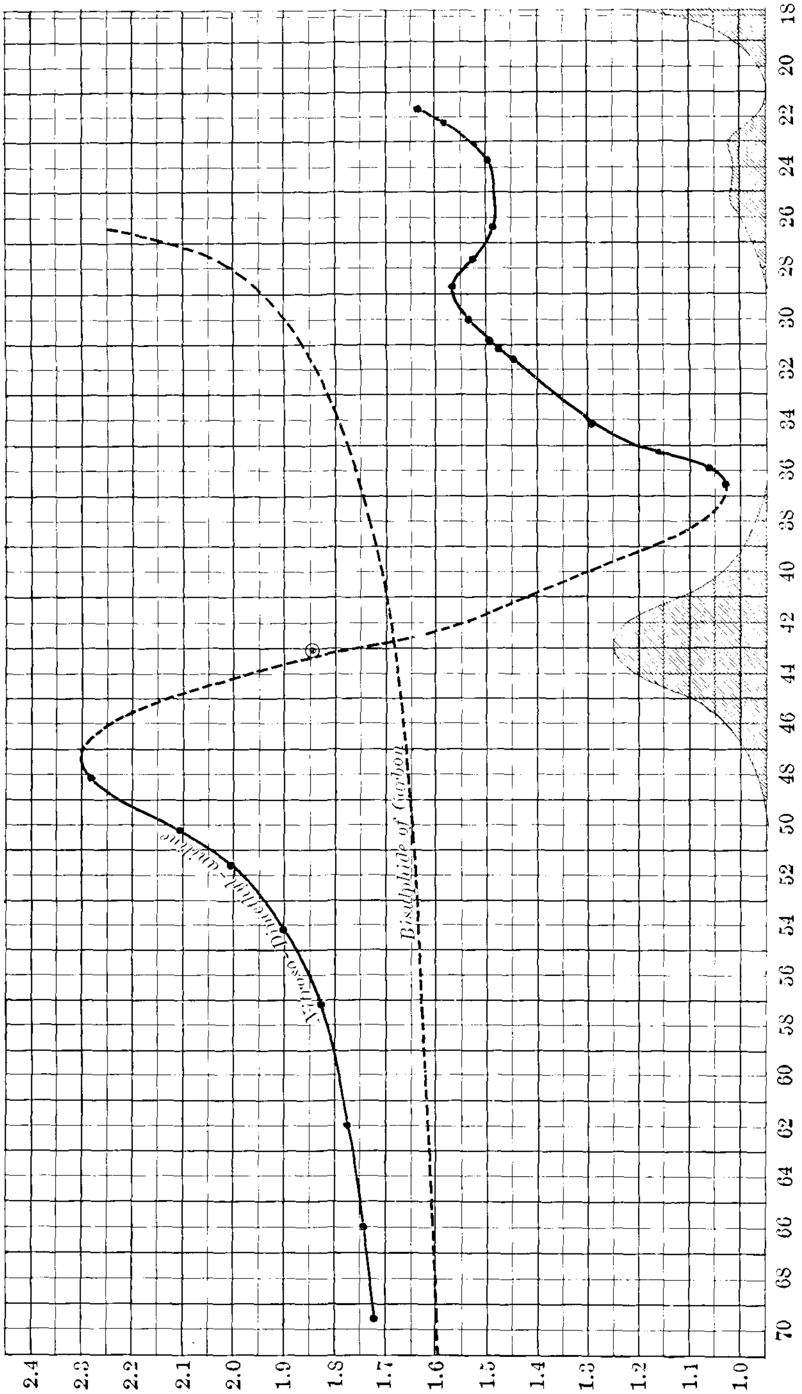
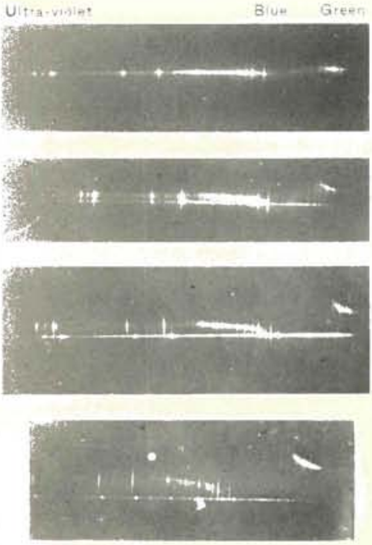
η
Nitroso-Dimethyl-aniline
Bisulphide of Carbon

WOOD.—OPTICAL PROPERTIES OF NITROSO DIMETHYLANILINE AND TOLUINE. PLATE **2.**



WOOD.—OPTICAL PROPERTIES OF NITROSO DIMETHYLANILINE AND TOLUINE. PLATE III.



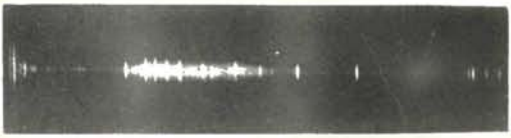


Fig. 4

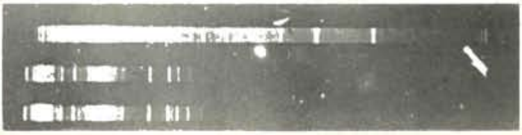


Fig. 3

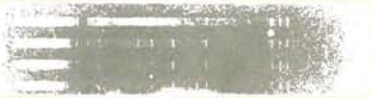


Fig. 5

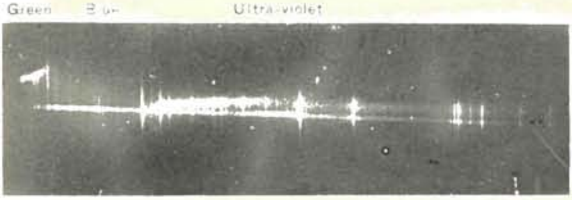


Fig. 1

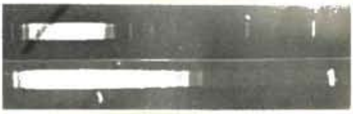


Fig. 2

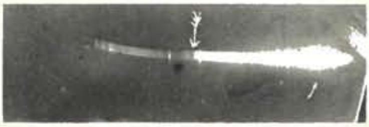


Fig. 6

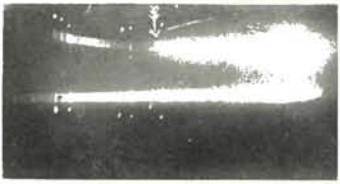


Fig. 7

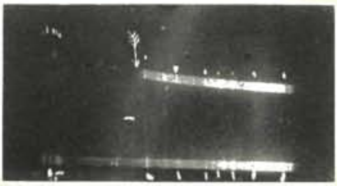


Fig. 8

HELIOTYPE CO., BOSTON.

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CONTRIBUTIONS FROM THE GRAY HERBARIUM OF HARVARD UNIVERSITY.

New Series. — No. XXV.

NEW AND OTHERWISE NOTEWORTHY ANGIOSPERMS FROM MEXICO AND CENTRAL AMERICA.

By J. M. Greenman.

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NEW AND OTHERWISE NOTEWORTHY ANGIOSPERMS FROM MEXICO AND CENTRAL AMERICA.

BY J. M. GREENMAN.

Presented May 13, 1903, by B. L. Robinson.

THE descriptions and notes in the present paper are the result of the study of several series of Mexican and Central American plants, chiefly from the recent collections of Cyrus Guernsey Pringle, Dr. Edward Palmer, Dr. Joseph Nelson Rose, Edward William Nelson, Señors Cassiano Conzatti and Valentin González, C. H. T. Townsend and C. M. Barber, E. W. D. Holway, Captain John Donnell Smith, Henri Pittier, and Adolfo Tonduz.

The herbarium in which the types and other mentioned specimens occur is indicated immediately after the citation of the numbered specimen; and the abbreviations for the different herbaria are as follows: hb. Gr. (Gray Herbarium), hb. U. S. Nat. Mus. (Herbarium of the United States National Museum), hb. J. D. S. (Herbarium of John Donnell Smith), hb. Inst. Phys.-Geog. Cost. Ri. (Herbarium of the National Physico-Geographical Institute of Costa Rica). For the most part the species represent the *Metachlamydeæ*, especially the *Compositæ;* although a few *Archichlamydeæ*, as well as *Monocotyledoneæ*, are herein described.

Rhynchospora Pringlei. Pale green: culms triangular, 1 m. or more high: stem-leaves remote; sheath slightly inflated, 5 to 7 cm. long; blade 2 to 4.5 dm. long, 8 to 10 mm. broad, hispidulous on the margins and along the midrib beneath: inflorescence cymose-paniculate; the terminal cymule subsessile or short pedunculate, the lateral on long slender peduncles 4 to 10 cm. in length; bracts lance-attenuate; scales of the spikelet broadly ovate, 1-nerved, terminated by a short awn, brown or somewhat chestnut-colored: achene flattened, oblong-obovate, 3 to 4 mm. long, bearing on either surface in the upper half near the margin a longitudinal ridge, minutely pitted, dark brown in color; tubercle or beak conical, about as long as the achene: setæ unequal, 6 in number and mostly shorter than the achene. — MEXICO. State of Michoacan: in swamps near Zamora, altitude 1500 m., 24 May, 1901, and 25 July, 1902, *C. G. Pringle*, nos. 9668, 8642 (hb. Gr.).

The affinity of *Rhynchospora Pringlei* is with *R. aurea*, Vahl., and *R. aristata*, Boeck., but it differs from both these species in the contracted inflorescence, and in the achenial characters.

Tradescantia dracænoides. *T. longifolia*, Greenm., Proc. Am. Acad. xxxiii. 471 (1898), not Small, Bull. Torr. Bot. Cl. xxiv. 233 (1897). *Dichorisandra longifolia*, Mart. & Gal. Bull. Acad. Brux. ix. 378 (reprint p. 7). *T. holosericea*, var. *dracænoides*, Clarke in DC. Monogr. Phanerog. 302; Hemsley, Biol. Cent.-Am. Bot. iii. 393.

Tradescantia saxicola. Stems decumbent or ascending, simple or branched, 1 to 2 dm. high, commonly rooting at the nodes, glabrous except along a line continuous with the short ciliated sheath opposite the leaf-blade, purplish: leaves narrowly lanceolate, 1.5 to 4 cm. long, 4 to 8 mm. broad, acute, slightly hirtellous-margined, sparingly pubescent above, glabrous beneath: inflorescence terminating the stem and branches in sessile subumbellate floral-clusters; pedicels reflexed and, as well as the calyx, hirsute-pubescent: sepals oblong-ovate, 4 to 4.5 mm. long, obtuse: petals white, somewhat exceeding the calyx: stamens unequal; the outer cycle shorter, and with glabrous filaments; the inner fully equalling the petals, with bearded filaments, and with anther-cells separated by rather broad connectives: capsule small, smooth and glabrous, 2 to 3 mm. in length. — MEXICO. State of Guerrero: on ledges, Iguala Cañon, near Iguala, 14 September, 1900, *C. G. Pringle*, no. 9270 (hb. Gr.).

This species belongs to the section *Descantaria*, and in general habit resembles *T. angustifolia*, Robinson, *T. disgrega*, Kunth, and *T. multiflora*, Swartz.

Tradescantia Urbinana. Roots tuberiform, fleshy: stems branching from the base; branches erect, 2 to 5 dm. high, glabrous: leaves lanceolate or lance-attenuate, 5 to 10 cm. long, 0.5 to 1.5 cm. broad, acute, smooth-margined, glabrous on both surfaces; sheath slightly inflated, 0.5 to 1 cm. long: inflorescence terminating the branches in compound pedunculate umbels; bracts small, 3 mm. or less in length; peduncles and pedicels glabrous and, as well as the calyx, purplish:

flowers small, 1 cm. or less in diameter: sepals oblong, somewhat navicular, 4 to 6 mm. long, glabrous: petals ovate or subrotund, 5 to 6 mm. long and broad, deep purple: stamens 6, equal; filaments bearded; anthers uniform: capsule oblong, obtusely triangular, 5 mm. long, hirtellous-puberulent above. — MEXICO. State of Morelos: Sierra de Tepoxtlan, near Cuernavaca, altitude 2300 m., 11 September, 1900, *C. G. Pringle*, no. 9250 (hb. Gr.).

This species belongs to the section *Eutradescantia*, and is seemingly most nearly related to *T. Karwinskyana*, Roem. & Schult., and *T. linearis*, Torr. The writer takes pleasure in dedicating the species to Señor Dr. Manuel Urbina, Director of the National Museum, City of Mexico.

Zygadenus mohinorensis. Bulbs scaly-fibrous coated, about 2 cm. in diameter: scape 10 dm. high, commonly bearing 2 remote leaves: basal leaves, 5 to 6 dm. long, 10 to 14 mm. broad, attenuate to an obtuse tip, dark green and glabrous on both surfaces: inflorescence 3 to 4 dm. in length, branched below; branches erect, subappressed against the main axis; bracts of the inflorescence conspicuous, green or purplish, exceeding the 0.5 to 2 cm. long pedicels: flowers white or greenish-white, about 2 cm. in diameter; perianth-divisions oblong, 10 mm. long, 5 to 7 mm. broad, bearing at their base an obcordate or somewhat Y-shaped erose-margined gland: stamens included, equalling the style-branches, about one-half as long as the perianth-segments: mature fruit not seen. — MEXICO. State of Chihuahua: Mt. Mohinora, 1 September, 1898, *E. W. Nelson*, no. 4875 (hb. Gr., and hb. U. S. Nat. Mus.).

This species is most nearly related to *Z. elegans*, Pursh, but differs in the somewhat larger flowers, the more conspicuous and longer bracts of the inflorescence, and in the nature of the glands at the base of the perianth-segments.

VERATRUM CALIFORNICUM, Durand, Journ. Acad. Nat. Sc. Phil. Ser. II. iii. 103; Watson, Bot. Calif. ii. 182. The geographical range of this species is extended considerably southward by the collection of excellent specimens in the Sierra Madres near Colonia Garcia, State of Chihuahua, MEXICO, altitude 2450 m., 7 August, 1899, *Townsend & Barber*, no. 235 (hb. Gr.).

Anthericum drepanoides. Roots fleshy-fibrous, sometimes tuberous, fascicled, crowned by a collar of brownish fibres 2 to 3 cm. in length, the remains of the bases of the older leaves: assimilating leaves rosulate, about 9, lance-attenuate, 0.5 to 1.5 dm. long, 8 to 18 mm. broad, acute, conspicuously ciliate on the subcartilaginous margins, more or less conduplicate, at least in the dried state, glabrous on both surfaces, rather prominently and numerously nerved: scape 5 to 6 dm. high, few-branched above, terete, glabrous, bearing about 4 bracts: pedicels 1 cm. or less in length, jointed below the middle: flowers yellow: perianth about 12 mm. long; divisions narrowly oblong, 3-nerved; mature fruit not seen. — MEXICO. State of Oaxaca: Sierra de San Felipe, altitude 1830 m., 6 August, 1894, *C. G. Pringle*, no. 5757 (hb. Gr.).

Mr. Pringle's plant above cited was in the Gray Herbarium referred by the writer to *Anthericum Humboldtii*, Hemsley (*Phalangium ciliatum*, HBK., Nov. Gen. & Sp. i. 276, & vii. t. 676), but that species is a native of South America, first collected near Caracas, Venezuela, and has straight leaves 10 to 13 cm. long, 6 to 7 mm. broad, and is said to have white flowers.

Anthericum durangense. Roots tuberous, fascicled: leaves basal, lanceolate, 1.5 to 2.5 dm. long, 1.5 to 2 cm. broad, acuminate, acute, ciliate, dark green and glabrous on both surfaces: scape 4 to 5 dm. high, simple or branched, glabrous, or slightly hirtellous, usually unibracteate: floral bracts ovate, mostly acuminate, hyaline, nerved, greenish-white: pedicels jointed: perianth yellow, or yellowish-white in the dried state; divisions at the period of anthesis about 1.5 cm. long, 3-nerved. — MEXICO. State of Durango: near El Salta, altitude 2440 to 2650 m., 12 July, 1898, *E. W. Nelson*, no. 4544 (hb. U. S. Nat. Mus., and hb. Gr.).

Var. **trachycaulum.** Leaves in outline and size like the type, yellowish-green, ciliolate-scabrellous on the margins: scape hirtellous even to the flowers: perianth somewhat smaller than in the species proper, about 1 cm. in length: capsule 1 cm. long, glabrous. — MEXICO. State of Durango: Ramos to Inde, 11–14 August, 1898, *E. W. Nelson*, no. 4706 (hb. U. S. Nat. Mus., and hb. Gr.).

Anthericum Nelsonii. Roots tuberous, fascicled: leaves basal, narrowly lanceolate, 4 to 9 cm. long, 3 to 7 mm. broad, glabrous on both surfaces, minutely scabrellous on the subcartilaginous margins: scape 1 to 2 dm. high, glabrous, naked or bearing a single greenish-white nerved bract near or somewhat above the middle, terminated by 1 to 3 flowers: floral bracts scarious, greenish-white, nerved: pedicels 0.5 to 1 cm. in length, jointed in the middle: perianth yellow or in the dried state yellowish-white; divisions 8 to 10 mm. long, 3-nerved, obtusish or somewhat rounded at the apex: stamens nearly or quite as long as the perianth; anthers free: capsule about 1 cm. long. — MEXICO. State of Chihuahua: base of Mt. Mohinora, 12.8 km. from Guadalupe y Calvo, altitude 2130 to 2285 m., 23–31 August, 1898, *E. W. Nelson*, no. 4844 (hb. U. S. Nat. Mus., and hb. Gr.).

Anthericum Nelsonii is apparently most nearly related to *A. nanum*, Baker, from which it differs in the glabrous character of the scape and by the twice longer capsule.

Echeandia flexuosa. Roots fascicled, tuberous, crowned by a collar 6 cm. in length, the persistent fibres of the bases of the older leaves: scape 7 to 9 dm. high, flexuous, glabrous, leafy at the base, bearing one or two reduced leaves on the main axis, few-branched above into arcuate-ascending more or less flexuous and somewhat compressed or subangulate branches: leaves linear-attenuate, 1.5 to 8 dm. long, 2 to 5 mm. broad, ciliolate to slightly roughened along the margins, thickish and firm in texture, closely parallel-nerved, without transverse anastomosing veinlets, glabrous above, scabrellous on the nerves of the outer or under surface: flowers 3 to 5 in the axils of short chartaceous bracts; pedicels jointed near or below the middle, 16 mm. or less long in fruiting specimens; perianth white or greenish-white, 10 to 12 mm. long; segments 3-nerved, the inner slightly broader than the outer: stamens about equalling the perianth; anthers coherent around the style; filaments plicated: capsule oblong, obtusely triangular, 7 to 10 mm. in length, valves somewhat rugose or transversely ridged: seeds black, about 1.5 mm. long and broad in the dried state. — MEXICO. State of Jalisco: Rio Blanco, July, 1886, *Dr. Edward Palmer*, no. 185 (hb. Gr.). State of Oaxaca: Mountains of Jayacatlan, altitude 1540 m., 10 September, 1894, *L. C. Smith*, no. 188 (hb. Gr.).

Very similar to this species is a specimen collected by Mr. C. G. Pringle on calcareous hills, Las Sedas, State of Oaxaca, altitude 1830 m., no. 5755 (hb. Gr.), but the anthers in Mr. Pringle's specimens are quite distinct, hence according to the present interpretation of the genera *Echeandia* and *Anthericum* it must be referred to the latter genus.

Echeandia macrocarpa. Scape 6 to 7 dm. high, leafy at the base, 2–3-bracteate above, glabrous: leaves lorate, 2 to 3 dm. long, 1.2 to 2 cm. broad, acute, entire or inconspicuously roughened along the margins, dark green and glabrous on both surfaces: inflorescence a glabrous panicle, about 3 dm. in length: pedicels jointed about one-third their length from the base, 2 cm. or less long: flowers 3 to 5 in the axils of chartaceous bracts: perianth white (?), 8 to 12 mm. long; segments 3-nerved; the outer 2 mm., the inner 3 mm. broad: stamens about equalling the perianth; anthers united or coherent into a tube around the style:

capsule 16 to 18 mm. long, prominently nerved. — MEXICO. State of San Luis Potosi: near Tancanhuitz, 2 May, 1898, *E. W. Nelson*, no. 4393 (hb. U. S. Nat. Mus., and hb. Gr.).

Hymenocallis graminifolia. Roots fleshy-fibrous: bulbs ovate-rotund, about 3 cm. high and nearly or quite as broad, covered with dark-brown scales: leaves lorate, 1.5 to 2 dm. long, 5 to 7 mm. broad, obtuse, entire, thin or submembranous: peduncle 1 to 1.5 dm. long, strongly 2-edged, especially above: flowers few (1 to 4), sessile: spathe-valves lance-attenuate, scarious: perianth-tube about 4 cm. long, greenish; segments lorate, narrowly lanceolate, slightly exceeding the tube: staminal-cup broadly funnel-shaped, about 3 cm. high; free portion of filament nearly or quite 2 cm. long: anthers versatile, 14 mm. long: style slender, greenish; stigma overtopping the anthers: cells of the ovary each containing 2 ovules. — MEXICO. State of Morelos: valley near Yantepec, altitude 1230 m., 6 July, 1901, *C. G. Pringle*, no. 8532 (hb. Gr.).

In habit this species approaches *H. humilis*, Watson, and *H. galvestonensis*, Baker, but differs materially in the size of the staminal-tube.

Hymenocallis Pringlei. Bulbs ovate-rotund, about 4 cm. long, equally broad, covered with dark-brown scales, which are continuous over the cylindrical 4 cm. long, and 1.8 cm. thick neck: leaves lanceolate, 1.2 dm. long, 1 to 2.5 cm. broad, narrowed at both ends, obtusish, entire, glabrous on both surfaces: peduncle 2 to 2.5 dm. long: spathe-valves lanceolate, attenuate, 4 to 5 cm. long, 12 mm. broad at the base, thin and scarious: flowers 4 to 6: perianth-tube 3.5 cm. long; segments somewhat exceeding the tube, narrow, 3 to 4 mm. broad: staminal-cup rather narrowly funnelform, 1.8 to 2 cm. high; free portion of filament about equalling the tube: style slender, greenish; stigma slightly overtopping the anthers: ovary sessile, bearing two ovules in each cell. — MEXICO. State of Hidalgo: hillsides near Tula, altitude 1100 m., 20 July, 1896, *C. G. Pringle*, no. 6371 (hb. Gr.).

Sisyrinchium bracteatum. Roots tuberous, fascicled: scapes 1 to 2 dm. high, simple below, branched above, erect or very slightly flexuous, hirsutish-pubescent to nearly glabrous: leaves linear-attenuate, 0.5 to 1.5 dm. in length: inflorescence branched, bearing several short-pedunculate spathes; peduncles mostly less than 1 cm. (rarely 1.5 cm.) long; bracts of the inflorescence foliaceous, 6 cm. or less in length, ciliate: spathe of two slightly unequal bracts, 13 to 16 mm. in length, rather strongly nerved, glabrous, the inner bract usually somewhat exceeding the outer: flowers of the spathe 2 to 4: perianth yellow; ovary at first glandular pubescent: capsule oblong, triangular, 5 to 8 mm. long, hispidulous: seeds small, round, about 1 mm. in diameter; surface slightly honeycombed. — MEXICO. State of Oaxaca: in dry thin soil, Cuesta de San Juan del Estado, altitude 1850 m., 20 August, 1894, *C. G. Pringle*, no. 4831 (hb. Gr.), distributed as "*Sisyrinchium Hartwegii*, Baker form"; Valley of Etla, altitude 1660 m., 2 July, 1895, *L. C. Smith*, no. 556 (hb. Gr.). Federal District: lava fields near Tlalpam, altitude 2250 m., 16 August, 1902, *C. G. Pringle*, no. 8656 (hb. Gr.). The species here described is distinguished from *Sisyrinchium Hartwegii*, Benth., which is apparently the same as *S. tenuifolium*, Humb. & Bonpl., by the less branched and less flexuous scapes, the short-pedunculate spathes, and finally by the hirsutish character of the upper part of the scape and inflorescence.

Sisyrinchium Palmeri. Roots tuberous, fascicled, 2 to 3 cm. long, 5 to 7 mm. thick in the dried state: leaves linear-attenuate, 0.5 to 2.5 dm. long, somewhat sheath-like below and more or less purplish just above the base, smooth and glabrous: scape 1.2 to 2 dm. high, flattened, usually narrowly winged, branched above, bearing 2 to 4 pedunculate inflorescences: peduncles 1.5 to 4 cm. in length, mostly exceeding the nodal bract: spathes of two slightly unequal bracts, 1.5 to 2 cm. long, 3–7-flowered: pedicels 2.5 cm. or less in length: perianth yellow, 1.5 to 2 cm. in diameter: ovary glabrous: capsule oblong, 6 mm. long, 4 mm. thick, rather sharply 3-angled, smooth and glabrous: seeds about 1 mm. long and broad. — MEXICO. State of Jalisco: Rio Blanco, June, 1886, *Dr. Edward Palmer*, no. 18 (hb. Gr.); gravelly plains near Guadalajara, 15 July, 1902, *C. G. Pringle*, no. 8644 (hb. Gr.).

SISYRINCHIUM QUADRANGULATUM, Klatt, Linnæa, xxxii. 284. Specimens secured by Mr. C. G. Pringle on Mt. Orizaba, Mexico, at an elevation of 4000 m., 1901, no. 8566 (hb. Gr.), accord well with the original characterization of the above species, which was based upon plants collected by Liebmann on Mt. Orizaba in 1841. It may be said, however, that the color of the perianth in Mr. Pringle's specimens is distinctly blue, not yellow as was stated by Klatt to be the color of the flowers in the Liebmann plant. There is in the Gray Herbarium a specimen from the Liebmann collection, which was obtained on Mt. Orizaba at an elevation of 3000 m., and the label accompanying this plant bears the name "*Sisyrinchium scabrum*, Schlecht., var. *exaltatum*." Although this specimen was determined by Klatt, it does not seem to have been cited by him in Linnæa xxxii. under the name which it bears. It seems altogether probable that the Liebmann plant in the Gray

Herbarium, here alluded to, represents a part of the type of *S. quadrangulatum*, Klatt, as it answers perfectly to the description. With it, moreover, Mr. Pringle's specimens are an excellent match. The species seems well characterized by the linear grass-like leaves, and the pyriform glabrous fruit, fully twice longer than in *S. angustifolium,* Mill., and its related forms.

SISYRINCHIUM TENUIFOLIUM, Humb. & Bonpl., var. **Seatoni.** Stems about 1 dm. high, much-branched, glabrous : leaves equalling or exceeding the inflorescence, slender, subfiliform : capsule pubescent. — MEXICO. State of Vera Cruz: Mt. Orizaba, altitude 2770 m., 9 August, 1891, *Henry E. Seaton,* no. 483 (hb. Gr.). State of Puebla : hillsides above Chalchicomula, altitude 2615 m., 27 July, 1901, *C. G. Pringle,* no. 9533 (hb. Gr.).

The variety *Seatoni* differs from the species chiefly in the lower stature, and in the filiform character of the leaves.

Habenaria oreophila. Glabrous throughout: stem erect, leafy, 7 to 8 dm. high : leaves oblong-lanceolate, 1 to 1.5 dm. long, 3 to 3.5 cm. broad, apiculate-acute, entire, membranous; the lowermost leaves reduced to mere sheaths, the uppermost gradually smaller: inflorescence a terminal elongated loose 2 to 3 dm. long raceme; bracts subfoliaceous, lanceolate-acuminate, about equalling the ovary : upper sepal ovate, cucullate, 8 mm. long, 3-nerved; lateral sepals oblong-lanceolate, slightly oblique, obtuse, 3-nerved : lateral petals deeply 2-parted ; the upper division lanceolate, upwardly arched, shorter than and coherent with the upper sepal; lower division bent downward, linear-attenuate and somewhat coiled at the tip : labellum deeply 3-parted ; the middle lobe ligulate, slightly thickened, 11 to 13 mm. long, obtuse ; lateral divisions linear-attenuate, 2.5 cm. long, more or less spirally coiled at the free ends : spur clavate, free, 4 to 4.5 cm. long : column bearing fleshy oblong appendages at the base : ovary narrowly winged. — MEXICO. State of Guerrero : mountains near Iguala, 15 September, 1900, *C. G. Pringle,* no. 9248 (hb. Gr.).

The affinity of this species is with *Habenaria jaliscana,* Watson, and *H. leucœcapensis,* Fernald. From the former it is readily distinguished by the height, the longer leaves, the longer spur, and by the entire absence of the falcate character of the lateral divisions of the labellum ; and from the latter it differs in having much longer and narrower leaves.

CORALLORHIZA INVOLUTA, Greenm., Proc. Am. Acad. xxxiii. 474. Specimens secured by Mr. C. G. Pringle on hills near Amozoc, State of Puebla, MEXICO, at an altitude of 2600 m., 10 September, 1901, no. 9632 (hb. Gr.), agree in all essential characters with the type of the above species, which was collected in the State of Oaxaca. The specimens obtained in Puebla, however, represent a more advanced stage of development, the plants being 3.5 dm. high, and the oldest flowers are reflexed, showing the immature 1.5 cm. long capsule.

Hexalectris mexicana. Glabrous and purplish throughout: stems 1 to 4.5 dm. high, bearing below 2 to 4 short sheathing bracts, simple and terminating in a racemose inflorescence: floral bracts ovate, 6 to 12 mm. long, short-acuminate, acute, one-half to one-third shorter than the ovary: flowers purple, at first erect-ascending, later reflexed; lateral sepals oblong-lanceolate, 12 to 14 mm. long, 4 to 6 mm. broad, obtuse, somewhat falcate, 5-nerved, upper sepal oblong-lanceolate, 14 to 18 mm. long, 4 to 5 mm. broad, obtuse; lateral petals oblong-oblanceolate, 12 to 16 mm. long, obtuse and slightly oblique: labellum obovate in general outline, rather deeply 3-lobed, abruptly narrowed at the base into a distinct claw, 5-nerved from the base, bearing a pair of partially united lamellæ on either side opposite the lateral sinus; lateral lobes obliquely ovate, obtuse; terminal lobe oblong-cuneate, 10 to 12 mm. long, rounded or retuse at the apex: immature capsule 12 to 16 mm. long, 8 to 10 mm. broad. *Hexalectris aphylla*, Hemsl. Biol. Cent.-Am. Bot. iii. 213 in part, not *Bletia aphylla*, Nutt.; Watson, Proc. Am. Acad. xviii. 159, excluding synonym. *H. aphyllus*, W. J. Robinson, Torreya. iii. 120 in part, as to plant of Schaffner. — MEXICO. State of San Luis Potosi: in mountains near San Miguelito, *Schaffner*, no. 316 (516?) (hb. Gr.); region of San Luis Potosi, altitude 1800–2400 m., *Parry & Palmer*, no. 861 (hb. Gr.). State of Coahuila: Sierra Madre, 6.5 kilometres south of Saltillo, July, 1880, *Palmer*, no. 2006 (hb. Gr.). State of Chihuahua: cañons of Sierra Madre, 5 October, 1887, *Pringle*, no. 1514 (hb. Gr.). State of Oaxaca: Cerro de Huauclilla, altitude 2500 m., 20 June, 1901, *Conzatti & González*, no. 1255 (hb. Gr.).

With the exception of the last, all the above cited specimens have been hitherto regarded as identical with *Hexalectris aphylla*, Raf., but all the Mexican material thus far examined by the writer proves to be quite distinct from that species. *Hexalectris mexicana* differs from *H. aphylla*, Raf., in its somewhat smaller average size, slightly smaller flowers, less conspicuously nerved parts of the perianth, shorter, smaller and more deeply 3-lobed labellum. The labellum, moreover, in *H. mexicana* bears only 2 to 4 lamellæ and these are opposite the lateral sinuses; while in *H. aphylla* the labellum is 5-(6-)lamellate from below the lateral sinuses, and the median fold is more or less continuous to the apex of the middle lobe.

BLETIA MACRISTHMOCHILA, Greenm., Proc. Am. Acad. xxxii, 297. Flowering and fruiting specimens of this attractive orchid were secured by Mr. C. G. Pringle during the past season, on banks of ravines near Guadalajara, MEXICO, no. 8640 (hb. Gr.). From Mr. Pringle's recently collected material it may be noted that the mature capsule is about 5 cm. in length and 1 cm. in diameter.

Celtis platycaulis. Fruticose: stems and branches flattened, covered with a reddish-brown bark, bearing single, curved, 5 to 6 mm. long spines: leaves ovate to obovate, 3 to 5 cm. long, 1.5 to 3 cm. broad, acute to rounded at the apex, usually serrate from near or just above the middle to the apex, entire in the lower half, rarely entire from base to apex, 3-nerved, in the very young stages puberulent, but soon becoming glabrous or essentially so; petioles 4 to 5 mm. long, and as well as the ultimate branches puberulent: the cymose inflorescences of the male flowers twice or more exceeding the petioles: fertile flowers few, borne at the tips of the branches; ovary puberulent; style branches bifurcated: mature fruit not seen. — MEXICO. State of Morelos: volcanic hills near Yantepec, altitude 1230 m., 5 July, 1901, *C. G. Pringle*, no. 8535 (hb. Gr.).

OXYBAPHUS VISCOSUS, L'Her., in Willd. Sp. i. 185; Choisy in DC. Prodr. xiii. 2, 430. Specimens collected by Mr. C. G. Pringle at Tehuacan, MEXICO, 1901, no. 8600 (hb. Gr.), agree in all important characters with typical representatives of the above species. In Mr. Pringle's specimens, however, the perigonium is considerably larger than is described and figured for *O. viscosus*, L'Her.; but this slight difference is supplemented by no other evident character, hence it seems best for the present at least to regard the Pringle plant as a mere form of the species here mentioned.

Ribes Dugesii. A much-branched shrub: stem covered with reddish-brown bark, glabrous; branchlets finely pubescent: leaves petiolate, ovate-rotund, shallowly 3–5-lobed, doubly crenate-serrate, 3–7-nerved from a cordate base, green and glabrous above, paler and somewhat glandular-pubescent beneath, especially on the prominent nerves; petioles 1 to 3 cm. long, and as well as the inflorescence minutely puberulent with stipitate glands intermixed, dilated at the base into a more or less glandular-ciliate expansion: racemes pedunculate, borne at the ends of young terminal branches and lateral shoots, 3 to 6 cm. long, suberect or somewhat drooping; pedicels 1 to 10 mm. long, jointed above, naked or bibracteate at the joint: flowers 1 cm. or less long: calyx subcampanulate, white or more or less roseate, 5-lobed for about one-half its length;

lobes oblong, 3 to 4.5 mm. long, slightly inflexed at the acutish tip, in the later stages recurved: petals subreniform, entire, 2 mm. long, equally broad, white: stamens as long as the petals; filaments about equalling the anthers: ovary glabrous: mature fruit not seen. — MEXICO. State of Guanajuato: mountains of Santa Rosa near the City of Guanajuato, March, 1901, and 1902, *Professor Alfredo Dugès*, no. 352*a* (hb. Gr.).

In the leaf-outline *R. Dugesii* suggests *R. viscosissimum*, Pursh, of western North America; but the two species have quite different flowers and pubescence. The affinity of *R. Dugesii* is evidently with *R. campanulatum*, Humb. & Bonpl. in Roem. & Schultes, Syst. v. 500 (1819); it differs, however, from this species in the character of the petioles, and in the absence of a hirsute pubescence on the under leaf-surface. From *R. ciliatum*, Humb. & Bonpl. l. c. (*R. jorullense*, HBK. Nov. Gen. & Sp. vi. 61) *R. Dugesii* is readily separated by the more rotund, less deeply and more obtusely lobed leaves.

Cassia longicoma. Stems herbaceous, erect or nearly so from a ligneous base, 1 to 3 dm. or more high, and as well as the petioles and peduncles pubescent with short reflexed hairs with which are intermixed long horizontally spreading pilose hairs: leaves 6-foliolate; petioles 1.5 to 3.5 cm. long; stipules linear-setaceous, 4 to 9 mm. in length, pubescent; leaflets short-petiolulate, oblong or obovate-oblong, more or less oblique, 1 to 4 cm. long, 0.5 to 2 cm. broad, rounded or obtusish at the apex and short-mucronate, entire, densely appressed-pubescent on both surfaces: peduncles axillary, 4 to 7.5 cm. long, mostly 3-flowered; pedicels about 1 cm. long: flowers nodding or erect, 1.5 to 2 cm. in diameter when fully expanded; sepals oblong, 8 mm. long, 3 mm. broad, obtuse or rounded at the apex, the outer surface pubescent; petals yellow, obovate, narrowed at the base into a short claw, rather conspicuously 3–5-nerved from the base; perfect stamens 7, subequal; staminodia 3; ovary densely sericeous-pubescent: legume 6 to 8 cm. long, 5 to 6 mm. broad, appressed-pubescent with somewhat scattered hairs: seeds ovate, short-acuminate, 3 to 4 mm. in length, slightly corrugate-pitted, and with a single central rather deep pit on either surface. — MEXICO. State of Morelos: Jojutla, altitude 1000 m., 12 June, 1901, and 30 August, 1902, *C. G. Pringle*, nos. 9674, 8665 (hb. Gr.).

Cassia morelensis. Fruticose: stem and branches covered with a grayish bark: ultimate branchlets terminating in a rather firm spine, glabrous or sparingly appressed-puberulent: leaves small, including the canaliculate subappressed-puberulent petiole 8 to 12 mm. long; leaflets 3–4-paired, short-petiolulate, obovate or oblong-obovate, 3 to 5 mm. long, two-thirds as broad, submucronate, essentially glabrous on both surfaces, or distinctly appressed-puberulent beneath: inflorescence paniculate or terminating the lateral branches in few-flowered corymbose clusters: flowers large, 5 cm. in diameter when fully expanded: sepals glabrous, strongly unequal, the larger about 1 cm. long, ovate-subrotund or slightly oblong, the smaller ovate-oblong, about 5 mm. in length, somewhat boat-shaped: petals yellow, obovate-oblong to distinctly oblong, 2.5 cm. long, 1.5 cm. broad, abruptly narrowed at the base into a 3 mm. long claw, 3-nerved from the base: stamens unequal: ovary glabrous, many-ovuled: legumes 2 dm. long, about 8 mm. wide, reticulate-veined, glabrous, somewhat granulose or dotted with minute blisters. — MEXICO. State of Morelos: near Jojutla, altitude 3000 feet, 12 May, 1901, *C. G. Pringle, no.* 9642 (hb. Gr.).

The affinity of the species here proposed is clearly with *C. Galeottiana*, Mart. & Gal., from which, however, it is distinguished by the armed character of the lateral branches, the smaller, nearly or quite glabrous leaflets, the appressed nature of the pubescence, and finally by the elongate-linear not oblong legume.

Desmodium Seatoni. *D. subsessile*, Seaton, Proc. Am. Acad. xxviii. 118 (1893), not Schlecht., in Linnæa, xii. 319.

Desmodium xylopodium. Stems several from a suffruticose base, ascending or suberect, 3 to 6 dm. in length: leaves unifoliolate; petioles 4 to 8 mm. long, and as well as the stem uncinulate-hispidulous; stipules obliquely ovate, acute, strongly nerved, caducous; the single terminal leaflet ovate to oblong, 8 to 27 mm. long, 5 to 15 mm. broad, rounded or retuse and often mucronate-apiculate at the apex, entire, slightly revolute-margined, rounded or subcordate at the base, nearly or quite glabrous above, prominently veined and slightly puberulent beneath; stipels subulate, persistent; petiolules scarcely more than 1 mm. in length: inflorescence racemose or paniculate, terminating the stem; pedicels slender, usually in pairs, 6 to 8 mm. long; bracts caducous: flowers small, about 5 mm. long, purple: legumes at first minutely puberulent, later becoming glabrous, 1–2-jointed; joints suborbicular or slightly oblong, 3 to 4 mm. long, nearly or quite as broad, rather strongly reticulate-veined. — MEXICO. State of Jalisco: near Guadalajara, altitude 1500 m., 13 May, 1901, *C. G. Pringle*, no. 9657 (hb. Gr.).

The affinity of this species seems to be with *Desmodium guadalajaranum*, Watson, from which it is readily distinguished by the smaller leaflets, and by the absence of spreading or subvillous hairs on the leaf-surface.

Croton flavescens. Fruticose, 2 to 2.5 m. high: stem branched, covered with a brownish bark; ultimate branches subangular-compressed, finely stellate-pubescent: leaves alternate, petiolate, ovate to somewhat rhombic-ovate, 4 to 13 cm. long, 3 to 7.5 cm. broad, subrotund to acuminate at the apex and terminated by a mucro, cordate to obtuse at the base, stellate-pubescent on both surfaces more densely so beneath, yellowish-green; petioles 1 cm. or less in length: inflorescence terminating the branchlets in monœcious spicate racemes, 5 to 6 cm. in length: pistillate flowers clustered at the base of the spike; calyx-divisions triangular-ovate, acute; style-branches deeply 2-parted, strongly recurved over the densely stellate-pubescent ovary: capsule nearly or quite 1 cm. long, 8 to 9 mm. in diameter, obtusely 3-angled, somewhat verrucose, closely stellate-pubescent; carpels slightly compressed above: seeds elliptic-oblong, 6 mm. long, 3 mm. broad, smooth; caruncle 1.5 mm. broad, 2-lobed. — MEXICO. State of Michoacan: volcanic hills, Monte Leon Station, altitude 1540 m., 29 August, 1902, *C. G. Pringle*, no. 8667 (hb. Gr.).

In general aspect *C. flavescens* resembles certain forms of *C. morifolius*, Willd., but it is readily distinguished by the longer less globose and more distinctly triangular capsule. The branches of the stellate hairs in *C. flavescens* are shorter than in *C. morifolius*, and the tomentum is more or less deciduous revealing the subverrucose character of the surface of the capsule.

Croton Gonzalezii. Fruticose: stem and branches covered with a grayish bark; branchlets cinereous-stellate-pubescent: leaves short-petiolate, ovate, 2 to 4 cm. long, one-half as broad, mostly acuminate, obtuse and short-mucronate, inconspicuously denticulate, rounded or subcordate at the base, at first minutely stellate-pubescent on both surfaces especially beneath, later glabrate, 3–5-nerved from the base; petioles 2 to 5 mm. long, densely stellate-pubescent: inflorescence terminating the stem and branches in monœcious 1.5 to 3 cm. long spicate racemes; bracts about 1 mm. long, acute, spreading and persistent: calyx at first stellate-pubescent, later glabrate: petals elliptic-oblong, narrowed and ciliate at the base, 1-nerved: stamens about 14; filaments glabrous: capsule 5 mm. long, closely stellate-pubescent. — MEXICO. State of Oaxaca: Cuicatlan, altitude 600 m., 16 September, 1899, *C. V. González*, no. 980 (hb. Gr.).

In general habit *Croton Gonzalezii* resembles *C. Sonoræ*, Torr. (*C. Pringlei*, Watson), but differs markedly in the finer stellate tomentum of leaves and fruit, and in the shorter petioles.

Acalypha oreopola. Stem branched, purplish, bifarious-puberulent: leaves petiolate, ovate, 1.5 to 5 cm. long, 1 to 3 cm. broad, acuminate, acute, shallowly serrate-dentate, short-cordate, glabrous upon both surfaces, pellucid-punctate, 3–7-nerved; petioles 0.5 to 3 cm. long, canaliculate, crisp-puberulent, purplish: inflorescence in terminal or axillary rather remotely flowered glabrous spikes: bracts of the fertile flowers 2 to 3 mm. long, glabrous and pellucid-punctate, 3–6-lobed with triangular-acute lobes, 1–3-flowered: calyx deeply 3-parted; divisions ovate, acute, sparingly pubescent: ovary hirsute above; styles pectinately divided: capsule triangularly and obtusely 3-lobed, about 2 mm. long, muricately hispid above: seeds about 1 mm. long, smooth. — MEXICO. State of Guerrero: mountains near Iguala, 10 October, 1900, *C. G. Pringle*, no. 9265 (hb. Gr.).

JATROPHA PSEUDO-CURCAS, Muell. Arg. in Linnæa, xxxiv. 208, and in DC. Prodr. xv. 2, 1080; Hemsl. Biol. Cent.-Am. Bot. iii. 108. Specimens agreeing well with the description of this species were collected by *C. Conzatti* and *V. González* at Cercanias de Oaxaca, State of Oaxaca, altitude 1550 m., June, 1901, no. 1207 (hb. Gr.). The leaves in the specimens secured by Señors Conzatti and González are somewhat larger than is ascribed to the species by Mueller, having in many cases leaves fully 2 dm. long and nearly or quite as broad. The fruit characters, which seem not to have been recorded hitherto, may be here presented, as follows: capsule oblong, 2 to 2.5 cm. long, 1 to 1.5 cm. in diameter, abruptly short-acuminate, subtriangular, smooth and glabrous: seed oblong-elliptic, rather conspicuously carunculate, smooth and shining.

Manihot caudata. A small tree, 4 to 5 m. high: stem covered with a brown or grayish-brown cortex: branches leafy at the summits: leaves alternate, petiolate, ovate-rotund in general outline, cordate, palmately and deeply 3–7-lobed, pale green and glabrous on both surfaces; lobes or divisions obovate to oblong, 3 to 9 cm. long, 2 to 4.5 cm. broad, abruptly and conspicuously caudate-acuminate, entire; the outer lower lobes more or less oblique and usually horizontally spreading; petioles 8 to 14 cm. long, terete or slightly canaliculate, and as well as the peduncles, pedicels and lower leaf-surface near the nerves more or less glaucous: racemes short, axillary, 5 to 6 cm. long in the fruiting stage; pedicels 1.5 to 3 cm. long: capsule 18 to 22 mm. in length: seeds oblong, flattened, 12 to 15 mm. long, 8 to 12 mm. broad, more or less mottled with brown or brownish-black subhorizontal markings; caruncle broader than long. — *Manihot* ———?, Watson, Proc. Amer. Acad. xxi. 440. —

MEXICO. State of Chihuahua: Hacienda San Miguel near Batopilas, September, 1885, *Dr. Edward Palmer*, no. 201 (hb. Gr.). State of Michoacan: volcanic hills, Monte Leon, 21 August, 1902, *C. G. Pringle*, no. 8687 (hb. Gr.).

EUPHORBIA CUMBRÆ, Boiss. Cent. Euph. 16, and in DC. Prodr. xv. 2, 48. Specimens agreeing in all details with this species, except that the glands of the involucre are either minutely appendaged or with the appendage wanting, were collected by Messrs. C. Conzatti and V. González in the State of Oaxaca, MEXICO, altitude 1900 m., July–August, 1900, no. 1040 (hb. Gr.).

Euphorbia (§ **Hypericifoliæ**) **rubida.** An herbaceous perennial: stems several, slender, 1.5 to 2.5 dm. long, ascending from a stout ligneous base, somewhat dichotomously branched, terete, reddish, crisp-puberulent: leaves opposite, short-petiolate, oblong, 5 to 23 mm. long, 2 to 8 mm. broad, rounded at the apex, sharply serrulate, oblique and subcordate at the base, sparingly pubescent on both surfaces with fine spreading hairs, often reddish, especially beneath; petioles 1 to 2 mm. long: inflorescence terminating the stem and branches in rather close cymules: involucre small, 1 to 1.5 mm. high; glands 4, bearing narrow crenate-margined appendages, these as well as the involucre mostly deep red in color: capsule obtusely triangular, 2 mm. long, equally broad, pubescent with somewhat spreading crisp-pilose hairs: seeds oblong-ovate, 1 mm. long, subquadrangular, grayish, slightly rugulose on the surfaces. — MEXICO. State of Mexico: dry ledges, Tultenango Cañon, 9 October, 1902, *C. G. Pringle*, no. 8673 (hb. Gr.).

E. rubida is perhaps most closely related to *E. Preslii*, Guss., but differs in having a distinctly perennial base, and in the pubescent character of the capsule.

CUPHEA ÆQUIPETALA, Cav., var. **epilosa.** Stems short-pubescent along decussating lines, not pilose-hispid. — MEXICO. State of Puebla: on hills near Amozoc, altitude 2500 m., 10 September, 1901, *C. G. Pringle*, no. 9628 (hb. Gr.). The variety *epilosa* differs from typical forms of the species in the absence of the purplish pilose-hispid hairs commonly found on the stem.

Erythræa micrantha. An erect, much-branched annual, 4 to 5 dm. high, glabrous throughout: stem and branches somewhat 4-angled: leaves sessile; the lower oblong-elliptic, 2 cm. long, 6 to 7 mm. broad, obtuse, entire, 3-nerved; the upper leaves similar or slightly spatulate, gradually reduced on the stem above: inflorescence terminating the dichotomously branched stem and branches in a compound cyme: pedicels

0.5 to 1.5 cm. long: flowers small, in anthesis 5 to 6 mm. long: calyx 2 to 3 mm. long; lobes lance-linear, acute: corolla purplish or purplish-crimson in the dried state, 5-lobed; lobes oblong-ovate or somewhat rotund, 2 mm. long, 1 to 1.5 mm. broad: both calyx and corolla somewhat elongating with the maturing fruit: capsule about 7 mm. long, twice as long as the persistent calyx. — MEXICO. State of Jalisco: wet soil, near Guadalajara, altitude 1540 m., 12 May, 1901, *C. G. Pringle*, no. 8482 (hb. Gr.).

Lachnostoma gonoloboides. Stem twining, densely pubescent with soft spreading or sometimes slightly reflexed hairs: leaves petiolate, oblong-ovate to lanceolate or somewhat halberd-shaped, 5 to 8 cm. long, 2 to 3.5 cm. broad, acuminate, acute, entire, cordate at the base with a broad open sinus, hirsute-pubescent on both surfaces, and especially on the prominent midrib and nerves beneath, dark green above, paler beneath; petioles 2 to 4 cm. long, pubescent: inflorescence axillary, umbellate; peduncles 12 to 15 mm. long; pedicels 2 to 3 cm. long: calyx divided nearly to the base; lobes oblong- to slightly rhombic-lanceolate, 8 to 12 mm. long, 3 to 4 mm. broad, obtuse or acutish, entire, externally hirsute-pubescent: corolla white, tubular or tubular-campanulate, 1.5 to 2 cm. high, pubescent in the throat; tube slightly exceeding the ovate obtuse externally pubescent greenish reticulate lobes: crown thickish, adnate to the corolla-tube, slightly crenate-margined: gynostegium 7 mm. high: fruit not seen. — MEXICO. State of Chihuahua: in the Sierra Madre, near Seven Star Mine, altitude 2450 m., 7 September, 1899, *Townsend & Barber*, no. 409 (hb. Gr.). A species with a distinctly gonolobus-like habit; but on account of the tubular-campanulate corolla, the elongated gynostegium, and the pendulous or somewhat oblique pollen-masses the plant is placed rather under the genus *Lachnostoma*.

IPOMŒA BARBATISEPALA, Gray, Syn. Fl. ii. pt. 1, 212. To this species are to be referred specimens collected in the State of Oaxaca, MEXICO, at an altitude of 1650 m., December, 1900, *Conzatti & González*, no. 1094 (hb. Gr.). The above species has been known hitherto only from the region of western Texas and Arizona.

IPOMŒA ORNITHOPODA, Robinson, Proc. Am. Acad. xxvii. 183. Specimens collected by Mr. C. G. Pringle in the Yantepec Valley, State of Morelos, MEXICO, altitude 1075 m., 17 June, 1901, no. 9645 (hb. Gr.), agree in all essential details with the original of the above species. The specimens from Yantepec Valley, however, show a considerable variation in the foliage, particularly as to the breadth of the leaf-lobes, which are frequently 1.5 cm. broad. To this species is also to be referred Dr. Edward Palmer's no. 781 (hb. Gr.), from Agiabampo, State of Sonora, collected in 1890.

CUSCUTA EPITHYMUM, Murr., in L. Syst., ed. 13, 140. This European species, collected in the Valley of Mexico, altitude 2250 m., 27 June, 1901, by *Mr. C. G. Pringle*, no. 8514 (hb. Gr.), seems not to have been hitherto reported from Mexico.

Phacelia madrensis. Perennial, subacaulescent, hirsute-pubescent throughout, not glandular-viscid: leaves mostly clustered at the end of an ascending or erect rootstock, pinnatifid, 1 to 4 dm. long; segments 7 to 9, obovate or cuneate, coarsely and rather deeply toothed, 8 to 22 mm. long, 5 to 15 mm. broad, terminal segment somewhat larger, the lowermost remote: inflorescence equalling or exceeding the leaves: flowers scattered, mostly remote, on pedicels 8 to 16 mm. long: calyx of 5 slightly unequal lanceolate, acute, or obtusish sepals: corolla campanulate, 10 to 12 mm. in diameter, pale blue or whitish; tube about equalling the broad semirotund lobes; folds of the corolla-tube not diverging at their free tips, puberulent along their margins: filaments hirsute-pubescent: ovary conspicuously hirsute, about 16-ovulate; style divided nearly to the base. — MEXICO. State of Chihuahua: Sierra Madre, near Colonia Garcia, altitude 2300 m., 2 July, 1899, *Townsend & Barber*, no. 100 (hb. Gr.).

The species here described belongs to the section *Eutoca*, and is apparently most nearly related to *Phacelia rupicola*, Rob. & Fern., and *P. pimpinelloides*, Gray. From the former it differs in the foliar characters, the shorter pubescence, smaller calyx-lobes, and finally by the approximate not divergent folds of the corolla-tube. From the latter species, namely, *P. pimpinelloides*, Gray, the species here proposed is readily distinguished by the spreading not appressed pubescence.

Nama parvifolium. Low depressed annual, 0.5 to 1.5 dm. high, cinereus-hirsute-pubescent throughout: stems prostrate or ascending, more or less dichotomously branched: leaves broadly spatulate, or somewhat oblong-spatulate, 0.5 to 1.5 cm. long, 3 to 6 mm. broad, obtuse, entire, contracted below into a narrowly winged petiolar base: flowers axillary, solitary or in pairs on unequal frequently curved slender 2 to 5 mm. long pedicels: calyx deeply 4–5-parted; divisions linear, acutish, 4 to 5 mm. in length, not dilated at the tip, hirsute-pubescent with hairs of unequal length: corolla 8 to 10 mm. long, white, or somewhat violet, subequally 5-lobed: ovary pubescent: mature capsule oblong, obtusish, 4 mm. long, pubescent. — *Nama rupicolum*, Pav., acc. to Hemsl. Biol.

Cent.-Am. Bot. ii. 362, in part. *N. rupicola*, Gray, Proc. Am. Acad. viii. 284, & xviii. 119, in part, not Bonpl. acc. to Choisy. *N. origanifolia,* Gray, Proc. Am. Acad. v. 337, in part, not HBK. *N. dichotoma*, var. *parvifolia*, Torr. Mex. Bound. 147. — MEXICO. State of Tamaulipas: de Santander à Victoria, *Berlandier*, nos. 834, 2254 (hb. Gr.); San Fernando to Jimeney, 26–27 February, 1902, *E. W. Nelson*, no. 6606 (hb. Gr., and hb. U. S. Nat. Mus.). State of Nuevo Leon: Monterey, *Eaton & Edwards* (hb. Gr.). State of Chihuahua: Santa Rosa, *Bigelow* (hb. Gr.). State of Coahuila: Soledad, *Palmer*, no. 2023 (Coll. of 1880, hb. Gr.); Valley of the Rio Grande, near Piedras Niegras, 19 April, 1900, *C. G. Pringle,* no. 9173 (hb. Gr.). Yucatan: without locality, *Schott* (hb. Gr.).

This species has been hitherto confused with *N. origanifolium*, HBK., and *N. rupicola*, Bonpl., which are both species with distinctly suffruticose stems, while *N. parvifolium* is an annual. From the former, namely, *N. origanifolium,* the species here proposed is readily distinguished by the broader leaves and larger flowers; and from the latter, namely, *N. rupicola,* by the cinereous-hirsute character of the leaves, not densely white-tomentose on the under surface, and by the larger flowers, lance-linear not spatulate calyx-divisions.

Cordia stellata. Shrub, much-branched; branchlets, as well as the foliage and inflorescence, densly covered with a canous-stellate tomentum: leaves petiolate, ovate, 2.5 to 10 cm. long, 1.5 to 8 cm. broad, obtuse, irregularly crenate-dentate, cuneate or subcordate at base, and the lamina usually short-decurrent on the petiole, densely canous-stellate-pubescent on both surfaces, paler and rather conspicuously reticule-nerved beneath; petioles 4 to 12 mm. long, channelled above, densely stellate-pubescent; inflorescence terminating the stem and branches in pedunculate spherical heads, 1.5 to 2 cm. in diameter; peduncles 2 to 4 cm. long: calyx obovoid, 5 to 6 mm. long, somewhat inflated, shortly and obtusely 5-dentate: corolla white, more or less salver-shaped, or subcampanulate; tube slightly exceeding the calyx; limb about 1 cm. in diameter, 5-lobed, strongly reticulate-veined. — MEXICO. State of Oaxaca: altitude 615 m., 27 August, 1894, *L. C. Smith,* no. 147 (hb. Gr.); Cuicatlan, altitude 600 m., 16 September, 1899, *V. González* no. 983 (hb. Gr.); about 9.5 kilometers above Dominguillo, altitude 640 m., October, 1894, *E. W. Nelson,* no. 1646 (hb. Gr., and hb. U. S. Nat. Mus.).

In general aspect *Cordia stellata* resembles *C. Pringlei,* Robinson, with which species it has been hitherto confused. The entire plant, however, is more canescent, the heads somewhat smaller, and the calyx and corolla shorter than in *C. Pringlei*. Moreover, the calyx-teeth in the species here proposed are not setaceous, but short and obtuse; on this character alone the two species can be readily separated.

Lippia chrysantha. Shrub, 1.5 to 2 m. high: stem much-branched, covered with grayish bark; ultimate branchlets hirsute-pubescent: leaves short-petiolate, lanceolate to lance-oblong, 2.5 to 8 cm. long, 1 to 3 cm. broad, acute or obtuse, finely and evenly crenate-dentate, cuneate or rounded at the base, rugose and hirsute-hispid above, soft-tomentose beneath, often showing tendency to become reflexed; petioles 5 mm. or less in length: inflorescence in axillary sessile crowded spikes, much shorter than the leaves, pubescent with spreading hairs intermixed with stipitate glands: floral bracts, lanceolate to somewhat oblanceolate, 6 to 7 mm. long, 2 to 2.5 mm. broad, acute: flowers solitary in the axils of the bracts, short-pedicellate: calyx 1 mm. long, 2-fid, pubescent with long pilose hairs; divisions entire or minutely 2-dentate: corolla 5 to 6 mm. long; tube slender, externally pubescent, gradually expanded above into a 4-lobed yellow or greenish-yellow limb: ovary and style glabrous. — MEXICO. State of Morelos: limestone hills near Yautepec, altitude 1385 m., 26 October, 1902, *C. G. Pringle*, no. 8679 (hb. Gr.).

A species well characterized by the sessile short inflorescences disposed in the axils of the upper leaves. *Lippia Chrysantha* is apparently most nearly related to *Lippia oaxacana*, Rob. & Greenm.

Stachys aristata. An herbaceous perennial: stem erect or ascending from a ligneous base, simple or branched, glabrous and more or less purplish below, puberulent above: leaves petiolate, triangular-lanceolate, 1.5 to 6 cm. long, 0.5 to 2.5 cm. broad, serrate-dentate, conspicuously aristate at the apex, cuneate to subcordate at the base and the lamina decurrent on the petiole, glabrous or slightly puberulent on both surfaces; petioles 0.5 to 4.5 cm. in length: verticillasters usually 6-flowered: calyx narrowly campanulate, puberulent; teeth subulate, nearly or quite equalling the calyx-tube: corolla light purple or whitish tinged with purple, somewhat maculate on the lower lip, about 1 cm. long, twice the length of the calyx; tube of the corolla projecting beyond the calyx; upper lip entire; lower lip 3-lobed: nutlets obovate, glabrous or slightly pubescent above. — MEXICO. State of Jalisco: on river-ledges, Falls of Guanacatlan, 6 August, 1902, *C. G. Pringle*, no. 8623 (hb. Gr.).

Stachys latipes. An erect or ascending herbaceous perennial: stem simple or branched, 2 to 3 dm. high, pubescent with scattered spreading stiff hairs: leaves oblong or slightly oblong-ovate, 1.5 to 3 cm. long, 1 to 1.5 cm. broad, obtuse, crenate-serrate, cuneate at the base into a flattish or narrowly winged ciliate petiole; petioles of the lower leaves nearly or quite equalling the blade, more or less continuous about the stem: verticillasters in the axils of the upper leaves, about 6-flowered: flowers 1 cm. long: calyx pubescent with spreading stiff hairs: corolla nearly twice exceeding the calyx, conspicuously 2-lipped; the 3-lobulate lower lip exceeding the galeate externally pubescent upper lip. — MEXICO. State of Chihuahua: in the Sierra Madre, near Colonia Garcia, altitude 2450 m., 17 July, 1899, *Townsend & Barber*, no. 128 (hb. Gr.).

A species resembling in general habit *S. agraria*, Cham. & Schlecht., from which, however, it is readily distinguished by the oblong cuneate leaves, and by the mostly spreading not reflexed hairs of the stem.

CAPSICUM FRUTESCENS, Linn., var. **lanicaule.** Stems more or less densely lanate-villous-pubescent: leaves ovate to ovate-lanceolate, 3 to 8 cm. long, 1 to 3.5 cm. broad, mostly villous-pubescent especially along the midrib and lateral veins beneath; petioles 0.5 to 3 cm. long: flowers solitary, axillary, nodding in anthesis: fruit oblong or oblong-lanceolate, 2 to 3 cm. long, 1 to 1.2 cm. in diameter. — MEXICO. State of Oaxaca: altitude 1550 m., 5 September, 1899, *V. González*, no. 975 (hb. Gr.). State of Jalisco: near Guadalajara, September, 1886, *Dr. Edward Palmer*, nos. 639, 640, 642 (hb. Gr.).

Señor González's specimens, on account of the copious pubescence of stem and foliage, seem at first sight to represent a distinct species, but the examination of a series of specimens representing *Capsicum frutescens*, L., shows very clearly that this species is sometimes rather conspicuously pubescent, hence it has seemed best for the present at least to regard the González plant as a variety to which may be referred, as somewhat less pubescent forms, the above numbers collected by Dr. Palmer.

BRACHISTUS PRINGLEI, Watson, Proc. Am. Acad. xxv. 159. Hereto should be referred specimens collected at Santa Rosa, Department of Santa Rosa, Guatemala, altitude about 1000 m., *Heyde & Lux*, nos. 3436, 4545 (exsic. John Donnell Smith), distributed as "*Capsicum frutescens*, L." The Guatemalan specimens here cited are somewhat less pubescent than in typical *B. Pringlei*, but in all essential characters they accord well with the original or type of that species. From descriptions of *B. diversifolius*, Miers, the *B. Pringlei* must be very closely related if indeed it does not represent the same species; but until a comparison of the original specimens of the two species can be made, it would not be wise to suggest more than a close relationship of the two described species.

Solanum polyadenium. Pubescent throughout with hirsute spreading hairs intermixed with densely crowded stipitate glands, heavy scented: tubers white: stems somewhat striate-angled: leaves pinnatisect, 5 to 12 cm. long, 4 to 8 cm. broad, usually auricled at the base by small subfalcate leaves of reduced axillary branches; segments 7 to 9, lance-oblong to ovate, 1.5 to 4 cm. long, 0.5 to 2 cm. broad, somewhat acuminate, obtuse, abruptly contracted below into an oblique subpetiolulate base; intermediate segments much smaller, very unequal, rarely more than a centimeter in length: inflorescence terminating the stem and branches in pedunculate false dichotomous more or less horizontally spreading cymes: flowers several: peduncles 1 to 2.5 cm. long, jointed: calyx 5-parted; segments sublanceolate to somewhat oblong, often abruptly contracted into an attenuated apex, persistent: corolla 5-angled, plicate, about 1 cm. high and 2 cm. broad, white: ovary and style glabrous: fruit conical-ovate, 10 to 13 mm. long, two-thirds to nearly as broad, glabrous. — MEXICO. State of Hidalgo: limestone hills, El Salto Station, 15 September, 1902, *C. G. Pringle*, no. 8692 (hb. Gr.).

Seymeria integrifolia. A much-branched annual, glabrous or slightly puberulent under a lens, somewhat viscid: leaves mostly opposite, linear to subulate-linear, 0.5 to 1.5 cm. long, about 1 mm. broad, entire, glabrous: flowers small, 7 to 8 mm. long, during anthesis nodding on slender naked recurved pedicels: calyx about one-half as long as the fully developed corolla, deeply 5-parted into 5 lance-linear acutish lobes: corolla glabrous; lobes ciliate. — MEXICO. State of Jalisco: on rocky hills near Guadalajara, 13 May, 1901, *C. G. Pringle*, no. 9660 (hb. Gr.).

STENANDRIUM BARBATUM, Torr. & Gray, Pacif. R. Rep. ii. 168, t. 4; Torr. Bot. Mex. Bound. 122. Specimens of this species were collected by Mr. C. G. Pringle on dry hills near Ciudad Juarez, State of Chihuahua, Mexico, at an altitude of 1140 m., 5 May, 1901, no. 8501 (hb. Gr.). Although this species is well known from Texas and New Mexico, through the collections of *Wright*, *Thurber*, *Pope*, *Lemmon*, and *Vasey*, yet it does not seem to have been hitherto reported from Mexico.

Anisacanthus Gonzalezii. Suffruticose: stem much-branched, below covered with a grayish bark, above striate and finely appressed-sericeous-pubescent to nearly glabrous: leaves opposite, short-petiolate, ovate-lanceolate, 2 to 4 cm. long, 0.5 to 1.5 cm. broad, acuminate, obtuse, entire, glabrous above, usually pubescent along the midrib and lateral veins beneath: inflorescence terminating the stem and branches in leafy-bracted racemose cymes: flowers 1 to 3 in the axils of the bracts: calyx 5-parted to below the middle; tube glabrous; lobes lance-attenuate, 4 to 5 mm. long, ciliate, otherwise glabrous: corolla scarlet, subcylindrical, 3 to 3.5 cm. long, 2-lipped to about one third its length, pubescent on the outer surface: ovary glabrous. — MEXICO. State of Oaxaca: Cuesta de Quiotepec, altitude 1200 m., 3 August, 1895, *L. C. Smith*, no. 695 (hb. Gr.); near Oaxaca City, *Conzatti & González*, June, 1900, no. 1231 (hb. Gr.).

The specimens collected by Mr. L. C. Smith, in the Gray Herbarium, were referred to *Anisacanthus virgularis*, Nees, from which species, however, *A. Gonzalezii* differs in having the pubescence of the stem scattered over the entire surface instead of restricted to definite lines, usually broader leaves which are pubescent along the midrib beneath instead of glabrous, and finally by having a glabrous instead of usually glandular calyx.

Dicliptera inequalis. Stem hirsute-pubescent, with glandular hairs intermixed especially throughout the inflorescence: leaves petiolate, ovate, 3 to 8 cm. long, 1 to 5 cm. broad, acuminate, acute, entire, narrowed at the base into the 0.5 to 2 cm. long petiole, hirsute-pubescent on both surfaces, somewhat paler beneath: inflorescence an elongated racemose-paniculate cyme: the flowers in small heads or floral clusters, terminating the primary and secondary branches and either subsessile or on peduncles 3 cm. or less in length: outer bracts of the involucre, unequal, lanceolate, acute; the inner bracts linear-lanceolate, acute, exceeding the 5-parted calyx: divisions of the calyx lanceolate, acute, about 3 mm. long, longer than the tube: corolla 12 to 20 mm. long, purplish or dark red; tube about twice exceeding the lip: fruit not seen. — MEXICO. State of Morelos: at Cuantla, near Cuernavaca, altitude 1250 m., 30 May, 1901, *C. G. Pringle*, no. 9665 (hb. Gr.).

Dicliptera nervata. Stem and branches obscurely hexagonal, sparingly pubescent: leaves ovate, including the petiole 3 to 6 cm. long, 1 to 2.5 cm. broad, short-acuminate, obtuse or acute, entire, narrowed at the base into a rather short petiole, sparingly pubescent on both surfaces: inflorescence in densely crowded axillary verticillasters: involucre 1-flowered; the two outer bracts of the involucre obovate to obovate-rotund, about 1 cm. long and two-thirds to equally broad, retuse and not unfrequently apiculate, entire, ciliate, pubescent with a few scattered hairs on the outer surface, glabrous on the inner surface, rather conspicuously nerved; the two inner bracts of the involucre narrowly lanceolate, slightly exceeding the deeply 5-parted calyx: divisions of the calyx lance-attenuate, inconspicuously ciliate: corolla about 1 cm. long, divided to near the middle: capsule 5 mm. in length, sparingly pubescent: seeds somewhat hispid over the entire surface. — MEXICO. State of Morelos: Cuernavaca, 8 January, 1866, *Bilimek*, no. 344; Cuantla, near Cuernavaca, altitude 1230 m., 31 May, 1901, *C. G. Pringle*, no. 9664 (hb. Gr.).

Jacobinia mollis. Fruticose: ultimate branches densely hirsute-pubescent: leaves petiolate, ovate to ovate-lanceolate, 2 to 9 cm. long, 1 to 4 cm. broad, short-acuminate or little narrowed toward the apex, obtuse or acute with a small mucro, entire or slightly repand, narrowed below into a 0.5 to 1.5 cm. long pubescent petiole, ciliate, dark green and hirsute-pubescent above, paler and subtomentose beneath: inflorescence in short axillary pubescent racemes, shorter than the leaves; bracts and bracteoles triangular-acute, shorter than the 5 to 6 mm. long, closely puberulent calyx; calyx-tube 1.5 mm. long, the posterior lobe much smaller than the other lobes, subulate: corolla 3.5 to 3.7 cm. long, purplish, 2-lipped for about one-half its length, pubescent on the outer surface; upper lip entire or slightly emarginate; lower or anterior lip shortly 3-lobed: style puberulent: mature capsule not seen. — MEXICO. State of Oaxaca: June, 1901, *Conzatti & González*, no. 1229 (hb. Gr.).

Jacobinia oaxacana. Fruticose: stem below covered with a grayish bark; branches and branchlets pubescent at the nodes and along descending lines between the nodes: leaves petiolate, ovate to ovate-lanceolate, 3 to 10 cm. long, 1.5 to 4 cm. broad, usually acuminate, obtusish or apiculate-acute, entire or slightly crenate, ciliate, green and glabrous above except on the nerves, paler and densely pilose on the conspicuous midrib and lateral nerves beneath, narrowed below into a ciliate 0.5 to 2 cm. long petiole; cistoliths lineate; inflorescence in axillary pedunculate few-flowered racemes, mostly shorter than the leaves; bracts and bracteoles small, triangular, acute, shorter than the calyx: flowers sessile: calyx deeply 5-parted, externally puberulent; tube 1 to 1.5 mm. long; lobes lanceolate, acute, the antero- and postero-lateral lobes about 4 mm. long, equal; the posterior lobe much smaller, subulate: corolla 3 cm. long, purple; tube equalling or slightly exceeding the limb; upper posterior lip entire or slightly emarginate, the lower anterior oblong, with short ciliated lobes: style puberulent: mature fruit not seen. — MEXICO. State of Oaxaca: June, 1901, *Conzatti & González*, no. 1230 (hb. Gr.).

RONDELETIA LEUCOPHYLLA, HBK., var. **calycosa.** Habit and foliar characters of the species: inflorescence much contracted, subcapitate; bracts 7 to 12 mm. long, 3 mm. or less broad, acute: calyx-lobes subequal, subfoliaceous, lance-oblong, 6 to 8 mm. long, 1 to 2 mm. broad, obtuse or obtusish. — MEXICO. State of Oaxaca: altitude 1650 m., December, 1900, *Conzatti & González*, no. 1127 (hb. Gr.); City of Oaxaca, altitude 1550 m., *Conzatti*, no. 970 (hb. Gr.). No differences other than the broader bracts of the inflorescence and the larger subfoliaceous calyx-lobes are found to distinguish the variety from the species.

The variety *calycosa* is cultivated in the Botanical Garden of the City of Oaxaca where it grows as a very graceful and decorative shrub.

Bouvardia Conzattii. Shrub: stem and branches covered with a grayish bark; ultimate branchlets smooth and glabrous or minutely puberulent: leaves opposite, ovate to ovate-lanceolate, 1.5 to 4 cm. long, 0.5 to 2 cm. broad, obtuse to subacuminate-acute, entire, slightly hirtellous on the margins, yellowish-green and puberulent above, paler and sparingly puberulent on the midrib and obliquely arched conspicuous lateral veins beneath, abruptly narrowed at the base into short densely puberulent 2 mm. long petioles; stipules lanceolate, 3 to 7 mm. long, lacerate-dentate: inflorescence terminating the stem and branches in close rather few-flowered corymbose cymes; bracts subulate, and as well as the unequal linear-acute 3 to 5 mm. long calyx-lobes, hirtellous-puberulent especially along the margins: corolla red or yellowish-red, 13 to 15 mm. long; tube cylindrical, glabrous on the outer surface, bearing within a ring of hairs one-third its length from the base; lobes short, ovate, acute, about 2 mm. long: stamens included; filaments adnate to the corolla-tube two-thirds its length from the base; anthers subsessile: style glabrous: mature fruit not seen. — MEXICO. State of Oaxaca: altitude 1750 m., July–August, 1900, *Conzatti & González*, no. 1076 (hb. Gr.).

Pæderia Pringlei. Stem woody, twining, 6 m. or more in length; ultimate branches subcompressed, glabrous: leaves opposite, petiolate, ovate or slightly ovate-oblong, 2.5 to 5 cm. long, 2 to 3 cm. broad, usually short-acuminate, acute, at first sparingly pubescent above, later more or less glabrate, hirsute-tomentulose beneath especially along the midrib and lateral nerves: flowers not seen: fruit elliptic-oblong, 10 to 12 mm. long, 6 to 8 mm. broad, smooth and glabrous, bearing above the persistent calyx-teeth; pyrenes separating by, and suspended on a filiform carpophore.—MEXICO. State of Morelos: lava fields near Yantepec, altitude 1385 m., 22 October, 1902, *C. G. Pringle*, no. 8682 (hb. Gr.).

Vernonia vernicosa, Klatt, var. **comosa.** Stem striate, densely pubescent: leaves pubescent on both surfaces with subappressed-hirsute hairs, somewhat glabrate above: other characters of the species.—Costa Rica. Buissons à Matambér près Nicoya, January, 1900, *Ad. Tonduz,* no. 13,607 (hb. Gr., and hb. Inst. Phys.-Geog. Cost. Ri.).

Eupatoriastrum, n. gen. of *Compositæ* (*Agerateæ*). Heads homogamous, many-flowered. Involucre campanulate; bracts 2–3-seriate. Receptacle convex, chaff-bearing throughout. Corollas equal, regular, shallowly 5-toothed. Pappus of numerous minutely barbellate setæ, disposed in a single row. Achenes linear-oblong, 5-angled. Leaves opposite, petiolate, palmately nerved. Inflorescence in terminal leafy panicles.

E. Nelsonii. Stem striate, slightly puberulent, somewhat stramineous in color: leaves petiolate, remote, broadly ovate, 1 to 2 dm. or more long, 4 to 16 cm. broad, 5–7-nerved from or just above the base, acute or short-acuminate, denticulate to shallowly dentate, rounded at the base, conspicuously netted-veined, sparingly pubescent with scattered crisp hirsutish hairs on both surfaces; petioles 1 to 3 cm. long: heads 1 to 1.5 cm. high, on axillary pedunculate and terminal clusters forming a leafy panicle: involucral scales lance-oblong, acute, externally pubescent: receptacle chaffy throughout with narrow pales slightly broadened toward the tip: corolla a little constricted above the slightly ampliated tube: pappus about equalling the corolla: achenes hirtellous-puberulent along the angles. — Mexico. State of Oaxaca: Santa Efigenia, altitude 150 m., 18 July, 1895, *E. W. Nelson,* no. 2827 (hb. Gr., and hb. U. S. Nat. Mus.).

In habit and in the character of the paleaceous receptacle this plant resembles *Schistocarpha,* but the style-branches are distinctly those of the subtribe *Agerateæ,* and closely resemble the style-branches in *Eupatorium* near which genus it is provisionally placed.

Stevia collodes. Shrub: ultimate branches reddish, puberulent: leaves linear, or linear-lanceolate, 1.5 to 3.5 cm. long, 1.5 to 3 mm. broad, obtuse, entire, glabrous on both surfaces: inflorescence terminating the stems and branches in few-headed umbellate-cymes: heads about 12 mm. high: involucral scales linear, 8 to 9 mm. long, acuminate, acute, obtusely keeled on the back and, as well as the leaves and young branches, glandular-viscid: flowers much exceeding the involucre: pappus coroniform, consisting of minute scales: corolla white, or slightly reddish-tinged, with a long slender glandular tube, and a rather deeply 5-lobed limb: achenes about 4 mm. long, slightly puberulent along the angles.—

Mexico. State of Puebla: calcareous hills near Tehuacan, altitude 1725 m., 22 August, 1901, *C. G. Pringle*, no. 8570 (hb. Gr.).

Archætogeron purpurascens. An herbaceous perennial, pubescent throughout with mostly appressed-hirsute hairs: stem erect or ascending, simple or branched, 2 to 4 dm. high: lower leaves oblanceolate, 2 to 6 cm. long, 3 to 10 mm. broad, obtuse, subentire to rather conspicuously toothed with remote spreading teeth, the uppermost leaves lanceolate and entire: heads terminating the stem and branches on rather long peduncles, including the rays about 2.5 cm. broad; involucral scales 2–3-seriate, narrowly lanceolate, acuminate, appressed-pubescent: ray-flowers in about 3 series, numerous, deep purple: disk-flowers numerous; pappus coroniform, of united more or less lacerated scales. — Mexico. State of Chihuahua: Sierra Madres, near Colonia Garcia, altitude 2450 m., 27 July, 1899, *Townsend* & *Barber*, no. 175 (hb. Gr.).

Erigeron adenophorus. Suffruticose: stem branched; branches hirsute, and more or less glandular-pubescent, leafy towards the tip: leaves sessile, crowded, amplexicaul, at first erect or spreading, later reflexed, linear-lanceolate, 1 to 2 cm. long, 2 to 5 mm. broad, mucronate-acute, sparingly dentate with minute spreading subcartilaginous teeth, densely glandular-pubescent intermixed with long jointed flaccid hairs; margins revolute: inflorescence terminating the stem and branches in dense round-topped corymbose cymes: heads about 1 cm. high: involucre campanulate; bracts 3–4-seriate, lance-linear, acute, the outer densely pubescent with long jointed subflaccid and somewhat matted hairs, the inner terminated by a glabrous purplish tip: ray-flowers mostly 2-seriate; achenes strongly compressed, with thick nerviform margins and a single nerve on either surface, puberulent; corolla-tube slender, puberulent, slightly ampliated and expanded above into a purplish minute inconspicuous ligule, or remaining essentially tubular; style included: disk-flowers numerous; the 5-toothed limb of the corolla deep purple at least in the dried state; achenes like those of the ray-flowers. — *Senecio eriocephalus*, Klatt, Bull. Soc. Bot. Belg. xxxi. 212 (1892), and Leopoldina xxxi. (1895) Beiblatt, p. 7. — Costa Rica. Entre le Rancho del Jaboncillal et le sommet de Cerro de Buenavista, et les roches, altitude 3100 m., 19 January, 1891, *H. Pittier*, no. 3425 (hb. Gr.).

A careful examination of the type of *Senecio eriocephalus*, Klatt, shows very clearly that the plant is widely remote from *Senecio;* its affinity is rather with *Erigeron* and *Conyza.* Because of the comparatively small number of ray-flowers and the character of the same, namely the included style, and the distinctly nerviform margins of the compressed achene, it is placed under *Erigeron*. Owing to the several discrepancies between Klatt's published description and the original specimen an entirely new characterization is here drawn.

ERIGERON REPENS, Gray, Syn. Fl. i. pt. 2, 17. Specimens collected by Mr. C. G. Pringle on sand dunes near the City of Vera Cruz, Mexico, 22 June, 1901, no. 9673 (hb. Gr.), are clearly referable to the above species, hitherto known only from the sandy sea-coast of Texas and Louisiana.

Gnaphalium Altamiranum. Densely lanate-tomentose throughout: stem 1.5 to 2 m. high, much-branched above: leaves sessile, auriculate-clasping, lanceolate, 8 to 16 cm. long, 0.5 to 1.5 cm. broad, acuminate, acute, entire, 3-nerved, broadest near the base: inflorescence a many-headed compound corymbose cyme: heads during anthesis about 6 mm. high, nearly or quite as broad: involucre campanulate, usually 5-seriate; scales of the involucre whitish or pale-stramineous, the outer broadly ovate, obtusish, the inner narrowly oblong, obtuse: flowers numerous: perfect flowers about 10. — MEXICO. State of Morelos: mountains above Cuernavaca, altitude 2460 m., 2 February, 1899, *C. G. Pringle*, no. 8041 (hb. Gr.).

The species here proposed is allied to *Gnaphalium Wrightii*, Gray, but differs in having larger leaves, which are relatively broad at the base, and gradually narrowed to the tip, also by somewhat larger and more numerously flowered heads, and broader outer involucral scales.

Gnaphalium chartaceum. Stem erect, 1 to nearly 2 m. high, freely branching, glandular-hirsute: leaves sessile, round-auricled, half-clasping the stem, lanceolate to somewhat oblong-lanceolate, 3.5 to 8 cm. long, 0.5 to 2 cm. broad, acute to acuminate-acute, entire or slightly undulate-margined, hirsute or glandular-hirsute and somewhat arachnoid in the early stages, especially on the under surface, hirsute on the midrib beneath: inflorescence a terminal compound many-headed corymbose cyme: heads 5 to 6 mm. high: involucre subcampanulate; scales imbricated in 4 series, ovate, obtuse or obtusish, almost pure white or somewhat cream-colored, firm or chartaceous in texture: perfect flowers 15 to 25; pistillate flowers numerous. — MEXICO. State of Jalisco: on hillsides and banks near Guadalajara, 22 November, 1888, *C. G. Pringle*, no. 1827 (hb. Gr.), distributed as "*Gnaphalium decurrens*, Ives, var." State of Puebla: near Tehuacan, 22 December, 1895, *C. G. Pringle*, no. 7033 (hb. Gr.).

G. chartaceum is readily distinguished from *G. decurrens*, Ives, with which species it has hitherto been confused, by the sessile auriculate half-clasping rather than decurrent leaves, the less lanate-tomentose nature of the under leaf-surface, the more glandular character of stem and foliage, and finally by the chartaceous, essentially white involucral scales.

Gnaphalium crenatum. Stem herbaceous, erect, 3 to 4 dm. high, branching from the base, leafy throughout, terete, white-tomentose with glandular-hirtellous hairs intermixed: leaves sessile, semiamplexicaul and often somewhat decurrent, spatulate to linear-lanceolate, 2.5 to 5 cm. long, 2 to 5 mm. broad, acute or mucronate-acute, more or less arachnoid and densely glandular-hirtellous above, lanate-tomentose beneath; margins revolute and mostly crenate: inflorescence a terminal corymbose cyme: heads about 5 mm. high, disposed in rather dense glomerules: involucre pale-stramineous, 4–5-seriate; scales of the involucre lanceolate, tomentose at the base and greenish along the median line, the outer acuminate, acute, the inner obtusish. — MEXICO. State of Jalisco: near Guadalajara, *C. G. Pringle*, no. 9524 (hb. Gr.).

The affinity of this species is apparently with *G. gracile*, and *G. stramineum*, HBK.

Gnaphalium jaliscense. Lanate-tomentose or densely arachnoid throughout: stem 5 to 7 dm. high, branched from near the base; branches arcuate-ascending, slender, usually simple: leaves narrowly oblanceolate to lance-attenuate, 3 to 8 cm. long, 2 to 8 mm. broad, acute, entire, commonly revolute-margined, short-decurrent: heads about 7 mm. high, disposed in few- to several-headed glomerules at the tip of stem and branches: involucre campanulate at least in the dried state; scales of the involucre white, narrowly ovate to elliptic-oblong, 3 to 5 mm. long, 1 to 2 mm. broad, acute or submucronate: flowers numerous: perfect flowers 20 to 30. — MEXICO. State of Jalisco: Guadalajara, July, 1886, *Dr. Edward Palmer*, no. 256 (hb. Gr.); gravelly soil near Guadalajara, 9 August, 1902, *C. G. Pringle*, no. 8628 (hb. Gr.).

Dr. Palmer's plant here cited was placed by the late Dr. Sereno Watson with *Gnaphalium semiamplexicaule*, DC., but with that species it has little in common. *G. jaliscense* differs from *G. semiamplexicaule* in having decurrent rather than half-clasping leaves, in having larger and more numerously flowered heads; and the scales of the involucre are more pointed and more numerously seriate.

Gnaphalium oaxacanum. Stem and branches densely white lanate-tomentose: basal leaves not seen: stem-leaves sessile, shortly decurrent, lance-attenuate, 2 to 5 cm. long, 2 to 6 mm. broad, entire, revolute-margined, arachnoid-tomentose above, densely white lanate-tomentose beneath, in the later stages becoming more or less recurved:

inflorescence corymbose; heads numerous, small, 3 mm. long, disposed in glomerules: involucral scales about 3-seriate, pale-stramineous, imbricated; the outer scales ovate, obtuse, brownish especially at the base, the inner linear-oblong or slightly oblanceolate, acutish: pistillate flowers about 14: perfect flowers few, usually two. — MEXICO. State of Oaxaca: altitude, 1750 m., — July–August, 1900, *Conzatti & González*, no. 1012 in part (hb. Gr.).

Gnaphalium oblanceolatum. Densely white lanate-tomentose throughout: stem erect, 4 to 5 dm. high, ligneous at the base, simple or branched, leafy: leaves sessile by a half-clasping base, oblanceolate, 2 to 7 cm. long, 0.5 to 1.5 cm. broad, entire, white lanate-tomentose on both surfaces or arachnoid above, also bearing on the upper surface crowded crisp hirsutish hairs: inflorescence terminating the stem and branches in a leafy corymbose panicle: heads crowded: involucre about 5 mm. high; scales imbricated, acute, pale yellow. — MEXICO. State of Oaxaca: Cerro de San Felipe, altitude 1800 m., 21 August, and 1st September, 1897, *Conzatti & González*, nos. 385 and 696 (hb. Gr.).

The species here proposed is described from young flowering specimens. In general appearance it approaches most closely *G. pannosum*, Gray, from which, however, it differs in the more leafy stem and in the oblanceolate leaf-outline.

GNAPHALIUM PURPUREUM, L., var. **macrophyllum.** Stem 6 dm. high, lanate-tomentose: leaves oblanceolate or oblanceolate-spatulate, 0.5 to 1.5 dm. long, 1 to 1.5 cm. broad, obtuse or submucronate-acute, arachnoid-pubescent above in the early stages and later glabrate, densely and permanently white-tomentose beneath: inflorescence racemose-paniculate: involucral scales brownish or with a green base and a rich brown tip. — COSTA RICA. El Copey, altitude 1800 m., February, 1898, *Ad. Tonduz*, no. 11,771 (hb. Gr., and hb. Inst. Phys.-Geog. Cost. Ri.).

It is quite possible that the variety here proposed may prove eventually to represent a distinct species, but in the present confused state of the genus, particularly the subtropical species, it seems to the writer best to regard the above plant as a variety of *G. purpureum*, L., especially as its chief distinguishing characters are the size of the plant, the very large leaves, and the paniculate inflorescence.

Clibadium anceps. Small tree, much-branched: ultimate branches somewhat flattened, decussately 2-edged, minutely strigose-pubescent: leaves opposite, petiolate, ovate, 8 to 12 cm. long, 4 to 6 cm. broad, acuminate, acute, remotely denticulate, more or less revolute-margined, cuneate at the base, hirtellous-pubescent upon both surfaces, glabrate and somewhat shining above, conspicuously 3–5-nerved: petioles 1.5 to 3 cm. long, and as well as the stem strigose-pubescent: inflorescence a terminal compound corymbose cyme: heads small, about 5 mm. high, disposed in glomerules: involucral bracts ovate, obtuse, appressed-cinereous-pubescent, the outer shorter: ray-flowers mostly 3: mature achenes obovate, about 2 mm. long, glabrous: sterile flowers few, 3 to 5. — COSTA RICA. Forêts de La Palina, altitude 1459 m., 8 September, 1898, *Ad. Tonduz*, no. 12,537 (hb. Gr., and hb. Inst. Phys.-Geog. Cost. Ri.).

CLIBADIUM CARACASANUM, DC. Prodr. vi. 506. To this species the following collections may be referred: VENEZUELA. Tovar, *Fendler*, no. 640 (hb. Gr.). PANAMA. Gatun Station, *Sutton Hayes*, no. 89 (hb. Gr.). COSTA RICA. Cañas Gordas, *H. Pittier*, no. 11,068 (hb. Gr., and hb. Inst. Phys.-Geog. Cost. Ri.).

Clibadium glomeratum. Stem stout, densely soft-tomentose: leaves opposite, long-petiolate, ovate, 1 to 2 dm. long, 0.5 to 1.5 dm. broad, acute, crenately and somewhat unequally dentate, obtuse to subcordate at the base, scabrous-pubescent above, densely soft-pubescent beneath, 3-nerved from a point considerably above the base below which are two or three pairs of less conspicuous lateral nerves: inflorescence terminating the stem and branches in compound corymbose pubescent cymes: heads small, few-flowered, about 5 mm. high, disposed in dense glomerules 5–8 (–10) mm. in diameter: involucral scales oblong or ovate-oblong, 2 to 3 mm. long, glabrous, ciliate: ray-flowers mostly 5, in 1 to 2 series: mature achenes narrowly obovate, about 2 mm. long, glabrous below, villous-pubescent above. — COSTA RICA. Forêts de Luis, altitude 650 m., November, 1897, *Ad. Tonduz*, no. 11,508 (hb. Gr., and hb. Inst. Phys.-Geog. Cost. Ri.).

Clibadium Pittieri. Stem terete, appressed-pubescent: leaves opposite, petiolate, ovate, 1 to 1.5 dm. long, 4 to 8 cm. broad, acuminate, acute, dentate above the middle, entire and narrowed below into the 1 to 6 cm. long petioles, hirsute-pubescent upon both surfaces, 3–5-nerved from just above the base: inflorescence terminating the stem and branches in corymbose clusters: heads about 5 mm. high: involucre of narrowly ovate acute strigose-pubescent bracts: ray-flowers several in about three series: mature achenes obovate, 2 mm. long, glabrous at the base, pubescent at the apex. — COSTA RICA. La Florida voie ferrée Atlantique, altitude 80 m., July, 1897, *H. Pittier*, no. 11,290 (hb. Gr., and hb. Inst. Phys.-Geog. Cost. Ri.). In general habit this species approaches most closely *C. surinamense*, L., but is readily distinguished from it by the conspicuous hirsute pubescence of the upper leaf-surface, the narrower and less striate involucral scales, and finally by the more numerous fertile flowers. From *C. fragiferum*, Griseb., which also has numerous disk-flowers, *C. Pittieri* differs in the more spreading pubescence of the leaves and in the less conspicuous pales of the receptacle.

Trigonospermum floribundum. Stem 2 to 3 m. high, purplish, striate, at first tawny puberulent later glabrate; branches and the many-headed compound corymbose cyme rather densely stipitate-glandular: leaves opposite, petiolate, broadly ovate, 0.5 to 1.8 dm. long, 2 to 14 cm. broad, acute, unequally cartilaginous-dentate, cuneate at the base into 1 to 5 cm. long petioles, densely subappressed-hispidulous-pubescent above, hirsute-tomentose beneath especially on the midrib and nerves, resiniferous-dotted on the surface, 3-nerved from just above the base: heads many, about 5 mm. high, 12 to 18 mm. in diameter including the rays: outer bracts of the involucre linear or linear-oblong, 4 mm. long, obtuse; inner involucral bracts broadly obovate, closely clasping the achene, appressed-puberulent: rays 5 to 8, yellow, 6 to 8 mm. long, somewhat fan-shaped, unequally dentate: mature achenes obovoid-triangular, smooth and shining. — MEXICO. State of Morelos: cliffs of the Sierra de Tepoxtlan, near Cuernavaca, altitude 2285–3000 m., 4 November, 1901, and 27 November, 1902, *C. G. Pringle*, nos. 9179, 8686 (hb. Gr.).

Polymnia Nelsonii. Stem striate, sparingly hirsute with horizontally spreading jointed hairs: leaves opposite, including the winged clasping petiole 1.5 to 2 dm. long, 1.2 to 1.5 dm. broad, mostly pinnately 5-lobed, thin, dark green above, paler beneath, sparingly pubescent on both surfaces; lobes ovate or rhombic-ovate, acuminate; margins remotely denticulate; petioles broadly winged, 0.8 to 2 cm. in breadth: heads about 3 cm. in diameter, on slender axillary 7 to 10 mm. long peduncles: involucre biseriate; bracts of the involucre oblong-lanceolate, 8 to 10 mm. long, sparingly hirsute, ciliate: ray-flowers 10 to 12; rays yellow. — MEXICO. State of Chiapas: near Santa Christobal, altitude 2150 to 2450 m., 18 September, 1895, *E. W. Nelson*, no. 3221*a* (hb. Gr., & hb. U. S. Nat. Mus.).

In general *P. Nelsonii* most closely resembles *P. oaxacana*, Schz. Bip. especially in foliar characters, but the yellow rays and the thin or membranous leaves and the sparing pubescence of the same readily distinguish it.

Rumfordia polymnioides. Stem purplish, striate, hirsute-pubescent: leaves opposite, subperfoliate, ovate, 0.5 to 1.5 dm. long, 2.5 to 8 cm. broad, acuminate, mucronate-acute, somewhat sinuate-dentate and bearing on the margins subcartilaginous mucronulations, abruptly narrowed below the middle into a winged petiole continuous about the stem, pubescent on both surfaces, 3-nerved, paler and conspicuously reticulate-veined beneath: inflorescence a terminal flat-topped cymose corymb: heads 1 to 1.5 cm. high, 2 to 2.5 cm. in diameter when fully expanded: involucre 2-seriate; outer scales of the involucre foliaceous, ovate, 1 to 1.5 cm. long, two-thirds as broad, acute or slightly acuminate-acute, pubescent with crisp hirsutish hairs; inner scales of the involucre smaller, externally pubescent with glandular and hirsute hairs intermixed, more or less surrounding the glabrous ray-achenes: ray-flowers subbiseriate; rays small, inconspicuous, yellow: disk-flowers numerous; corolla-tube long, slender, externally pubescent, ampliated above into an elongated campanulate limb: achenes smooth and glabrous, slightly obovate-oblique, and more or less angular. — COSTA RICA. Plantations de maïs du Copey, April, 1898, *Ad. Tonduz*, no. 11,947 (hb. Gr., & hb. Inst. Phys.-Geog. Cost. Ri.).

In general habit this species closely resembles the genus *Polymnia*, and more particularly *P. edulis*, Wedd., but the involucral characters and the perfect disk-flowers place it readily in the genus *Rumfordia*.

MONTANOA MACROLEPIS, Rob. & Greenm. Proc. Am. Acad. xxxii. 44, & xxxiv. 512. *Polymnia Liebmannii*, Schz. Bip. acc. to Klatt in Leopoldina xxiii. 89 (reprint, p. 2). A fragment and a good drawing of the original or type of *Polymnia Liebmannii*, collected by Liebmann at Cumbre de Estepa, Mexico, show very distinctly that the plant has its affinity rather with *Montanoa* than with *Polymnia*, and moreover is identical with the above species *Montanoa macrolepis*, Rob. & Greenm.

GYMNOLOMIA MICROCEPHALA, Less., Linnæa, v. 153. In the original description of this species it is stated that the heads are discoid, but an examination of the type specimen shows the heads to be distinctly radiate. A comparison of the type of *Gymnolomia patens*, Gray, with the original of *G. microcephala*, Less., also shows that the two plants are absolutely identical. The combination made by Dr. Gray thus becomes a synonym, and the varieties now under *G. patens* should be transferred to *G. microcephala*, Less., hence the synonymy briefly stated is as follows:

GYMNOLOMIA MICROCEPHALA, Less., l. c. *G. patens*, Gray, Proc. Am. Acad. v. 182; Hemsl. Biol. Cent.-Am. Bot. ii. 163; Rob. & Greenm., Proc. Bost. Soc. Nat. Hist. xxix. 94, incl. synonymy.

Var. **abbreviata**, Rob. & Greemn. *G. patens*, var. *abbreviata*, Rob. & Greenm., Proc. Am. Acad. xxix. 387; Proc. Bost. Soc. Nat. Hist. xxix. 94.

Var. **guatemalensis**, Rob. & Greenm. G. *patens*, var. *guatemalensis*, Rob. & Greenm., Proc. Bost. Soc. Nat. Hist. l. c.

Var. **brachypoda**, Rob. & Greenm. *G. patens*, var. *brachypoda*, Rob. & Greenm., l. c. 95.

Var. **macrophylla**, Rob. & Greenm. *G. patens*, var. *macrophylla*, Rob. &. Greenm. l. c.

Gymnolomia Pittieri. Stem somewhat striate, reddish-brown; the younger branches densely short-hirtellous-pubescent, with few scattered hirsutish hairs intermixed: leaves ovate-lanceolate, 4 to 8 cm. long, 1 to 4 cm. broad, acuminate, acute, crenate-serrate, narrowed below into a subpetiolate base, hirsute-hispid above, paler and densely pubescent beneath, 3-nerved from just above the base: heads, including the rays, 5 to 6 cm. in diameter, terminating the stem and branches on upwardly thickened conspicuously striate short-hirtellous-pubescent peduncles: involucre 3-seriate; outer bracts pubescent, shorter than the oblong somewhat petaloid inner ones: ray-flowers conspicuous; rays narrowly oblong, 2 to 2.5 cm. long, 0.5 cm. broad; ray-achenes triangular, glabrous, neutral: disk-flowers numerous; corolla short-tubular, abruptly expanded into a tubular-campanulate limb, externally pubescent: achenes of the disk-flowers glabrous. *G. platylepis*, Rob. & Greenm., Proc. Bost. Soc. Nat. Hist. xxix. 102, in part. — COSTA RICA. Défrichements le long du Rio Giliri, pres de San José, 5 December, 1890, *H. Pittier*, no. 3136 in part (hb. Gr); bords du Rio Ceibo à Buenos Aires, altitude 200 m., February, 1891, *H. Pittier*, no. 3735 (hb. Gr.); Ujarras de Buenas Aires, February, 1897, *H. Pittier*, no. 10, 631 (hb. Gr., and hb. Inst. Phys.-Geog. Cost. Ri.).

This species has hitherto been confused with *Gymnolomia platylepis*, Gray, from which, however, it is well distinguished by the smaller leaves and heads, and by the character of the pubescence. The stem and branches of *G. platylepis* are usually pubescent with long spreading pilose or pilose-hirsute hairs, while *G. Pittieri* is densely pubescent with short somewhat hirtellous pubescence.

Zaluzania Pringlei. Shrub: stems covered with a grayish bark; ultimate branches striate, appressed-puberulent, purplish: leaves alternate, petiolate, ovate, 3.5 to 5 cm. long, 2 to 3.5 cm. broad, short-acuminate, obtusish or submucronate-acute, subentire to somewhat crenate-denate, cuneate to subcordate at the base, slightly hispidulous above, puberulent beneath especially on the midrib and nerves; petioles slender, canaliculate, appressed-puberulent, 1 to 1.5 cm. in length: inflorescence a terminal round-topped paniculately corymbose cyme:

heads ovate, 6 to 7 mm. high, discoid: involucre campanulate; bracts of the involucre about 3-seriate, lance-oblong, acutish, appressed-canescent-puberulent: achenes glabrous. — Mexico. State of Morelos: limestone hills near Jojutla, altitude 925 m., 18 October, 1902, *C. G. Pringle*, no. 8710 (hb. Gr.).

The species here proposed is most nearly related to *Zaluzania discoidea*, Gray, but is readily distinguished by the smaller leaves, more slender petioles; the slight puberulence is not dense and soft-canous-tomentose on the lower leaf-surface.

Aspilia (?) **aggregata.** Perennial: stem terete, covered with a grayish bark; ultimate branches, upwardly appressed-hispid: leaves opposite, oblong-oblanceolate, 2 to 4 cm. long, 0.5 to 1 cm. broad, acute, entire or inconspicuously denticulate, revolute-margined, narrowed below into a subpetiolate base, tuberculate-hispid above and on the midrib and nerves beneath: heads discoid, about 1 cm. high, 0.5 cm. in diameter, aggregated at the ends of long slender almost naked peduncles: involucres cylindrico-campanulate, 8–10-flowered; bracts of the involucre 4–5-seriate, ovate, short-acuminate and acute to oblong and obtuse or rounded at the apex, more or less strigose-hispid on the outer surface; the outer or lowermost bracts shorter and not unfrequently slightly herbaceous, the inner purplish or somewhat magenta-colored, noticeably ciliate toward the apex with brownish hairs: corollas glabrous; tube slender, gradually ampliated above into a 5-dentate limb: achenes subterete or slightly compressed laterally, constricted above and bearing a lacerate-margined cup-shaped persistent pappus: mature achenes 2 to 3 mm. long, hispidulous under a lens. — Mexico. State of Jalisco: between Balaños and Guadalajara, 20 September, 1897, *Dr. J. N. Rose*, no. 3029 (hb. Gr., and hb. U. S. Nat. Mus.)

The plant here described has, as far as examined, constantly discoid heads, and in the character of achene and pappus might be equally well referred to either *Wedelia* or *Aspilia.* In habit, however, it seems to approach rather more closely the latter genus, and in specific relationship suggesting the Brazilian species *Aspilia floribunda*, Baker.

Aspilia stenophylla. An herbaceous perennial: stems erect, 5 to 7 dm. high, slender much-branched, appressed-hirtellous-pubescent, in the dried state reddish: leaves lanceolate or linear, 8 to 12 cm. long, 3 to 10 mm. broad, acute, entire or inconspicuously denticulate, slightly revolute-margined, subsessile or narrowed at the base into a short petiole, appressed-hispidulous-pubescent on both surfaces: heads about 1 cm. high, including the rays 1.5 to 1.8 cm. in diameter, terminating the stems and branches, usually on long slender peduncles: involucre 3-seriate; bracts of the involucre narrowly ovate-lanceolate, about 8 mm. long, acute, appressed-pubescent; the outer sometimes a little shorter; the innermost more oblong and thinner: ray-flowers neutral, few, 5 to 8, yellow: disk-flowers 20 to 30; pales of the receptacle somewhat keeled, and purplish along the median line: achenes pubescent, maculate with purple-magenta patches. — MEXICO. State of Chihuahua: in the Sierra Madre, near Seven Star Mine, altitude 2450 m., 28 August, 1899, *Townsend & Barber*, no. 380 (hb. Gr.). This species is nearly allied to *Aspilia angustifolia*, Gray, but it differs in the longer, less pubescent leaves, and in the more acuminate outer bracts of the involucre.

Viguiera brevifolia. A much-branched shrub, 1 m. or less in height: stems terete, covered with a grayish bark; ultimate branches canous-strigose: leaves opposite below, alternate above, lanceolate-triangular to somewhat ovate, acute to obtuse, entire or subcrenate and revolute-margined, mostly abruptly narrowed at the base into a short petiole, densely appressed-canous-pubescent on both surfaces, especially beneath and on the short petioles: heads 1 cm. or less high, terminating the stem and branches, on long slender peduncles, radiate, including the rays about 1.5 cm. in diameter: involucre campanulate; bracts of the involucre 2–3-seriate, lanceolate to ovate, acute, about 4 mm. long, externally canous-appressed-pubescent: ray-flowers 8 to 10; rays oblong, 6 to 7 mm. long, yellow: disk-flowers numerous, 50 to 60: pappus of two slender aristæ, equalling or slightly exceeding the intermediate lacerated scales: mature achenes 3 mm. long, sparingly pubescent. — MEXICO. State of Durango: Mapimi, 21 to 23 October, 1898, *Dr. Edward Palmer*, no. 527 (hb. Gr., and hb. U. S. Nat. Mus.).

In general habit this species most resembles the Lower Californian *Viguiera microphylla*, Vasey & Rose, but is readily distinguished from it by the character of the inflorescence.

VIGUIERA CORDIFOLIA, Gray, var. **latisquama.** Habit, cauline, foliar and inflorescence characters of the type of the species: involucral bracts imbricated, 3-seriate, linear-oblong, 2 to 3.5 mm. broad, somewhat rounded or very abruptly narrowed at the tip and terminated by a mucro; the outer bracts shorter than the inner ones: floral and achenial characters of the species proper. — MEXICO. State of Durango: vicinity of the City of Durango, September, 1896, *Dr. Edward Palmer*, no. 667, and no. 747 (hb. Gr.).

A very distinct variety, and possibly worthy of specific rank.

Viguiera eriophora. Stem somewhat ligneous, covered with a brownish bark; branches and branchlets reddish-brown, striate, more or less compressed, tomentulose, lanate at the nodes as well as on the petioles with long woolly hairs: leaves mostly opposite, ovate to ovate-lanceolate, including the petiole 5 to 16.5 cm. long, 1 to 8 cm. broad, acuminate, acute, coarsely and irregularly dentate, abruptly contracted at the base and more or less decurrent on the petiole, tuberculate-hispid above, hirsute-pubescent beneath, 3-nerved, rather prominently reticulate-veined on the under surface: inflorescence a terminal many-headed sub-corymbose cyme; peduncles 0.5 to 3 cm. in length, hirsute-pubescent: heads large, including the bright yellow rays fully 4 cm. in diameter, 1 to 1.5 cm. high; involucral scales 3–4-seriate, lance-oblong, short-acuminate, acute, tomentulose to nearly or quite glabrous, conspicuously ciliate, the outer scales somewhat shorter: ray-flowers 6 to 8: disk-flowers numerous: achenes densely appressed-sericeous-villous. — MEXICO. State of Oaxaca: hills of Telixtlahuaca, altitude 2000 m., 18 October, 1895, *Rev. Lucius C. Smith*, no. 971 (hb. Gr.); without locality, 21 October, 1899, *E. W. D. Holway*, no. 3689 (hb. Gr.), July–August, 1900, *Conzatti & González*, no. 987 (hb. Gr.).

In general appearance *Virguiera eriophora* resembles certain species of *Helianthi*, but in the technical characters of the head it is distinctly a *Viguiera*.

Viguiera Goldmanii. Stem erect, strict, striate, brownish, glabrous or slightly puberulent above: leaves opposite, subsessile, oblong-lanceolate, 5 to 8 cm. long, 1 to 2.5 cm. broad, acuminate, acute, entire or inconspicuously denticulate, subcordate at the base, sparingly hispidulous on both surfaces, distinctly 3-nerved: inflorescence a loose di-trichotomously branched corymbose cyme: heads 10 to 12 mm. high, radiate, including the rays about 2.5 cm. in diameter: involucre subcylindrical, slightly shorter than the flowers of the disk: bracts of the involucre 4–5-seriate, ovate-oblong to typically oblong, obtuse to rounded at the apex, yellowish green, glabrous and striate, short-ciliate: ray-flowers commonly 8; rays oblong, about 1 cm. long: pappus of two paleo-aristate awns, much longer than the intermediate lacerated scales: achenes densely appressed-sericeous-pubescent. — MEXICO. State of Durango: Chalco, altitude about 1000 m., 7 March, 1899, *E. A. Goldman*, no. 359 (hb. Gr.).

A species resembling *Viguiera montana*, Rose, but readily distinguished by the more rounded and less ciliated involucral scales, and by the less attenuated foliage.

Viguiera hypargyrea. Stem terete, substriate, upwardly appressed-pubescent: leaves alternate, sessile or subsessile, lanceolate, acuminate, acute, 2.5 to 6 cm. long, 0.5 to 1.5 cm. broad, entire, rounded or obtuse at the base, indistinctly 3-nerved, appressed-hirsute above, densely soft-sericeous beneath: heads solitary, terminating the stems, 1.5 to 2 cm. high, excluding the rays 2 to nearly 3 cm. broad at least in the later stages: involucre campanulate, 2–3-seriate; bracts of involucre imbricated, lance-oblong, acute, externally sericeous-pubescent, the outer bracts a little shorter than the inner: ray-flowers neutral; rays oblong, 12 mm. long, 4 to 5 mm. broad: disk-flowers numerous: corollas 6 mm. long, abruptly and conspicuously ampliated above the short pubescent tube: mature achenes about 5 mm. long, densely appressed-sericeous-pubescent; pappus of two subpaleaceous awns with intermediate erose scales, persistent. — MEXICO. State of Durango: City of Durango and vicinity, April to November, 1896, *Dr. Edward Palmer*, no. 816 (hb. Gr., and hb. U. S. Nat. Mus.).

Viguiera microcephala. Stems herbaceous, 1 to 2 m. high, slender, erect, much-branched, striate, reddish-brown, strigose-pubescent: leaves alternate, at least the upper ones, petiolate, ovate, 3 to 6 cm. long, 1 to 3.5 cm. broad, acuminate, acute, somewhat sinuate-dentate with more or less spreading teeth, cuneate to subcordate at the base, appressed-pubescent on both surfaces; petioles 12 mm. or less in length, canous-strigose: heads small, 6 to 7 mm. high, radiate, including the rays 1 cm. or less broad, disposed in few-headed cymes at the ends of the slender branchlets: involucre 1–2-seriate; bracts lanceolate or narrowly ovate-acuminate, 4 to 5 mm. long, appressed-pubescent: ray-flowers 5, neutral; rays yellow, broadly ovate or subrotund, 3 to 4 mm. long, nearly or quite as broad, 4-nerved: disk-flowers 12 to 14: pappus of 2 aristæ with 2 intermediate laciniate scales on either side of the achene: achenes thickish, sub-4-angled, 2 mm. long, at first appressed-silky-villous, later glabrous or nearly so. — MEXICO. State of Morelos: limestone hills near Yantepec, altitude 1385 m., 7 November, 1902, *C. G. Pringle*, no. 8717 (hb. Gr.).

In general aspect *V. microcephala* suggests *V. tenuis*, Gray, and *V. helianthoides*, HBK., but the heads are more numerous and smaller than in either of these species.

Viguiera Rosei. Stems densely white-lanate-tomentose: leaves alternate, sessile, half clasping the stem by a broad auriculate base, oblong-lanceolate, attenuate, 1 to 2.2 dm. or more long, 1.5 to 6 cm. broad, acute, entire to remotely denticulate, conspicuously reticulate-veined and appressed-hirsute-pubescent above, rather densely soft-tomentose beneath: heads large, about 2 cm. high, 4 to 5 cm. broad, terminating the stem and branches on 2 to 6 cm. long slightly thickened upwardly densely white-lanate peduncles: involucre campanulate; bracts of the involucre 2–3-seriate, lance-attenuate, 3 cm. or less in length, about 5 cm. in breadth, more or less foliaceous and recurved at the tip, sericeous-pubescent: ray-flowers about 16, neutral; rays 12 to 15 mm. long, 5 mm. broad: disk-flowers numerous: pappus of 1 or 2 unequal subpaleaceous deciduous awns and shorter deeply lacerated intermediate persistent scales: corollas apparently roseate or somewhat pink in color: mature achenes about 6 mm. long, densely appressed-pubescent. — MEXICO. State of Zacatecas: near Plateado on the road from Colatlan, 31 August, 1897, *Dr. J. N. Rose*, no. 2710 (hb. Gr., and hb. U. S. Nat. Mus.).

Notwithstanding the superficial resemblance this plant bears to the genus *Tithonia*, on account of the robust habit and large heads, the real affinity seems to be with the genus *Viguiera*, and is closely related to *V. decurrens*, Gray, and *V. excelsa*, Benth. & Hook. The writer takes pleasure in dedicating the species to its collector, Dr. J. N. Rose of the U. S. National Museum, Washington, D. C.

Altamirania, n. gen. of *Compositæ* (*Verbesineæ*). Heads heterogamous, radiate. Involucre campanulate; bracts 2–3-seriate. Receptacle convex or conical; pales of the receptacle conduplicate, keeled. Ray-flowers uniseriate, fertile; achenes triangular. Corollas of the disk-flowers regular; tube short, ampliated above into an elongated-cylindrical 5-toothed limb. Anthers minutely dentate at the base, appendaged at the apex. Style-branches obtuse. Achenes of the disk-flowers laterally compressed-4-angled. Pappus of 2 to 3 paleaceous awns, with 1 to 3 lacerated scales between, persistent. Erect suffruticose (?) plants with opposite leaves and terminal cymose inflorescence.

A. pachyphylla. An erect suffruticose (?) plant, glabrous throughout, or slightly puberulent on the peduncles: stem striate, purplish: leaves lanceolate or somewhat oblanceolate, 5 to 12 cm. long, 1.5 to 3.5 cm. broad, acute or acuminate-acute, dentate, narrowed below to a subpetiolate base, smooth on both surfaces, dark- or somewhat olive-green above, paler beneath: heads 1 cm. or less high, including the rays 3 to 4 cm. in diameter, disposed in terminal compound corymbose cymes: involucral scales linear, obtusish, slightly thickened and more or less spreading: ray-flowers 8 to 10; rays narrowly oblong about 1.5 cm. long, 3 mm. broad: disk-flowers numerous; the paleaceous awns about one-half as long as the corolla; achenes tapering toward the base, ciliolate on the margins, glabrous on the surface, at maturity 3 to 4 mm. in length. ? *Aspilia pachyphylla*, Klatt, Leopoldina, xxiii. 143 (reprint, p. 4). — MEXICO. State of Oaxaca: near Totontepec, altitude 1125 to 1625 m., 15 to 28 July, 1894, *E. W. Nelson*, no 829 (hb. Gr., and hb. U. S. Nat. Mus.).

But for the fertile ray-flowers and the diverse habit the plant here described might be referred to the genus *Viguiera*. From *Encelia* it differs in the character of the ray-flowers, and in the distinctly 4-angled achenes of the disk-flowers. From *Verbesina* it differs in the achenial characters, namely, in being 4-angled and not distinctly winged, and also in the nature of the pappus. Although bearing a certain affinity with the three genera here mentioned, it differs in the important characters enumerated, hence it seems best to give it generic rank with a position in the natural sequence near *Viguiera*.

The genus is dedicated to Dr. Fernando Altamirano, Professor at the School of Medicine, and Director of the National Medical Institute, City of Mexico.

Helianthus oaxacanus. Stems striate, terete and glabrous below, subangled and minutely appressed-puberulent above, especially in the inflorescence: leaves alternate (at least the upper ones), petiolate, broadly ovate, acute, 4 to 14 cm. long, 2 to 8 cm. broad, dentate to subentire, often revolute-margined, rounded or abruptly narrowed at the base into a 1 to 3.5 cm. long petiole, glabrous or essentially so on both surfaces, 3-nerved: inflorescence a terminal comparatively few-headed leafy paniculate cyme: heads large, 1.5 to 2 cm. high, including the rays 4 to 5 cm. in diameter: involucre campanulate; bracts of the involucre 2–3-seriate, imbricated, lanceolate, acuminate, acute; the outer bracts shorter than the inner ones; ray-flowers 12 to 15, neutral; rays yellow, oblong, 1.5 cm. or more long, 6 to 7 mm. broad: disk-flowers numerous: pappus of two rather large unequal paleaceous awns with smaller intermediate persistent erose scales: achenes appressed-sericeous pubescent. — MEXICO. Between Huajuapam, State of Oaxaca, and Retlatzingo, State of Puebla, altitude 1450 to 2000 m., 19 November, 1894, *E. W. Nelson*, no. 1985 (hb. Gr., and hb. U. S. Nat. Mus.).

HELIANTHUS PETIOLARIS, Nutt. Jour. Acad. Phil. ii. 115. Specimens referable to this species were collected by *Townsend & Barber* in the Sierra Madre near Rio de Aroz, State of Chihuahua, Mexico, altitude 2300 m., 3 September, 1899, no. 403 (hb. Gr.). The specimens at hand agree well with the more northern representatives of the species, except for the somewhat narrower involucral bracts in which character they approach the variety *canescens*.

HELIANTHUS PETIOLARIS, Nutt., var. CANESCENS, Gray, Pl. Wright. i. 108; Syn. Fl. i. pt. 2, 272. Mr. E. W. Nelson collected near Casas Grandes, State of Chihuahua, Mexico, 30 August, 1899, specimens under no. 6328 (hb. Gr., and hb. U. S. Nat. Mus.), which, except for a slightly more pronounced dentation of the leaves, accord well with the type of the above variety.

The locality not only extends considerably the geographical range of the variety *canescens*, but adds another *Helianthus* not hitherto recorded from Mexico.

Perymenium macrocephalum. Stout herbaceous perennial, pubescent throughout with more or less spreading stiff hairs; stem and branches mottled with reddish-brown patches: leaves opposite, petiolate, ovate, 5 to 10 cm. long, 3 to 7 cm. broad, acuminate, acute, rather remotely and shallowly dentate, rounded or cuneate at the base, hispid above, paler and hirsute-hispid on the conspicuously reticulated veins beneath; petioles 0.5 to 2.5 cm. long, narrowly winged by a continuous prolongation of the leaf-blade: heads few, on long peduncles 1 to 10 cm. in length; scales of the involucre 3–4-seriate, ovate to oblong, obtuse, externally appressed-pubescent, ciliate, the outer shorter: ray-flowers about 13, fertile; rays yellow: disk-flowers numerous: mature achenes 4 to 5 mm. long, somewhat roughened on either surface: pales of the receptacle conspicuous, especially in the later stages. — MEXICO. State of Guerrero: limestone mountains above Iguala, altitude 1230 m., 27 September, 1900, *C. G. Pringle*, no. 8416 (hb. Gr.). This species suggests in some respects *Montanoa macrolepis*, Rob. & Greenm., but in the technical characters it is quite distinct.

PERYMENIUM RUDE, Rob. & Greenm., Proc. Am. Acad. xxxiv. 526. Specimens collected near Tequila, State of Jalisco, Mexico, 5–6 July, 1899, by *Dr. J. N. Rose & Mr. Walter Hough*, no. 4749 (hb. Gr., and hb. U. S. Nat. Mus.), agree in all essential details with the original specimen, and with the first published description of this species. In the above cited number, however, the leaves are often 4 cm. in length, and 3 cm. in breadth, thus somewhat larger than was indicated in the original description.

SPILANTHES DISCIFORMIS, Rob., var. **phaneractis.** Stems ascending, 2.5 to 8 dm. long, rooting at the lower nodes: leaves narrowly lanceolate, 8 cm. or less in length, 8 mm. in breadth: peduncles slender, 7 to 13 cm. long: rays conspicuous, sulphur-yellow, 5 to 6 mm. long, 2 to 3 mm. broad. — MEXICO. State of Michoacan: swamps, Zamora, altitude 1540 m., 1901, and 24 July, 1902, *C. G. Pringle*, nos. 9539, 8637 (hb. Gr.).

Spilanthes macrophylla. Stems erect or nearly so, 2.5 to 6 dm. high, striate, sparingly pubescent with spreading hirsute hairs, glabrate: leaves opposite, petiolate, ovate-lanceolate, 5 to 15 cm. long including the petiole, 1.5 to 4 cm. wide, acuminate, acute, entire to rather conspicuously sinuate-dentate, narrowed at the base into a slender ciliate petiole, glabrous on both surfaces or with a few scattered hirsute hairs on the midrib and veins beneath, membranous: heads radiate, 8 to 10 mm. high, on long axillary and terminal somewhat sulcate sparingly subappressed-hirsute 2 to 15 cm. long peduncles: involucre, 2-seriate; scales of the involucre linear-lanceolate, 4 to 7 mm. long, acute, externally hirsute-pubescent: ray-flowers about 10; rays yellow: disk-flowers numerous: achenes about 2 mm. long, bearing two slender pappus-awns, ciliate on the margins, sparingly appressed-pubescent on both surfaces. — COSTA RICA. Bord du Rio del Volcan, 2 February, 1891, *H. Pittier*, no. 3717 (hb. Gr.); Salutre de San Murios, altitude 1355 m., *Ad. Tonduz*, no. 7831 (hb. Gr.); Alto del Sacatæ, Cerro de Buena Vista, January, 1897, *H. Pittier*, no. 10,546 (hb. Gr., and hb. Inst. Phys.-Geog. Cost. Ri.). *Pittier's* no. 3717 and *Tonduz's* no. 7831, both of the Pittier and Durand, "*Plantæ costaricensis exiccatæ*," were referred by Klatt to *Spilanthes Mutisii*, HBK., a South American species from which, however, the Costa Rican plant differs materially in the character of the involucral scales.

SPILANTHES URENS, Jacq. Am. Stirp. 214, t. 126, f. 1; HBK. Nov. Gen. & Sp. iv. 211; DC. Prodr. v. 625. Specimens collected at Altata, State of Sinaloa, Mexico, by *Dr. J. N. Rose*, 15 June, 1897, without number (hb. Gr., and hb. U. S. Nat. Mus.), accord in all important details with the above maritime species, which has not been recorded hitherto from Mexico.

Encelia adenophora. Stout herb, 1 to 2.5 m. high, pubescent throughout with stipitate glands intermixed with horizontally spreading hirsute hairs: lower leaves opposite, petiolate, ovate to deltoid-ovate, 5 to 12 cm. long, 3 to 10 cm. broad, more or less acuminate, acute, rather coarsely and somewhat unequally dentate, 3-nerved from or just above the base; petioles 6 cm. or less in length, naked; upper leaves gradually smaller, shorter-petioled, becoming lanceolate-oblong, more acuminate and remotely dentate to subentire: inflorescence a subcorymbose, or paniculate cyme; peduncles 0.5 to 7 cm. long: heads 12 to 17 mm. high, 1.5 to 2.5 cm. in diameter: involucre campanulate; bracts of the involucre lance-attenuate, 10 to 12 mm. long, subequal or the outer bracts slightly exceeding the inner: ray-flowers about 12; rays pale yellow, 8 mm. long, one-half as broad, longer than the slender sterile achene: disk-flowers numerous, about 50; corollas pale yellow or sometimes slightly tinged with purple: mature achenes 5 mm. long, 3 mm. broad. — MEXICO. State of Jalisco: fields and copses, Tequila, September–October, 1893, *C. G. Pringle,* no. 4602 (hb. Gr.). State of Guerrero: between Tlapa and Tlaliscatilla, altitude 1200 to 1400 m., 5 December, 1894, *E. W. Nelson,* no. 2045 (hb. Gr., and hb. U. S. Nat. Mus.). State of Oaxaca: hills of Soledad de Etla, altitude 1850 m., 19 November, 1895, *L. C. Smith*, no. 894 (hb. Gr.).

This species has hitherto been confused with *Encelia mexicana*, Mart., and Mr. Pringle's specimen above cited was distributed as "*Encelia mexicana*, Mart., var." From that species, however, *E. adenophora* is amply distinct, being readily separated by the pale yellow color of the entire plant, the larger, more numerously flowered heads, the attenuated involucral scales, and finally by the copious stipitate-glandular hairs of stem, leaves and inflorescence.

Encelia angustifolia. An herbaceous perennial: stem simple, erect from a ligneous base, 3 to 4 dm. high, glabrous, striate, more or less purplish: leaves alternate or scattered, linear or linear-lanceolate, 1.5 to 8 cm. long, 1.5 to 3 mm. broad, acute or acutish, remotely and inconspicuously serrulate, appressed-hirtellous above, glabrous beneath, 3-nerved; nerves rather prominent especially on the lower leaf-surface: heads radiate, about 1 cm. high, excluding the rays about 6 mm. in diameter, terminating the stems on slender peduncles: involucre 1–2-seriate; bracts of the involucre lance-linear, subequal, shorter than the flowers of the disk: ray-flowers about 5; rays narrowly oblong, 10 to 12 mm. long, yellow: disk-flowers comparatively few: achenes strongly compressed laterally, ciliate-margined. — MEXICO. Territory of Tepic: in the Sierra Madre, 13 August, 1897, *Dr. J. N. Rose*, no. 3453 (hb. Gr., and hb. U. S. Nat. Mus.).

A very characteristic species, which on account of the narrow attenuated leaves and the few heads, is easily distinguished from all the described species of the genus.

Encelia collodes. Suffruticose: stem covered with a brownish bark: branches striate, or striate-angled puberulent or essentially glabrous: leaves alternate, petiolate, lanceolate or oblong-lanceolate, 5 to 12 cm. long, 1.5 to 5 cm. broad, acuminate, acute, entire, narrowed at the base into a 0.5 to 1.5 cm. long petiole, glabrous above, puberulent on the prominent midrib and nerves beneath, firm in texture: inflorescence a terminal few-headed subcorymbose cyme: heads rather large, pedunculate, 1.5 to 2 cm. high, 5 cm. in diameter: involucre about 4-seriate; bracts of the involucre lance-oblong, acute to obtuse, thickish, striate, externally somewhat tawny pubescent and as well as the corollas and pales of the receptacle more or less glutinous: ray-flowers 12 to 16, conspicuous: disk-flowers numerous: achenes strongly compressed laterally, sparingly hirsute-pubescent on both surfaces, long-ciliate on the margins. — MEXICO. State of Chiapas: along road from Ocuilapa to Tuxtla, altitude 600 to 1000 m., 29 August, 1895, *E. W. Nelson*, no. 307, (hb. Gr., and hb. U. S. Nat. Mus.).

The affinity of *E. collodes* seems to be with *E. glutinosa*, Rob. & Greenm., and *E. Pringlei*, Fernald. From the former it is distinguished by the larger and fewer heads, longer involucral scales, and by the absence of the long sericeous hairs on the young shoots; from the latter it is readily separated by the foliar characters.

Encelia Conzattii. Perennial: stem covered with a grayish bark; branches and branchlets purplish, slightly compressed particularly at the nodes, striate, closely and minutely appressed-strigulose: leaves opposite, short-petiolate, lanceolate, 5 to 10 cm. long, 1 to 2.5 cm. broad, acuminate, acute, remotely denticulate, narrowed below into a crisp-undulate base, dark green and puberulent above, pale and closely canous-pubescent beneath: inflorescence a terminal leafy subcorymbose or paniculate cyme: heads including the sulphur-yellow rays 2 to 2.5 cm. in diameter, 1 cm. or less high, on densely canous-pubescent peduncles 0.5 to 3 cm. in length; involucral scales 2–3-seriate, the outer scales subfoliaceous; ray-flowers usually 6: disk-flowers about 35: achenes sparingly pubescent on both surfaces, conspicuously ciliate on the margins. — MEXICO. State of Oaxaca: altitude 1750 m., July–August, 1900, *Conzatti* & *González*, no. 1007 (hb. Gr.).

Encelia pilosa. Annual: stem erect, purplish or stramineous, hispidulous-pubescent with long pilose hairs intermixed, somewhat stipitate-glandular especially in the inflorescence: lower leaves opposite, petiolate, broadly ovate, 3 to 14 cm. long, fully two-thirds as broad, acute, subentire to crenate-dentate, hirsute-hispid above, hirtellous-pubescent beneath, 3-nerved from the base; petioles neither winged nor auricled, usually bearing long pilose hairs; upper leaves gradually reduced and shorter-petioled: inflorescence a many-headed nearly naked subcorymbose or paniculate cyme: heads radiate, 10 to 12 mm. high: scales of the involucre imbricated in 3 to 4 series, more or less purplish colored, externally pubescent, the outer scales ovate, acute, shorter than the inner lanceolate ones: ray-flowers, commonly 5; rays yellow, elliptic-oblong, excluding the short tube 6 mm. long, 4 mm. broad; ray-achenes slender, 6 mm. long: disk-flowers 20 to 25; corolla yellow or purplish: mature achenes 4 to 5 mm. long, appressed-pubescent on both surfaces, ciliate. — MEXICO. State of Oaxaca: 17 October, 1899, *E. W. D. Holway*, no. 3747 (hb. Gr.), and 25 October, 1899, no. 3740 (hb. Gr.); altitude 1750 m., July–August, 1900, *Conzatti & González*, no. 1002 (hb. Gr.); Ocotlan, December, 1901, *Conzatti & González*, no. 1263 (hb. Gr.); between Coixtlahuaca and Tamazulapam, altitude 2000 to 2500 m., 12 November, 1894, *E. W. Nelson*, no. 1937 (hb. Gr., and hb. U. S. Nat. Mus.); Valley of Etla, altitude 1700 m., 23 October, 1895, *L. C. Smith*, no. 854 (hb. Gr.).

Encelia squarrosa. Perennial, 4 to 6 m. high: stem slightly furrowed, tawny pubescent: leaves opposite, petiolate, ovate, 8 to 15 cm. long, 5 to 8 cm. broad, acuminate, dentate, cordate, dark green and hirsute-hispid above, paler and hirsute-tomentose beneath, 3–5-nerved from or just above the base; petioles 2 to 3 cm. in length: inflorescence a terminal corymbose cyme: heads large, about 1.5. cm. high, including the rays 5 to 6 cm. broad: involucre 3–4-seriate; scales of the involucre lanceolate, shorter than the flowers of the disk, acute, strongly squarrose, thickened at the base, ciliate, pubescent on the outer surface: ray-flowers neutral, about 20, yellow; pappus of slightly unequal scales, more or less united, shorter than the triangular sterile achenes; rays oblong, 1.5 cm. long, 6 mm. broad: disk-flowers numerous; pappus of two paleaceous awns with two or three intermediate scales, persistent: achenes strongly compressed, pubescent on both surfaces, ciliate. — MEXICO. State of Guerrero: mountains above Iguala, altitude 1230 m., 11 October 1900, *C. G. Pringle*, no. 8390 (hb. Gr.).

HELIANTHELLA QUINQUENERVIS, var. ARIZONICA, Gray, Syn. Fl. i. pt. 2, 284. Specimens collected by E. W. Nelson, on the Sierra Madre of S. W. Chihuahua, Mexico, about 95 km. south of Guadalupe and Calvo, altitude 2000 to 2500 m., August, 1898, no. 4808 (hb. Gr., and hb. U. S. Nat. Mus.), are clearly to be referred to the variety *arizonica* of the above species. This variety does not seem to have been hitherto reported from Mexico.

Verbesina madrensis. An herbaceous perennial: stems 4 to 6 dm. high, winged below, naked or nearly so above: lower leaves opposite, the upper alternate, elliptic-oblong to oblong-lanceolate, 3 to 10 cm. long, 1 to 4 cm. broad, obtuse, crenate-dentate, decurrent on the stem, hirsute-hispid on both surfaces especially above, prominently reticulate-nerved: heads one to few, terminating the stem and branches on long naked or subnaked, hirsute-hispid peduncles, 1 to 1.5 cm. high, including the rays 3 to 4 cm. in diameter: involucre 2–3-seriate; bracts of the involucre subequal, linear-oblong, obtuse or obtusish, a little shorter than the flowers of the disk, externally hirsute-pubescent: ray-flowers about 12, neutral; rays conspicuous, yellow, 1 to 1.5 cm. long, 4 to 6 mm. broad: disk-flowers numerous; pappus reduced to two inconspicuous knobs: achenes glabrous. — MEXICO. Territory of Tepic: in the Sierra Madre, between Santa Gertrudés and Santa Teresa, 8 August, 1897, *Dr. J. N. Rose*, no. 3378 (hb. Gr. & hb. U. S. Nat. Mus.).

A species belonging to the § *Pterophyton*, and most nearly related to *V. capitaneja*, Nees.

Eryngiophyllum, n. gen. of *Compositæ* (*Coreopsideæ*). Heads heterogamous, radiate. Involucre campanulate; bracts 2–3-seriate, free or very slightly united at the base. Receptacle flat or somewhat convex, paleaceous; pales thin, membranous. Ray-flowers uniseriate, fertile; achenes subtriangular. Disk-flowers regular; corolla-tube short, gradually ampliate above into an elongated-campanulate 5-toothed limb; achenes densely compressed. Anthers obtuse at the base, appendaged at the apex. Style-branches hirtellous-elongate-appendaged; stigmatic-lines short. Pappus obsolete. Herbaceous perennials with radical leaves and cymose inflorescence.

E. Rosei. Leaves densely rosulate from the crown of a deep perennial root, often somewhat falcate, linear-lanceolate, 3 to 12 cm. long, 5 to 8 mm. wide, undivided or bearing one or more arcuate-ascending lateral divisions, mucronate-acute, entire, somewhat cartilaginous-margined, ciliate, glabrous on both surfaces, more or less areolate: inflorescence axillary scapoid compound bracteate cymes, exceeding the leaves: heads about 1 cm. high, 2.5 cm. broad, on glabrous or slightly hirtellous peduncles 2 to 8 cm. ong: involucre 2–3-seriate; bracts of the involucre subequal; those of the two outer series subherbaceous, lanceolate, acute, ciliate, much thickened and gibbous at the base: ray-flowers about 20, rays yellow, conspicuously nerved: disk-flowers numerous: pappus quite obsolete or represented by very short protuberances: apical appendages of the anthers short, broad, obtuse: achenes glabrous. — MEXICO. State of Sinaloa; between Rosario and Calomas, 12 July, 1897, *Dr. J. N. Rose*, no. 1618 (hb. Gr., and hb. U. S. Nat. Mus.).

The natural affinity of the genus here proposed is apparently with *Isostigma* and *Chrysanthellum*. In the technical characters of the head, particularly the style-branches, it approaches more closely the latter genus.

Chrysanthellum mexicanum. Annual: stem simple or much-branched, erect or ascending, 1 to 3 dm. high, glabrous or sparingly hirtellous, apparently somewhat glaucous, often purplish at the base: leaves petiolate, tripinnately parted, glabrous on both surfaces, prominently veined; divisions narrow, apiculate-acute: inflorescence a subcorymbose cyme: heads small, 3 to 4 mm. high: involucre subcampanulate, glabrous; scales of the involucre ovate-oblong, acute: outer achenes subterete, the inner strongly ob-compressed, narrowly cartilaginous-margined and conspicuously ciliate. — MEXICO. State of Jalisco: banks of ravines near Guadalajara, 10 September, 1890, *C. G. Pringle*, no. 3259 (hb. Gr.), distributed as "*Chrysanthellum procumbens*, Rich."; Tequila, August, 1886, *Dr. Edward Palmer*, no. 364 (hb. Gr.). State of Chiapas: tableland about Ocuilapa, altitude 1000 to 1200 m., 21 August, 1895, *E. W. Nelson*, no. 3065 (hb. Gr., and hb. U. S. Nat. Mus.). State of Vera Cruz: Orizaba, *Botteri*, without number (hb. Gr.). State of San Luis Potosi, altitude 2000 to 2450 m., *C. C. Parry & Dr. Edward Palmer*, without number, coll. of 1878 (hb. Gr.).

The Mexican and Central American representatives of this genus hitherto have been referred to *C. procumbens*, Rich., which species, as far as the material at hand shows, is confined to the West Indies, Venezuela and Guatemala. *C. mexicanum* is readily distinguished from *C. procumbens* by its more erect, stricter habit, and by the uniformly tripinnatisect leaves. In habit and in foliar characters *C. mexicanum* bears a close resemblance to *C. Indicum*, DC. from which it differs in having smaller, narrowly cartilaginous-margined and conspicuously ciliated achenes.

Bidens bicolor. Stem stramineous, striate, hirsute-pubescent to essentially glabrous, rather freely branched above: leaves pinnately 3–5-parted; divisions ovate to ovate-lanceolate, 1.5 to 5 cm. long, 0.5 to 2 cm. broad, acute to acuminate-acute, dentate, sparingly pubescent to quite glabrous, paler beneath: inflorescence a loose subcorymbose cyme: heads, including the rays, 2.5 to 6 cm. in diameter: involucre biseriate, more or less hirsute; outer involucral scales linear-oblong, subfoliaceous, usually spreading: ray-flowers 5 or 6, neutral; rays oblong, 1.5 to nearly 3 cm. long, dark-nerved in the earlier stages, conspicuously purple-maculate at the base: disk-flowers numerous: pappus retrorsely few-barbed: mature achenes not seen. — MEXICO. State of Oaxaca: altitude 1750 m., July–August, 1900, *C. Conzatti & V. González*, nos. 1008, 1009 (hb. Gr.). Apparently most nearly related to *Bidens grandiflora*, Balb.

BIDENS TERETICAULIS, DC., var. **sordida.** Leaves mostly simple and undivided, rarely deeply 3-parted, sordid-pubescent upon both surfaces more densely so beneath: outer scales of the involucre rusty tomentose. — COSTA RICA. Thal des rio Segando, *Hoffmann*, no. 383 (hb. Gr.); forêts de la Mala Via au Copey, April, 1898, *Ad. Tonduz*, no. 12,284 (hb. Gr., and hb. Inst. Phys.-Geog. Cost. Ri.).

Arnicastrum, n. gen. of *Compositæ* (*Helenoideæ*). Heads heterogamous, many-flowered. Involucre campanulate; bracts 2–3-seriate, the outer herbaceous. Receptacle convex, naked. Ray-flowers yellow, ligulate, 2–3-dentate, fertile, epappose; mature achenes numerously ribbed. Disk-flowers with a rather slender tube dilated above into an equally 5-toothed limb: pappus of numerous somewhat unequal barbellate bristles: achenes slightly compressed, 4-angled, many-ribbed. Herbaceous perennials with opposite leaves and rather large heads.

A. glandulosum. Stems erect, 3 to 5 dm. high, mostly branched from the base, striate, hispidulous-pubescent, and above especially on the usually long peduncles bearing long-stipitate purplish glandular hairs: leaves sessile, subamplexicaul, 3–5-nerved from the base, narrowly lanceolate, 2 to 8 cm. long, 3 to 15 mm. broad, entire or sparingly denticulate, hispidulous-pubescent on both surfaces: heads about 1 cm. high, including the rays 3 to 4.5 cm. in diameter; outer involucral scales oblong or oblong-ovate, 10 to 12 mm. long, 5 to 8 mm. broad, obtuse, entire, hirtellous-pubescent on the outer surface; the inner scales mostly longer, sometimes acuminate, membranous: ray-flowers about 10; tube 2 to 3 mm. long, externally pubescent; rays elliptic-oblong, 1.5 cm. long, 6 mm. broad: disk-flowers numerous: pappus-bristles unequal, the longer equalling or exceeding the tube of the corolla: achenes hirtellous-puberulent above, many-ribbed. — MEXICO. State of Chihuahua: Sierra Madre, 21 June, 29 July, 1899, *E. W. Nelson*, no. 6102 (hb. Gr., and hb. U. S. Nat. Mus.); Sierra Madre, near Colonia Garcia, altitude 2450 m., 26 July, 1899, *Townsend & Barber*, no. 169 (hb. Gr.).

Var. **vestitum.** Outer scales of the involucre 7 to 9 mm. long, and closely covered with purplish glandular hairs; otherwise like the species proper. — MEXICO. State of Durango: 48 km. north of Guanacevi, altitude 2450 to 2775 m., 18 August, 1898, *E. W. Nelson*, no. 4778 (hb. Gr., and hb. U. S. Nat. Mus.). In general habit *Arnicastrum glandulosum*, and the variety *vestitum* resemble certain species of *Arnica* more particularly in the opposite leaves and the rather long internodes. In the technical characters of the head, however, they seem to have their nearest affinity with *Jaumea*, *Bæria* and *Whitneya*.

Porophyllum calcicola, Rob. & Greenm. Annual, glabrous throughout: stem erect, usually much-branched, green or purplish, more or less glaucous: leaves petiolate, oblong to obovate-subrotund, 0.5 to 3.5 cm. long, 0.5 to 1.5 cm. broad, obtuse or rounded at the apex, entire to somewhat sinuate, bearing 3 to several elliptic-oblong marginal glands, cuneate at the base into the slender 0.5 to 2.5 cm. long petioles: heads about 1.5 cm. high on upwardly enlarged 1 to 3 cm. long peduncles: involucre of 5 linear acute bracts about 13 mm. in length, indigo-blue or plum-colored: flowers 18 to 20, slightly exceeding the involucre; pappus about equalling the slender puberulent greenish corolla-tube; the campanulate limb of the corolla rather deeply 5-lobed, deep purple: mature achenes 6 to 7 mm. long, puberulent. — MEXICO. State of Guerrero: on limestone ledges of mountains above Iguala, altitude 920 m., 24 October, 1900, *C. G. Pringle*, no. 9059 (hb. Gr.), distributed as "*Porophyllum Pringlei*, Rob." State of Morelos: on limestone hills near Yantepec, altitude 1385 m., 8 November, 1902, *C. G. Pringle*, no. 8720 (hb. Gr.). Most nearly related to *Porophyllum Pringlei*, Rob., but differing by the more numerously flowered heads, shorter involucres and flowers, more deeply lobed corolla-tube and shorter mature achenes. The flowers moreover in *P. calcicola* are deep purple not white as in *P. Pringlei*.

Bahia glandulosa. An herbaceous perennial (?), glandular-pubescent throughout: stem erect, 7 dm. or more high, freely branching above, striate: leaves alternate, petiolate, tri- or quaternately compound, including the petiole 2 to 12 cm. long, 10 cm. or less broad; divisions linear to linear-oblong, obtuse; the uppermost leaves often reduced to individual spatulate-linear or linear-obtuse bracts: inflorescence terminating the stem and branches in a many-headed flat-topped corymbose cyme: heads 1 cm. or less high, including the rays about 1.5 cm. in diameter: involucre of 10 to 14 lance-oblong obtuse uniseriate bracts: ray-flowers 15 to 20, pale yellow or whitish: disk-flowers numerous: pappus of 10 to 12 obovate-spatulate and muticus to lanceolate paleo-aristate scales, somewhat variable in length: mature achenes about 3 mm. long, puberulent to essentially glabrous. — MEXICO. State of Durango: vicinity of the City of Durango, April to November, 1896, *Dr. Edward Parker*, no. 492 (hb. Gr.). State of Chihuahua, on hills and plains near the City of Chihuahua, 24 October, 1885, *C. G. Pringle*, no. 308 (hb. Gr.), distributed as "*Bahia biternata*, Gray."

A species on the one hand resembling *Bahia chrysanthemoides*, Gray, and on the other *B. biternata*, Gray. From the former it is distinguished by the presence of the pappus, and the short corolla-lobes; and from the latter by the glandular pubescence throughout instead of being densely hirtellous on stem and leaves with only a few glandular hairs above in the inflorescence.

Tagetes Nelsonii. Stem striate, purplish, glabrous: leaves opposite, petiolate, pinnatisect; divisions 3 to 7, elliptic-lanceolate, 1 to 6 cm. long, 0.5 to 2 cm. broad, acute, sharply and evenly dentate, each tooth bearing a single gland, dark green above, paler beneath, glabrous or slightly puberulent on the upper surface, distinctly crisp-puberulent on the lower surface, evenly glandular-punctate: inflorescence a terminal many-headed corymbose cyme: heads about 1 cm. high, radiate: involucre cylindrical, 5–7-dentate, numerously punctate with oblong-elliptic glands: ray-flowers usually 5, occasionally 6; tube slender, nearly equalling the oblong 0.5 cm. long lamina: disk-flowers 9 to 12: pappus of short, unequal scales: achenes 4 to 5 mm. long, glabrous. — MEXICO. State of Chiapas: near Tumbala, altitude 1200 to 1650 m., 20 October, 1895, *N. W. Nelson*, no. 3314 (hb. Gr., and hb. U. S. Nat. Mus.).

Urbinella, n. gen. of *Compositæ* (*Tagetineæ*). Heads heterogamous, radiate. Involucre campanulate; bracts 1-seriate, free or very slightly united at the base. Receptacle conical, naked. Ray-flowers uniseriate, fertile. Disk-flowers regular; tube slender, gradually ampliated above into a 5-toothed limb. Anthers minutely toothed at the base, appendaged at the apex. Style-branches sublanceolate-appendaged. Achenes of ray- and disk-flowers uniform, terete, many-striate. Pappus paleaceous of 5 to 7 scales, some or none awn-bearing. Small annuals with alternate leaves.

U. Palmeri. Glabrous throughout: stem 0.5 to 2 dm. high, simple or much-branched, more or less purplish: leaves simple and undivided, linear or subterete, 0.5 to 2.5 cm. long, obtusish, bearing 1–several rather conspicuous glands, one of which usually occurs near the tip: heads nodding or erect, 5 to 9 mm. high, including the rays 1 to 1.5 cm. in diameter, terminating the stem and branches on slender peduncles: involucral bracts mostly 5, oblong or obovate-oblong, about 5 mm. long, externally obtusely keeled along the median line, scarious-margined, slightly lacerated or unevenly margined at the rounded tip, bearing 1–several roundish or elliptic-oblong glands: ray-flowers 5; tube slender, 2 mm. long; rays white with a yellowish base, obovate or obovate-cuneate, 5 to 6 mm. long, nearly or quite as broad: disk-flowers few to many (40 or less), light yellow: anthers narrowly appendaged at the tips: scales of the pappus obovate-cuneate to lanceolate, one or two awn-bearing:

achenes 3 mm. long, glabrous or sparingly puberulent. — MEXICO. State of Durango: vicinity of the City of Durango, April to November, 1896, *Dr. Edward Palmer*, no. 793 (hb. Gr., and hb. U. S. Nat. Mus.).

The genus here proposed is related to *Dysodia* and *Hymenatherum*. From the former it differs in the character of the pappus, and from the latter in the fewer scales of the pappus, as well as in the character of the involucre.

The genus is dedicated to Dr. Manuel Urbina, Director of the National Museum, City of Mexico.

Pectis latisquama, Schz. Bip. A low subprostrate herbaceous perennial, numerously branched from a suffruticose base: stems leafy, hirtellous-pubescent: leaves linear or linear-lanceolate, 1 to 2 cm. long, 2 to 3 mm. broad, acute, somewhat revolute-margined, bearing 3 to 5 pairs of cilia near the base, glabrous on both surfaces: heads 10 to 12 mm. high, terminating the stem and branches on 1 to 3 mm. long puberulent naked or bracteate peduncles: scales of the involucre 5, oblong-elliptic or obovate, 6 to 7 mm. long, 4 to 5 mm. broad, rounded at the apex, obtusely keeled, ciliate, more or less purplish-tinged, glabrous: ray-flowers usually 5; tube 2 to 3 mm. long, shorter than the longest of the numerous unequal pappus-bristles; rays orange-yellow, oblong-ovate, 6 to 7 mm. long, 3 to 5 mm. broad: disk-flowers 10 to 20; pappus of numerous bristles, the longer two-thirds as long as the corolla: achenes of both ray- and disk-flowers pubescent. — Acc. to Gray, Proc. Am. Acad. v. 181, name only. *Pectis canescens*, Gray, l. c. xix. 47, in part, as to citation in synonomy, not HBK. *P. canescens*, Fernald, l. c. xxxiii. 82, in part, as to *Schaffner's* no. 85 (hb. Gr.), not HBK. — MEXICO. State of Mexico: Valley of Mexico, *Schaffner*, no. 85 (hb. Gr.). State of Puebla: thin soil, Amozoc, altitude 2100 m., 9 September, 1902, *C. G. Pringle*, no. 8604 (hb. Gr.).

Neurolæna macrocephala, Schz. Bip. acc. to Hemsley, Biol. Cent.-Am. Bot. ii. 233. *Neurolæna semidentata*, Klatt, in Leopoldina, xx. 94 (reprint, p. 7). The type specimens of these two described species are both in the Gray Herbarium, and the identity of the two plants is without question, hence it becomes necessary to retain the older combination and treat the other as a synonym.

Neurolæna macrophylla. Stem sulcate, striate, reddish-brown, appressed-puberulent: leaves alternate, thin or membranous, somewhat rhombic-lanceolate, including the petiole 1.5 to 4 dm. long, 4 to 9 cm. broad, acuminate, shallowly dentate to denticulate, gradually narrowed below into a 1 to 3 cm. long petiole, dark green and sparingly puberulent on the upper surface, paler and essentially glabrous beneath: inflorescence a terminal compound many-headed corymb: heads discoid, about 1 cm. high, and 0.5 cm. thick, about 20-flowered: involucre 4–5-seriate; bracts of the involucre ovate to ovate-oblong, obtuse, imbricated, the outer bracts shorter than the inner: achenes 2.5 mm. long, somewhat pubescent above. — MEXICO. State of Chiapas: Chicarras, altitude 1000 to 2000 m., 6 February, 1896, *E. W. Nelson*, no. 3766 (hb. Gr., and hb. U. S. Nat. Mus.).

Senecio firmipes. Fruticose: stem leafy, glabrous: leaves scattered, crowded, lanceolate, 2 to 3 cm. long, 4 to 6 mm. wide, acuminate, acute, somewhat pungent tipped, entire, slightly hirtellous on the margins, narrowed below into an articulate base, glabrous on both surfaces, and somewhat glaucous beneath; the persistent petiole-like portion of the base of the leaves later more or less covering the stem: inflorescence a terminal compound corymbose cyme: heads discoid, 5 to 6 mm. high, 15–20-flowered; involucre of 8 oblong-ovate glabrous thickish obtuse bracts, shorter than the flowers; bracteoles few and appressed to involucre: achenes glabrous. — COSTA RICA. Cerro de la Muerte, altitude 3100 m., January, 1897, *H. Pittier*, no. 10,472 (hb. Gr., and hb. Inst. Phys.-Geog. Cost. Ri.); Vueltas, *H. Pittier*, no. 10,505 (hb. Gr., and hb. Inst. Phys.-Geog. Cost. Ri.).

This species belongs to the § *Fruticosi*, and of the North American species it is nearest related to *S. cernicosus*, var. *major*, Wedd. In South America, however, it has several congeners, namely, *S. glacialis*, Wedd., *S. pulchellus*, DC., and *S. abietinus*, Willd.

Senecio (§ Mulgedifolii) pterocaulis. Stem erect, striate, arachnoid-tomentose: leaves alternate, lanceolate or lance-attenuate, 0.5 to 1.5 dm. long, 3 to 13 mm. broad, acute, cartilaginous-denticulate, conspicuously decurrent on the stem, arachnoid-tomentose on both surfaces more densely so beneath; inflorescence terminating the stem and branches in a paniculate many-headed cyme; bracts of the inflorescence linear-attenuate to setaceous: heads 10 to 12 mm. high, discoid, 40–50-flowered: involucre calyculate with minute setaceous bracteoles; bracts of the involucre usually 21, linear, acute, glabrous and stramineous: achenes appressed-sericeous-pubescent. — MEXICO. State of Morelos: in lava fields above Cuernavaca, altitude 2155 m., 24 November, 1902, *C. G. Pringle*, no. 8704 (hb. Gr.).

Lycoseris macrocephala. Stem leafy to the inflorescence, subangulate, striate, at first arachnoid-tomentulose, later glabrate: leaves alternate, short-petiolate, elliptic-lanceolate or somewhat ovate-oblong, 1 to 2 dm. long, 2 to 8 cm. broad, acuminate, acute, serrulate, narrowed at the base into a 3 to 5 mm. long petiole, dark green and glabrous above, arachnoid-tomentulose beneath, pellucid-punctate, subparallel 5-nerved; midrib and the two pairs of lateral nerves prominent on the under side of the leaf-blade: heads of pistillate flowers large, 4 to 5 cm. high, 5 to 7 cm. broad, terminating the stem and branches on short stout peduncles: involucre about 8-seriate, arachnoid-tomentose; scales of the involucre lance-acuminate, appressed or slightly squarrose, the outer successively shorter: rays about 1 cm. long, 2 mm. broad: disk-flowers numerous; pappus of many slender setæ: heads of staminate flowers about 2 cm. high and broad, on short peduncles: involucre 4–5-seriate, arachnoid-tomentose; scales of the involucre lance-oblong, obtuse, appressed or very slightly spreading, the outer shorter than the inner; rays narrowly oblong, 10 to 12 mm. long, 3 mm. broad. — COSTA RICA. Forêts des collines de Nicoya, February, 1900, *Ad. Tonduz*, no. 13,602 (hb. Gr., and hb. Inst. Phys.-Geog. Cost. Ri.), pistillate plant; boussailles a Nicoya, January, 1900, *Ad. Tonduz*, no. 13,620 (hb. Gr., and hb. Inst. Phys.-Geog. Cost. Ri.), staminate plant.

Notwithstanding the considerable difference in the size of the heads of the two specimens above cited, the writer has no doubt that they represent the pistillate and staminate plants of one species. In the stem and foliar characters there is absolute identity.

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QUANTITATIVE STUDIES IN THE EVOLUTION OF PECTEN. — III.

COMPARISON OF PECTEN OPERCULARIS FROM THREE LOCALITIES OF THE BRITISH ISLES.

By C. B. Davenport.

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I. Introduction.

This investigation was undertaken with the aim of finding out quantitatively the way in which lots of individuals belonging to one species but living in widely separated localities differ. The fact that such lots differ is recognized by many naturalists and has been designated "geographical variation." Hitherto attention has been directed especially toward the geographical variation of land animals, especially mammals and birds. Above all, in America, ever since Allen's work on the subject, has the study of the geographical variation of mammals and birds of North America been pursued with energy. The studies made have, it must be confessed, rarely been undertaken in the spirit of great precision which characterizes modern biometry. Not only is the fact of geographical variation thus recognized, but this class of variations is regarded by many as being more important than any other in the origin of species. Under these circumstances it becomes interesting to examine a case of geographical variation biometrically.

II. Material.

The material used in this investigation has been the shells of *Pecten opercularis*, a common bivalve mollusc. Visiting Europe in the autumn of 1902, with the aim of gathering material for this study, I obtained sufficiently large numbers of this species from four localities, as follows: —

1. The Firth of Forth, two miles north of Newhaven on the "city of Edinburgh grounds," living in six fathoms (12 metres) of very clear water, on hard, clean shell-bottom. The salt content of the estuary at this point must be much diluted below that of the adjacent North Sea, where it is about 3.4 per cent.*

2. The Irish Sea, in latitude 54° 18′ N and longitude 4° 12′ W. The salt content of the water in which these shells were living is probably between 3.4 and 3.5 per cent. The bottom is sand and mud. The depth was about 20 metres. The July temperature of the surface of the sea is about 57° F. (14° C.).†

3. The northwestern part of the English Channel, in the vicinity of the Eddystone Lighthouse, within two miles of the Light to the north, and within five miles to the east and to the west. The depth of the water is 30 to 35 fathoms (60 to 70 metres). The mean density of the

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* Mr. Thomas Devlin of Newhaven kindly arranged for the collection and shipment of these shells and gave me the data concerning the collecting ground.

† For the collection of shells from the Irish Sea, I am indebted to the Liverpool Marine Biology Committee, whose well-equipped new laboratory I visited in August, 1902. Especial thanks are due Professor Herdman, director, and Mr. H. E. Chadwick, curator, for interesting themselves in my search for pectens. Most of the data concerning the place of collection were sent me by Mr. Chadwick.

sea-water in this part of the Channel is 3.5 to 3.55 per cent, and the mean surface July temperature is about 62° F. (16.7° C.)*

4. The fourth collection was obtained from the Bay of Naples in 30 metres, off the Posilipo. These were of small size, probably not fully grown, and are not included in the present study.†

Before attempting to compare the shells from different localities, it is essential to consider whether they are comparable. Age is an important factor of variation that must not be neglected. Over 99 per cent of the shells measured had a dorso-ventral diameter between the limits of 40 and 75 mm. As they were all collected in the autumn, at which time the new brood is very small, they were clearly at least one year old. From analogy with *Pecten irradians* they do not live to be three years old. Consequently they must be either one or two years old. While the pectens of the new brood may increase in diameter as much as a millimetre a day, the growth after the first year is exceedingly slow, averaging from 5 to 10 mm. for the entire second year and much less in the third year. Consequently the range in diameter (as given in Table I) can be attributed in only small part to differences in age. The material was all taken by dredging, and consequently quite at random. There is nearly perfect correlation between the right and the left valves in the dorso-ventral and the antero-posterior diameters as well as in the number of rays. So that, for most studies, one valve alone need be considered. The lower valve has usually been chosen.

The geographical range of *Pecten opercularis* is, according to Locard (1898), as follows. In the east Atlantic, to the north (at a depth of between 9 and 187 metres), as far as the Lofoten Islands off the coast of Norway. It is found on the coasts of Great Britain and Ireland, France, Spain, Portugal, Morocco, and south to the Azores, Madeira, and Canary Islands. To the west it has been dredged (by the "Light-ning ") off the Hebrides and Faroe Islands at a depth of 670 metres. In the Mediterranean it occurs from Gibraltar to Asia Minor on the coasts of Europe, Africa, and Asia, and most of the isles, also in the Adriatic and Ægean seas. The species is found fossil from the Miocene on.

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* These shells were obtained for me in September and October, 1902, through the generous assistance of the Marine Biological Association's laboratory at Plymouth. I take this opportunity to express my obligation to Director E. J. Allen, D.Sc., to Mr. R. A. Todd, who was in charge at the time of our visit and advanced our work in every way, and to the other members of the laboratory staff.

† To the Director, Professor Dohrn, Professor Eisig, Dr. LoBianco, and the other members of that Mecca of zoölogists, I am glad to return thanks for the opportunity to study at the laboratory and for assistance received there.

I gladly acknowledge, moreover, my indebtedness to the Smithsonian Institution, under the direction of Professor Langley, for an appointment to the table of the Institution at the Naples laboratory.

III. METHODS.

The two valves of *Pecten* are respectively right and left; they are also upper and lower, for the mollusc, during early life at least, lies upon its right side. The right valve is notched near the hinge to permit the passage of the byssus, the tough, threadlike secretion by which the shell is attached to the substratum. The two valves can be readily distinguished by the presence or absence of this notch, and it will be convenient to use preferably the terms "lower" "and upper." Six dimensions were taken on each valve; namely, first the dorso-ventral diameter, perpendicular to the hinge line at the beak; second and third, the antero-posterior distance in front of and behind this line (the sum of these, naturally, gives the total antero-posterior dimension); fourth and fifth, the length of the anterior and of the posterior part of the hinge, measuring from the beak (the sum gives the total hinge length); and sixth, the partial dextro-sinistral diameter, or the greatest perpendicular distance from the plane passing between the two valves to the outside of the valve. The measurements were made with a special instrument, which might be called a valvameter, constructed as follows: At a point on the circumference of a circular planed steel base 11 mm. thick and 170 mm. in diameter, there rises perpendicularly a scale, graduated to millimetres. Upon this scale slides an arm, which is parallel to the base and extends out over its centre and also bears a vernier, by which distances on the scale may be read, from the base, to tenths of a millimetre. Upon the base is pasted a piece of metric quadruple-ruled paper so that one system of lines runs parallel to the arm. The valve to be measured is placed flat on the basal plate with its beak touching the vertical scale and with the hinge line strictly tangential. The zero lines having been carefully adjusted at the time of affixing the quadruple-ruled paper, one can read off at once the dorso-ventral and the two parts of the antero-posterior diameter and the two partial hinge lengths. By lowering the arm until it touches the highest part of the shell, one can read off on the vertical scale the transverse diameter of the valve (partial transverse diameter of the animal).

IV. Results.

1. *Change of Proportions with Age.*

In order to make use of the dimensions of shells of different sizes, it is important that we should know how and to what extent the shells change in proportions as they grow larger. The change in the proportions of the two chief axes perpendicular to the transverse interests us most at this time, and are given in Table I for all localities.

TABLE I.

For each Class of Dorso-ventral Diameters are given the Frequency and the Ratio of Antero-posterior Diameter to Dorso-ventral Diameter for the Right Valves of all Sizes from the Three Localities.

Dorso-ventral	Eddystone.		Irish Sea.		Firth of Forth.	
diam. in mm.	*f.*	Ratio.	*f.*	Ratio.	*f.*	Ratio.
25–29			1	0.926		
30–34			0			
35–39	4	1.001	2	0.987		
40–44	48	1.007	5	0.991		
45–49	90	1.019	4	1.014	15	1.006
50–54	137	1.041	14	1.025	33	1.013
55–59	212	1.049	80	1.040	66	1.026
60–64	40	1.052	272	1.049	112	1.034
65–69	4	1.067	194	1.061	130	1.039
70–74	2	1.114	40	1.065	126	1.044
75–79			1	1.026	23	1.058
80–84					2	1.050
N	537		613		507	

This table shows, first, that the Eddystone shells are the smallest, and the Firth of Forth shells the largest, the shells from the Irish Sea being intermediate in size; secondly, that there is an increase in the ratio from near or even below unity in the smaller shells to a point much above unity, 1.06–1.10, in the largest shells; and thirdly, that the shells of a given dorso-ventral diameter are longest at Eddystone and roundest at the Firth of Forth. Thus for shells having a length of 67 mm. the ratios are: 1.067, 1.061, and 1.039 in the three localities.

A similar change of proportion with size I have found in *Pecten irradians* from the east coast of the United States and in *Pecten ventricosus* from the coast of southern California. Assuming that the small shells retain embryonic characters more than the large ones do, then in accordance with the biogenetic law we would be led to conclude that the ancestral pectens had a relatively greater dorso-ventral diameter, and that modern pectens are becoming longer. This must, however, remain an hypothesis until measurements have been made on the British fossil *Pecten opercularis.*

2. *Symmetry in Pecten.*

Pecten belongs to a group (*Lamellibranchiata*) which is characterized, on the whole, by a high degree of bilateral symmetry; i.e., the right and the left valves tend to be equal in all their axes. Some remarkable exceptions to this rule, however, occur; as in the case of the oyster, the family *Chamidæ,* and the now extinct family of *Hippuritidæ.* In all such cases the mollusc rests on one valve, either the right or the left, and it is usually stated that "the fixed valve, whether right or left, is always deep, and the free valve flat." * *Pecten* rests invariably upon the right valve, and this is (Tryon, 1882, p. 288) in most species the more convex. This is, indeed, true for *Pecten maximus* and even *Pecten irradians,* but it is not true for *Pecten opercularis,* despite its close relationship to the latter species; for in *Pecten opercularis* the lower valve is the flatter.† The ratio of the breadth of the upper valve to the lower valve is, however, not constant for shells of all sizes. The facts are shown in Table II.

TABLE II.

Ratio of Breadth of Upper Valve divided by Breadth of Lower Valve for Shells of all Sizes from the Irish Sea.

Breadth of lower valve in mm.	$\frac{\text{Upper valve breadth}}{\text{Lower valve breadth}}$.	Breadth of lower valve in mm.	$\frac{\text{Upper valve breadth}}{\text{Lower valve breadth}}$.
40–45	1.375 (1 case)	90– 94	1.485
60–64	1.356	95– 99	1.455
65–69	1.450	100–104	1.448
70–74	1.548	105–109	1.437
75–79	1.542	110–114	1.416
80–84	1.514	115–119	1.357
85–89	1.507	120–121	1.463 (1 case)

This table shows that in the smaller shells, of 62 mm. breadth, the ratio is 1.35, that it rises from this to 1.55 at about 72 mm., and then falls to 1.36 in shells whose lower valve has a breadth of 117 mm. The meaning of this series is quite obscure. Some would see in it evidence of selection tending in later life to weed out the more unsymmetrical individuals; others would draw the conclusion that the ancestors of

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* Cooke (1895), 271; Tryon (1882), 53.

† This fact was noted by Lamarck (1819).

Pecten were more nearly equivalve than their later descendants, but that in very recent times the species has tended to become symmetrical again. Again it is quite possible that the series has a physiological explanation. It would be interesting to know whether those pectens whose lower valve has a breadth under 75 mm. rest on the *upper* or left valve while the larger ones have the normal position. Whatever the explanation of the series, the main fact stands out clearly that *Pecten opercularis* is bilaterally unsymmetrical.

We have next to consider whether with the loss of symmetry there has also gone a loss of correlation in the variation of the valves. It is perfectly possible, of course, to have perfect correlation in variation of two unequal dimensions. And the two sides of the body in most Bilateralia have a very high correlation, as shown below.

TABLE III.

SOME COEFFICIENTS OF CORRELATION OF CORRESPONDING RIGHT AND LEFT ORGANS IN BILATERALLY SYMMETRICAL ANIMALS.

R. femur and l. femur,	man	0.9765 ± .0032	Warren, 1898.
R. tibia and l. tibia,	“	0.9634 ± .0044	“ “
R. humerus and l. humerus,	“	0.9454 ± .0091	“ “
R. radius and l. radius,	“	0.9246 ± .0151	“ “
R. clavicle and l. clavicle,	“	0.9317 ± .0115	“ “
Metacarpal			
of thumb on r. and l. hand,	man	0.974 ± 006	Lewenz & Whitely, 1902.
of index finger on r. and l. hand,	“	0.990 ± .002	“ “ “
of middle finger on r. and l. hand,	“	0.985 ± .003	“ “ “
of ring finger on r. and l. hand,	“	0.946 ± .011	“ “ “
of little finger on r. and l. hand,	“	0.955 ± .010	“ “ “
No. r. and l. pectoral fin-rays, Acerina,		0.700	Duncker, 1897.
No. r. and l. pectoral fin-rays, Cottus,		0.700	“ “
No. glands, r. and l. foreleg, Sus,		0.79	Davenport & Bullard, 1896.
No. teeth, r. and l. jaws, Nereis,		0.820 ± .008	Hefferan, 1900.

We have already some data upon the correlation of bilaterally homologous organs occurring in unsymmetrical species. Duncker has made a study of such a case in the flounder, *Pleuronectes flesus*, L. This fish lies upon the bottom on its left side, with both eyes on the right side. In this case the lateral correlation is strikingly small. Thus between the right and left pectoral fin-rays it is 0.590 ± .013; between the right and left ventral fin-rays it is 0.228 ± .020. Another instance has been afforded by Yerkes (1901) in the case of the fiddler crab, *Gelasimus pugilator*, Latr. These crabs have a symmetrical body except that the first of the walking appendages bears a large terminal claw on the one side and only a small one on the other side. The side is indeterminate, about fifty per cent of the individuals being right-handed and fifty per cent left-handed. The great claw is nearly thrice as large as its fellow of the opposite side. Nevertheless, the correlation is as high as in some cases of symmetrical organ; it is, namely, 0.774.

In the case of *Pecten opercularis* from the Irish Sea the correlation between the breadth of the right valve and that of the left valve is $r = 0.8578 \pm .0056$. This correlation is higher than between some symmetrical organs in vertebrates and in the lower vertebrates. It is, however, lower than the symmetrical correlations in man.

The question arises whether in the new position of a species the parts arranged about the newly assumed vertical axis may not have a high correlation. Duncker has found this to be the case in the flounder. Thus the correlation between the dorsal and the ventral fin-rays is 0.672 ± .011, so that the morphologically dissimilar but now antimerically placed organs are more closely correlated in their variations than the bilaterally homologous organs. The same thing is found in *Pecten.* In the normal position of *Pecten*, in which it lies upon its right side, so that its transverse axis is vertical, the antero-posterior and the dorso-ventral axes are horizontal. What is the correlation between the lengths of these horizontal axes?

TABLE IV.

Correlation Coefficients between the Antero-posterior Diameter and the Dorso-ventral Diameter of the Lower Valve of Pecten Opercularis from Three Localities.

Locality.	Number of Individuals.	r.	P. E. r.
Eddystone (Appendix A)	537	0.96997	± .00124
Irish Sea (Appendix B)	613	0.97596	± .00093
Firth of Forth (Appendix C)	507	0.98495	± .00063

Here the correlation coefficients of non-bilateral dimensions are extremely high, as high as in many of the highest human coefficients between bilateral dimensions. As a result of the newly assumed position of the scallop, two formerly largely independent axes have come to vary simultaneously just because they have similar relations to the bottom. *Pecten* has gained a new kind of symmetry; namely, a radial symmetry. The fact points very forcibly to another conclusion; namely, that physiological factors are much more important in determining correlation than morphological relationship when the two come in conflict.

We have considered the variation and correlation of the transverse diameters of the two valves and also the correlation between dorso-ventral and antero-posterior axes. We must now combine these two systems by determining the relation of the dorso-ventral diameter and the transverse. This relation is set forth in Table V.

TABLE V.

Ratio of Dorso-ventral Diameter to "Partial" Breadth of Right Valve.

Dorso-ventral diameters in mm.	Ratios. Eddystone.	 Irish Sea.	 Firth of Forth.
25–29		.146	
30–39	.146	.161	
40–44	.145	.156	
45–49	.146	.155	.132
50–54	.151	.151	.132
55–59	.149	.152	.138
60–64	.145	.155	.136
65–69	.148	.153	.134
70–74	.175	.153	.134
75–79		138	.133
80–84			.139

The ratios given in Table V above register the (half) globosity of the mollusc. The difference in globosity between the Eddystone and the Irish Sea forms is not marked, but the Forth shells are decidedly flat. There is no clear change in globosity with age.

All axes of the molluscan body are of course correlated. We have seen that this correlation becomes extraordinarily high in axes that play a physiologically similar part. What is the correlation between the dorso-ventral and transverse diameter of the right valve, which play very dissimilar parts?

TABLE VI.

Correlations between Dorso-ventral Diameter and Partial Transverse Diameter of Right Valve (Appendices D, E, F)

Locality.	Number of Individuals.	*r*.	P. E. *r*.
Eddystone	537	0.8446	± .0063
Irish Sea	614	0.7840	± .0081
Firth of Forth	508	0.8497	± .0064

The difference between the correlation coefficient of Eddystone and that of the Firth of Forth shells is probably not significant; but the Irish Sea shells have sensibly the least correlation between the axes in question.

3. *Ray Frequency.*

a. Correlation between Ray Frequency and the Dimensions of the Shell. — The variability of the rays was determined by counting the grooves on the inner face of each valve. This ordinarily presents no difficulty, but occasionally the terminal groove of a series is so obscure that it is doubtful whether it should be counted or not. The rule adopted was to count any groove that had an appreciable elevation on *two* sides. An elevation is determined by its ability to cast a shadow or to jar a pencil point drawn across the inner face of the shell.

The number of rays varies from 14 to 22, the prevailing numbers being 17 and 18. It is important to determine whether the variability in rays is correlated with the size of the shell, so that there are more or possibly fewer rays as the shells become larger. The rays run roughly dorso-ventrally in an antero-posterior series, much as a mammal's ribs run, except that they start from a single dorsal point, the beak. It might be expected that the greater the antero-posterior axis the greater the number of rays. An inquiry into this relation will be at the same time an inquiry into the relation between number of rays and size, because the length of the antero-posterior axis is one index of size. Again the breadth of the valve is, to a certain extent, independent of the antero-posterior length; and since broad shells have a greater area than narrow shells of the same length of median axes, we might expect the number of rays to be correlated with breadth. The appropriate correlation surfaces having been prepared (Appendices G, H), the following correlation coefficients were calculated: —

Between number of rays and antero-posterior diameter . $r = -0.074 \pm .027$
Between number of rays and breadth, right valve . . . $r = -0.057 \pm .027$

The departure of r from 0 is negative, the relation being the reverse of our expectations; but as the departure from 0 is only about twice the probable error, it is quite probable that no relation whatever exists between the number of rays and the size and form of the shell. The number of rays is determined very early in the life of the individual and is independent of environment. It follows from this that, for the comparison of races and the study of the effects of isolation, the number of rays is an ideally good character.

b. Variation in the Number of Rays from the Three Localities. — The seriations of the number of rays for the lower valves from the three localities are given in Table VII. (See also Fig. 1.)

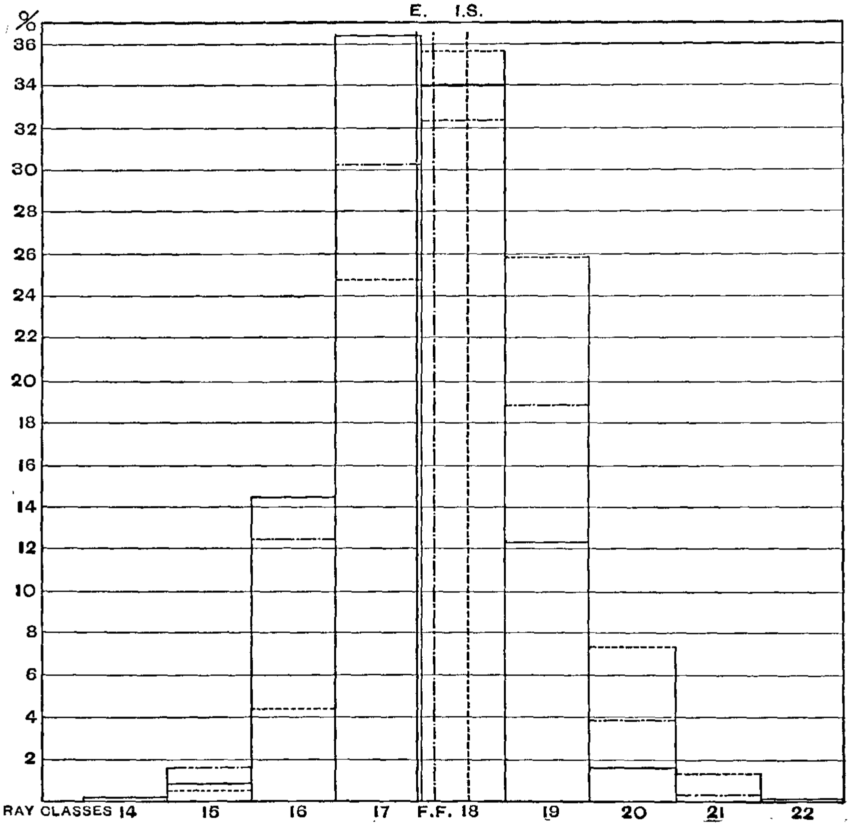

FIGURE 1.

Polygons of percentage frequencies of rays.

———— Eddystone.
- - - - - - - - - - Irish Sea.
—·—·—·— Firth of Forth.

The centroid verticals are drawn at E., I. S., and F. F., respectively.

TABLE VII.

Number of Rays.	FREQUENCIES. Eddystone.		Irish Sea.		Firth of Forth.	
14	1	0.2%	0		1	0.2%
15	5	0.9	3	0.5%	8	1.6
16	77	14.4	27	4.4	63	12.4
17	195	36.4	152	24.8	154	30.3
18	182	34.0.	219	35.7	164	32.3
19	66	12.3	159	25.9	96	18.9
20	9	1.7	45	7.3	20	3.9
21	0		8	1.3	2	0.3
22	1	0.2	1	0.2		
	536		614		508	

From these seriations the constants have been calculated as follows: —

Eddystone Light. Mean, 17.478 ± .029 rays; stand. dev., 1.000 ± .020 rays; coef. of var., 5.72 ± .12%.

$\mu_2 = 0.999495$ $\mu_3 = 0.201477$ $\mu_4 = 3.474913$

$\beta_1 = 0.040654$ $\beta_2 = 3.47842$ Crit. funct. $(\kappa_2) = 0.037046$

$\therefore$ Type IV. Skewness $= 0.08015$; mode at 16.6955.

COMPARISON OF EMPIRICAL AND THEORETICAL POLYGONS.

$$y_0 = 165.89 \, (\cos\theta)^{19.51944} \cdot e^{0.000016\dot{6}\,\theta''}.$$

Class.	Observed.	Theoretical.	Class.	Observed.	Theoretical.
14	1	0.6	18	182	182.3
15	5	8.2	19	66	60.5
16	77	68.4	20	9	10.9
17	195	203.7	21	0	1.5
			22	1	0.0

Irish Sea. Mean, 18.101 ± .029 rays; stand. dev., 1.074 ± .021 rays; coef. of var., 5.931 ± .114%.

$\mu_2 = 1.152672$ $\mu_3 = 0.219822$ $\mu_4 = 4.00141$

$\beta_1 = 0.031552$ $\beta_2 = 3.079312$ Crit. funct. $(\kappa_1) = 0.06398$

Type IV, or normal type. (Skewness, 0.08698.)

COMPARISON OF EMPIRICAL AND NORMAL POLYGONS.

Class	Observed.	Theoretical.	Class.	Observed.	Theoretical.
15	0.5%	0.7	19	25.9%	25.9
16	4.4	5.9	20	7.3	8.4
17	24.8	22.1	21	1.3	1.2
18	35.7	35.7	22	0.2	0.1
				100.1	100.0

Firth of Forth. Mean, 17.673 ± .027 rays; stand. dev., 1.117 ± .019 rays; coef. of var., 6.32 ± .11%.

$\mu_2 = 1.247553$ $\mu_3 = 0.021655$ $\mu_4 = 4.422348$

$\beta_1 = 0.000241$ $\beta_2 = 2.841431$ Crit. funct. $(\kappa_2) = -0.000060$

Normal type. (Skewness, Type I., 0.0069.)

Comparison of Empirical and Normal Polygons.

Class.	Observed.	Theoretical.	Class.	Observed.	Theoretical.
14	0.2%	0.2	18	32.3%	33.2
15	1.6	2.4	19	18.9	17.9
16	12.4	12.1	20	3.9	4.5
17	30.3	29.1	21	0.3	0.6
				99.9	100 0

In all cases the relation between the observed and the calculated frequencies is fairly close.

4. *Individual and Specific Variability.*

In *The Origin of Species* Darwin refers to the "notorious" fact that specific characters are more variable than generic ones. The evidence he presents is, naturally enough, wholly qualitative. Brewster (1897) first tested this law quantitatively, making use of various data derived from mammals, and found it to be confirmed. Heincke (1898) finds the law to hold good for races of herring. He says: "Die Rassen des Herings unterscheiden sich in sehr vielen und im allgemeinen in denselben Eigenschaften von einander, in denen die Species der Gattung Clupea von einander verschieden sind. Nur sind die Unterschiede der Rassen meistens, aber nicht immer, kleiner als die der Species" (quotation from Duncker, 1899, p. 366). The genus *Pecten* affords a good opportunity to test this, especially in the case of the relative variability of the anterior and the posterior "ear," or half, of the hinge line. For in some species of *Pecten* the posterior ear is nearly equal to the anterior ear. In other species it is very much less. The anterior, or notched, ear is the more constant; it may be regarded as a generic character. The posterior ear may be regarded as a specific character; is it more variable than the anterior ear?

a. Geographical Variation of the Ears.—First, a brief consideration of the differences in hinge proportions in the different localities. The different species of *Pecten* differ greatly in the length of the hinge in relation to that of the main part of the body. It has seemed worth while to calculate the average ratio of length of hinge joint to length of shell for each locality.

The results are as follows: —

Locality.	$\frac{\text{Length hinge joint}}{\text{Antero-posterior length}}$.
Eddystone Light	.507
Irish Sea	.483
Firth of Forth	.473

Thus the Firth of Forth shells have relatively the shortest hinge joint; those from Eddystone the longest.

We have now to consider the portions that the anterior and posterior ears, respectively, bear to the entire hinge joint. This proportion is important because in many species, e. g., *Pecten varius*, the posterior ear is only half as long as the anterior, or even less; that is, the proportions are 33 per cent and 67 per cent, respectively. In *Pecten opercularis*, also, the posterior ear is the shorter; the proportions for the three localities are as follows: —

Locality.	Anterior %.	Posterior %.
Eddystone	.538	.462
Irish Sea	.520	.480
Firth of Forth	.533	.467

That is to say, near the Eddystone Light *Pecten opercularis* has the posterior ear proportionately shortest (46: 54), while in the Irish Sea the two ears are most nearly equal (48: 52).

b. Variability of the Ears and their Correlation with the Antero-posterior Diameter. — The correlation between the antero-posterior diameter on the one hand and the anterior ear and the posterior ear on the other has been worked out for 541 lower valves from the Irish Sea. The correlation is sensibly the same in both cases; for the anterior ear, $r = 0.768 \pm .009$, and for the posterior ear $r = 0.779 \pm .009$ (Appendices I, K.)

The standard deviation for the length of the anterior ear is 1.375, and for the posterior ear, 1.378; the coefficient of variation is in the two cases 7.97 ± .16 and 9.30 ± .19, respectively. Since these are graduated variates, the coefficient is the more significant index of variability.* The coefficient of variation is 17 per cent higher in the posterior ear than in the anterior ear. So far as this case goes, then, it confirms for a mollusc the results of Brewster for man and mammals and Heincke for fish.

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* See my 1900 paper, p 962.

5. *Color Variations and Abnormalities.*

The three collections differ in general appearance as much as in dimensions; that is, they have certain features in common and others that are peculiar. The upper valves from all localities are more highly colored than the lower valves. This condition is found in pectens in general and is probably due to the greater exposure of the upper valves to the light. The lower valves, however, sometimes exhibit concentric rings of color. *Pecten opercularis* from the three localities tends to have on the upper valve a median light-colored rib and a submarginal one on each side. Concentric rings of color are common, especially in the Irish Sea and Firth of Forth shells. Antero-posterior stripes of white are frequently present in the young shells from all localities, but especially in those from the Forth.

The upper valves from Eddystone are the lightest in color of all in the collection, probably due to their living at a greater depth, where less sun light penetrates. Relatively few barnacles, but many serpula tubes, adhere to them. The Firth of Forth valves are very muddy, indicating that there was some mud on the "shelly" bottom. They are densely covered with barnacles and bear a few serpulas. They vary in color from white to dark red-brown. The Irish Sea valves are, on the whole, the darkest of all, being nearly uniformly red-brown. Of the variations in ground color, yellow is the commonest, being found in all localities. From the Irish Sea I have noticed also orange, pink, salmon, and lavender. The color variation from yellow to red is that characteristic of lipochromes, whose presence has been detected in the shells of *Pecten.**

The *abnormalities* which I have come across may be classified as follows: (*a*) Interference with the development of the shell by organisms growing on it; and (*b*) abnormal rays, due to abnormal crenulations of the mantle.

a. Interference with Development by Growth of Organisms on the Shell. — The effect was repeatedly noticed in valves from Eddystone, and only from here. Numerous *Serpulæ* grew on the valves, and as they reached the free margin of the shell turned toward the opposite valve. Thus a barrier was interposed at the margin to the further enlargement of the shell, which is, consequently, notched at this point. That this peculiar modification of the shell was found in this locality only, is correlated with the fact that there are more *Serpulæ* on the shells here than elsewhere. In a shell from the Firth of Forth the growing edge has been interrupted by a mat of escharine *Bryozoa.*†

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* *Teste*, Newbigin (1898), 189.

† No. 15775, University of Chicago collection.

b. Abnormal rays are found in all three collections and not more frequently in one valve than the other. Not all rays are equally apt to be affected, but only those near the middle or near the extreme margin. It is especially the two rays at the ends of the series and five on each side of the middle that are affected. The following cases may be cited.

Type 1. An additional (single) ridge appears on the side of a normal double ridge* (No. 15821). The extra ridge is of ontogenetically recent origin and cannot be traced to the young shell, but is, as it were, budded off from the side of the adjacent normal ridge.

Type 2. An additional (single) ridge is interpolated midway between two normal double ridges (No. 14553). In the case given there is only a ridge on the inside of the valve and no corresponding groove on the outer side.

Type 3. The lateral ridge or one that occupies the middle of the furrow shows various gradations from a single ridge to a more or less completely double one (Nos. 13665, 16258). The groove showing the extra ridge lies in a ray that was originally typical but suffered an injury when 20 mm. in diameter. Since then there has been a tendance toward bifurcation of the ray (No. 8140). In No. 13665 the evidence of injury is more doubtful; but the tendency to double began at a corresponding age of the mollusc.

Type 4. Two narrow but normally double ridges are in contact, or the groove between them is abnormally narrow (Nos. 13594, 13759, 14034). In No. 13794 an injury to the edge of the shell and mantle occurred when the pecten was of 10 mm. diameter, and twelve rays were laid down as a more or less unconformable continuation of the young rays. In the readjustment of new rays to each other two rays come to lie close together. A similar accident had occurred in No. 13759, but in No. 14034 a groove has been plainly interpolated.

Type 5. A lateral or median interpolated ridge may become obsolescent, and a groove of unusual width result (Nos. 13555, 14610, 13912, 13760, 15691). In No. 13555 three grooves arose, after an accident (?), from four, and one of them gradually broadened preparatory to division. Nos. 13912, 14610, 13760, and 15691 have had a similar history.

To sum up, the abnormal rays are induced by abnormal conditions, as follows: (1) A rib may differentiate off from one of the main ridges as an independent ray; (2) a rib may widen and divide into two equal ribs; (3) a rib may arise late in life independent of other ribs and gradually gain a size coördinate with the others; (4) a ridge may appear on the inside of a valve without a corresponding groove on the outside (a condition that is normal in the genus *Amussium*); (5) after an injury, or when new shell is laid down unconformably upon old, one or more rays may be incompletely or abnormally regenerated.

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* A rib, as seen from the inside face of the shell, has square, angular edges which usually are elevated as ridges. There is such a ridge running along each edge of the rib; i. e., the complete normal rib has a double ridge.

6. *Definition of the Form Units.*

It is clear from the foregoing sections that the pectens from the different localities are sensibly different. Upon just such differences as these, varieties have, in the past, been named and described at length, but totally inadequately, by means of adjectives. I have no interest in applying a name to the form units, but I think it is worth while to define them quantitively, as in Table VIII. To be precise, this definition holds only for the right valve of *Pecten opercularis* coming from the localities which have been defined in detail and collected in October or November. Indeed these definitions present only the condition, as near as half a thousand random samples can give it, of the form units in the autumn of 1902. They afford standards of comparison with future years, which cannot be said of qualitative definitions.

TABLE VIII.

PROPORTIONS.

	Eddystone.	Irish Sea.	Firth of Forth.
Maximum dorso-ventral diameter	†70 mm.	77 mm.	*80 mm.
Ratio a-p. to d-v. diam. when d-v. diam. = 67 mm.	*1.067	1.061	†1.039
Ratio hinge length to a-p. diameter	*0.507	0.483	†0.473
Ratio anterior ear length to hinge length . . .	*0.538	†0.520	0.533
Ratio posterior ear length to hinge length . . .	†0.462	*0.480	0.467
Half globosity at length of 53 mm.	*0.151	*0.151	†0.132

RAYS.

	Eddystone.	Irish Sea.	Firth of Forth.
Average number of rays . . .	†17.478 ± .029	*18.101 ± .029	17.673 ± .027
Standard deviation, rays . . .	† 1.000 ± .020	1.074 ± .021	* 1.117 ± .019
Coefficient of variability . . .	† 5.72 ± .12%	5.93 ± .11%	* 6.32 ± .11%
Theoretical mode	†16.606	*18.101	17.673
Skewness of distribution polygon	0.0802	* 0.0870	† 0.0069

CORRELATION COEFFICIENTS.

	Eddystone.	Irish Sea.	Firth of Forth.
A-p. diam. and d-v. diam.	†0.96997 ± .00124	0.97596 ± .0093	*0.98495 ± .00063
D-v. diam. and partial transverse diam. . . .	0.8446 ± .0063	†0.7840 ± .0081	*0.8497 ± .0064
Sum of symbols . .	*4 + †7 = 11	*5 + †2 = 7	*5 + †5 = 10

Form unit with the greatest number of extreme quantities: Eddystone.

Form unit with the next greatest number of extreme quantities: Firth of Forth.

Form unit with the least number of extreme quantities: Irish Sea.

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* Maximum. † Minimum.

a. Eddystone. — This form unit is, on the whole, the most extreme, being such in eleven out of the thirteen (not always independent) measurements. It has the least minimum dorso-ventral diameter, and therefore the greatest ratio of a-p. to d-v. diameter, when the d-v. diameter equals 67 mm. It has the smallest ratio of posterior ear length to total hinge length, and consequently it has proportionately the longest anterior ear. Its globosity is high. Its average as well as modal number of rays is least. The index and coefficient of variability are both least. On the other hand it has relatively the longest hinge joint. In the skewness of ray distribution and in the correlation of d-v. diameter and partial transverse diameter it is intermediate. And, it may be added, the upper valves are lightest in color. In general, the right valves are the smallest, longest, most globose, and lightest in color of all the lots; they have the longest hinge line and the beak lies farthest posteriorly on the hinge. The number of rays is smallest and least variable. The d-v. and a-p. diameters are least correlated.

b. Firth of Forth. — This form unit is least like the Eddystone form unit. The right valves are largest and roundest and flattest of all the lots; they also have the shortest hinge line. The variability of the rays is greatest, but their skewness least. Their correlations are high. The average and modal number of rays and the position of the beak on the hinge are intermediate.

c. Irish Sea. — This form unit is, in most respects, intermediate between the others. It has, however, the greatest mean and modal number of rays, with the greatest skewness of their distribution; also its beak lies farthest forward on the hinge. Its correlations are relatively slight.

Can we correlate these peculiarities of the form units with any facts of physiography? The correlation that appears at once is with latitude. Eddystone is at N 50° 15′; the Irish Sea at 54° 18′; the Firth of Forth at 56° 5′. The shells *occurring at extremes of latitude are extreme in* qualities also; and difference of latitude means difference of temperature. Thus the mean annual marine isotherm of 57° F. for the surface in September runs through the Irish Sea near the Isle of Man; it runs 500 kilometres south of the Firth of Forth in the North Sea and 300 kilometres north of Land's End. Clearly, then, the sea temperature of the Irish Sea is intermediate between that of the Firth of Forth and that of Eddystone. Likewise in density of water and depth the Irish Sea locality is intermediate between the other two.

Again, regarding *Pecten opercularis*, which is closely allied to *P. irradians* of the western Atlantic and *P. ventricosus* of the eastern

Pacific, to be, like them, semi-tropical species in origin, then the Eddystone scallops will lie nearest the centre of distribution and the Firth of Forth will be in an extreme position; i. e., nearest to the northern limit of the species, — the Lofoten Islands (where it occurs, however, only at a great depth). The high correlations of the Forth form unit likewise indicate that it is highly specialized; hence not primitive but an outlying branch.

V. General Discussion.

1. *The Inconstancy of the Coefficient of Correlation in Allied Races.*

The view that the coefficients of correlation of any two organs of a species will be constant for all form units of the species, or even for distinct but closely related species, has been long maintained. Weldon (1892, p. 9) was led by a suggestion of Galton to compare the coefficient of correlation between length of carapace and length of post-spinous portion of *Crangon vulgaris* from five localities, and he found that they varied between .81 and .85, which, considering the roughness of the methods then in use, was an approximately uniform result. In 1893 he determined a large number of coefficients between organs in *Carcinus mœnas* from Naples and from Plymouth. Here the results were not very close (e. g., .60 ± .01 : .70 ± .01 ; .50 : .55 ; .71 : .78, etc.), but Weldon concluded that, on the whole, they spoke for the identity of the coefficients. Nevertheless, the differences were often greater than the probable errors, as calculated by Warren (1896) ; the Plymouth shells appearing, on the whole, the more closely correlated. Warren finds, moreover, that the correlations of corresponding organs of a different genus (*Portunus*) hardly differ more from those of *Carcinus mœnas* than do the correlations of the *Carcinus* from the two localities.

Pearson (1896, pp. 267, 280) pointed out that Weldon's values of r proved rather the fact of variation of the correlation coefficient; and he obtained the following values of r between the length and the breadth of human skulls: Naquada race, $r = 0.2705$; modern Germans, $r = 0.2849$; modern Parisians, $r = 0.0474$; modern French peasants, $r = 0.1265$. Also in Dr. Lee's paper on the skull (1901) we find such discrepancies as these: capacity and length: r for Aino, 0.89 ± .01 ; for Germans, 0.52 ± .05; length and height: r for Aino, 0.50 ± .05 ; for Germans, —.10 ± .07.

In the present paper the coefficients run quite close, but sometimes the difference is several times the probable error. Thus between d-v.

diameter and transverse diameter, r is for Eddystone 0.845 ± .006, and for the Irish Sea 0.784 ± .008. The sum of the probable errors is .014, but the difference in the coefficients is .061, or four times the sum of the probable errors. The results obtained for man are therefore confirmed in these *Mollusca*. The coefficient of correlation of allied races is no more constant than the means of their organs.

2. *Geographical Variation in Relation to Evolution.*

In a letter to Moritz Wagner, Darwin declares that he early appreciated the importance of isolation in the origin of species, and the case of the Galapagos Archipelago was one which led him to that appreciation. And later students of island faunas have been struck by the change of type in going from one constituent to another of any archipelago. The literature on isolation as an evolutionary factor is indeed immense, but only recently have biometric studies been directed to the variation of geographically separated form units. Some of the results of these studies may be considered here.

Geographic races are of two principal kinds: one results from isolation combined with mutation; such are found under essentially *similar* conditions of life. The other kind results from isolation combined with dissimilar conditions of life, and mutation may not occur. The first kind is illustrated, apparently, in the Galapagos Islands. Each island has its peculiar form of lizards and sparrows belonging to genera that are spread over the whole archipelago. Yet the conditions of life on adjacent islands, remnants as they probably are (Baur, 1892) of a single island, must be remarkably similar — as similar as the parts of one of the larger islands, in which, however, only one form occurs. Again, the successive valleys of the island of Oahu, as we know from Gulick (1873) and others, are remarkably alike in environmental conditions, yet the snails of the genus *Achatinella* inhabiting them are very unlike. Mayer (1902) has shown that the same thing is true for the partulas of Tahiti. He says: —

> "It is probable that geographical isolation plays a most important part in the formation of new species. If two valleys be adjacent, their snails are closely related each to each, whereas the wider the separation between any two valleys, the more distant the relationship between their snails. The ridges between the valleys, being either barren or covered with vegetation unsuitable to the snails, afford barriers over which the animals must find it more or less difficult to pass. . . . As far as the very limited observation of the writer goes, there appears to be no difference in the character of the

snails in different parts of the same valley. The difference between any two adjacent valleys is, however, very marked.

"All the snails of Tipærui valley are dextral, while all of the same species in Piræ valley are sinistral. In the two intermediate valleys of Hamuta and Fautaua some individuals are dextral and some are sinistral."

Now it is clearly absurd to ascribe the dextrality of all the snails of one valley to one condition of life and the sinistrality of all the snails of another valley to a different condition of life. Sinistrality and dextrality arise suddenly in various groups and are strongly inherited qualities having no close relation to environmental conditions.

The opposite extreme of geographic races is induced, or at least influenced, by environmental conditions. Such is the case with some species of North American birds which are dark in the moist northwest of the United States and ash-colored in the southwest. Years ago Allen (1871) pointed out that northern mammals spreading southward tend to diminish in size with latitude, or as the temperature increases.

The foregoing types of geographic races are extremes. There will be in the different localities all grades of diversity of environmental effect from zero up. In the case of the *Pecten opercularis* studied by me there are diverse environmental conditions of temperature, depth, and density of the water in the three localities, and these may all have contributed to making the shells from these localities dissimilar. It will be worth while to compare the results here gained with those derived from other quantitative studies on geographic races.

One of the first to study race differentiation quantitatively has been Heincke. His great work (1898) is, unfortunately, little known to the mass of naturalists.* His measurements of thousands of herrings of the North Sea and elsewhere led him to recognize that races of herrings exist of which two main divisions are physiologically isolated by reproducing in the spring and in the autumn respectively, while subdivisions of these breed in particular and widely separated localities. Moreover the young develop in different conditions of water density; some in the open sea and others in brackish-water harbors. Heincke is convinced that the cause of the origin of the diverse herring races is a direct action of dissimilar conditions of life. Similarly, Kyle (1900) has found by quantitative methods that the plaice (*Pleuronectes platessa*, L.) from the Baltic and the North Sea are distinct races, and he has been able, moreover, to distinguish a northern and a southern form in the North Sea. Kyle attributes the differences to the dissimilar temperatures and salt content of the water. The brackish-water forms from the Baltic and the Zuider Zee have a smaller number of vertebræ than that from the North Sea. An increase in temperature brings about a decrease in the number of vertebræ and fin-rays. Jordan (1893) had previously called attention to the law that in fish in general the number of vertebræ tend to decrease toward the tropics. In our pectens the most southern form unit has fewest average rays; but this is not universal for pectens.

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* For reviews of this rather inaccessible work see Duncker (1899) and Redeke (1902).

The mackerel (*Scomber scomber*) of the British Isles has also been studied quantitatively. Garstang (1898), working on a rather too small number of fish from a variety of British localities, distinguished an "Irish" race and a second from the North Sea and English Channel. More extensive is the work of Williamson (1900) on mackerel from the east and west coasts of Scotland. His fish came from three localities forming the angles of a nearly isosceles triangle: Aberdeen on the northeast, Barra and Stornoway on the northwest, and Clyde at the south. While the fin-rays, finlets, and vertebræ do not differ enough to form racial distinctions (on Heincke's method), "the differences in the length of the head, skull, and pectoral fin may with some probability be granted racial distinction."

Among fresh-water fishes also, a comparison of lots from different localities shows clearly marked racial differences. Eigenmann (1895) has shown in a roughly quantitative fashion that in *Leuciscus balteatus* from near the mouth of the Frazer River of the Pacific coast of North America the empirical mode of anal rays is 15; at Sicamous, altitude 1300, it is 17; and at Griffin Lake in the same basin, but at 1900 feet, there are 19 rays. Yet the anal rays of fishes have usually so constant a number that they are used as specific or even generic criteria. Even in adjacent lakes two lots of a species have been found by Moenkhaus (1895) to have an average difference of one in the number of dorsal rays. Thus *Etheostoma caprodes* has in Turkey Lake, Indiana, an average of 14.80 ± .02 dorsal rays; in Tippecanoe Lake 15.81 ± .03 dorsal rays. These two lakes are only six miles apart, but the former drains into the Great Lakes and the latter into the Wabash and Mississippi rivers. The lakes are essentially similar in size and origin. The difference in the number of fin-rays of their fish is probably due to the preservation of mutations by isolation.

In the marine gastropod *Mollusca* similar geographic races have been studied quantitatively. Thus Bumpus (1898) has shown that the shore snails, *Littorina littorea*, from different localities of Great Britain and North America, have a proportion of breadth to height of shell varying from 88 to 92.5 per cent, and as Duncker (1898) has pointed out, the shells from Great Britain are in all cases much less variable ($\sigma = 2.3$) than those from North America ($\sigma = 2.4$ to 3.0). Recently Miss Dimon (1902) has shown in *Biometrika* that the absolute dimensions and proportions of the mud snail, *Nassa*, vary in different localities of the same harbor, and in this case the shells are subjected to diverse environmental conditions.

Bateson (1889), studying cockles from old beaches of southwest Asiatic seas that are gradually becoming salter through evaporation, has demonstrated most clearly a change in proportions of the shell with increasing salinity.

In some studies on *Pecten irradians* of our coast from Long Island, New York, to Pensacola, Florida, I was struck by the gradual change of the shells from place to place; a change of such a nature that one might say that the difference in the place modes was a function of the spacial interval between the places in question. This first study on the pectens of Europe yields a similar result. Three lots of one species collected at three points on the coast of Great Britain are measurably unlike in size, proportions, and average number of rays. And when the lots are arranged in the order *a*, *b*, *c*, in which *a* and *c* are the geographical extremes, they are found to be the biological extremes also.

Lastly, the evidence from the shells we have examined bears upon De Vries's law of mutation. Where the environmental conditions of the isolated form units are similar the differences met with are easily accounted for on the assumption of mutations which are preserved. Where, on the other hand, the environmental conditions are dissimilar it is obvious that they must produce a change either through their "direct and definite" action * or possibly by selection. To deny that environment may act directly to produce profound, eventually *specific* changes is to deny the evidence of some of the best experimental work in evolution, and this experimental work has also proved the inheritance of these environmentally induced changes. The mutation theory errs, then, in stating only a half truth. Through mutation, and also through the direct action of environment, specific changes may be produced.

HULL ZOÖLOGICAL LABORATORY,
UNIVERSITY OF CHICAGO, April 25, 1903.

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* In the sense of Darwin in his Origin of Species, chapters I and V.

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APPENDIX A.

Correlation Surface: Antero-posterior Diameter and Dorso-ventral Diameter.

EDDYSTONE LIGHT, LOWER VALVE.

Antero-posterior Diameter. $M = 55.251 \pm 0.201$; $\sigma = 6.916 \pm 0.142$.

Dorso-ventral Diameter. $M = 53.080 \pm 0.166$; $\sigma = 5.689 \pm 0.117$.

	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	77	78		
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57	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	9	8	9	13	3	1	.	.	.	.	.	.	.	.	.	.	.	44
58	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	3	8	12	6	9	.	.	.	.	.	.	.	.	.	.	.	39
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.	1	1	3	2	7	4	13	9	20	21	18	10	17	13	14	14	9	24	30	32	41	45	33	34	41	23	24	10	7	3	1	5	2	1	2	1	2	=	537

APPENDIX B.

Correlation Surface: Antero-posterior Diameter and Dorso-ventral Diameter.

IRISH SEA, LOWER VALVE.

Antero-posterior Diameter. $M = 66.493 \pm 0.173$; $\sigma = 6.361 \pm 0.123$.

Dorso-ventral Diameter. $M = 62.969 \pm 0.141$; $\sigma = 5.189 \pm 0.100$.

	25	37	38	40	41	42	44	45	46	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80		
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APPENDIX C.

Correlation Surface: Antero-posterior Diameter and Dorso-ventral Diameter.

FIRTH OF FORTH, LOWER VALVE.

Antero-posterior Diameter. $M = 67.181 \pm 0.245$; $\sigma = 8.152 \pm 0.174$.

Dorso-ventral Diameter. $M = 64.780 \pm 0.214$; $\sigma = 7.104 \pm 0.151$.

	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	86		
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APPEN

Correlation Surface: Transverse

EDDYSTONE LIGHT,

Transverse diameter in tenths of millimetres.

Dorso-ventral diameter in millimetres. $M = 53.084 \pm 0.166$; $\sigma = 5.691 \pm 0.117$.

	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82
37	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
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DIX D.

Diameter and Dorso-ventral Diameter.

LOWER VALVE.

$M = 79.493 \pm 0.324$; $\sigma = 11.127 \pm 0.229$.

83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	110	116	122	124		
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.	1	2	1	.	2	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	30
4	2	2	2	1	2	1	1	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	43
4	4	1	2	3	1	1	3	.	1	1	1	1	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	40
3	4	4	5	4	4	2	5	.	1	2	.	2	.	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	63
3	6	1	7	4	1	3	1	1	1	.	2	.	2	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	45
4	4	1	2	6	2	.	1	1	1	.	1	.	3	.	.	2	1	.	.	.	.	.	.	.	.	.	.	.	.	39
2	1	1	2	.	3	2	2	.	.	1	.	2	3	1	1	.	.	.	.	.	1	.	.	.	.	.	.	.	.	26
1	.	.	1	.	1	1	1	2	1	1	2	1	1	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	15
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23	22	13	24	23	22	10	16	5	8	6	8	7	12	7	3	2	2	.	1	.	3	.	1	1	1	1	1	1		= 537

APPEN

Correlation Surface: Transverse

Irish Sea,

Transverse diameter in tenths of millimetres.

Dorso-ventral diameter in millimetres. $M = 62.979 \pm 0.141$; $\sigma = 5.190 \pm 0.100$.

	40	61	63	65	66	67	68	69	71	72	73	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97
27	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
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57	.	.	.	.	.	.	.	.	.	.	.	1	1	.	.	2	.	.	1	.	1	.	1	2	.	1	.	2	.	.	.	.	1
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59	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	1	.	2	.	1	3	4	1	.	2	4	1	1	1	.	.	.
60	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	1	.	4	4	6	6	3	3	5	1	4	1	.
61	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	2	2	2	5	1	2	2	7	1	2	3
62	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	2	.	3	2	1	3	4	2	4	6	7	1	5
63	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	2	1	1	2	1	3	3	1	4	4	3	2
64	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	1	1	2	1	2	5	1	2	5	4	1	2	.
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67	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	1	1	1	1	.	2
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	1	2	1	1	2	1	1	1	1	1	2	4	4	4	3	4	1	1	6	6	10	11	24	23	14	30	24	18	25	30	23	18	18

DIX E.

Diameter and Dorso-ventral Diameter

Lower Valve.

$M = 96.607 \pm 0.272$; $\sigma = 10.014 \pm 0.193$.

98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	124		
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2	1	2	1	1	1	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	48
.	2	1	1	.	1	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	36
2	4	5	2	1	3	3	2	1	1	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	66
6	4	5	4	1	.	1	.	2	3	2	.	2	.	.	.	2	.	.	.	.	.	.	.	.	.	60
4	5	3	4	3	2	3	2	.	.	1	1	1	.	2	.	.	.	.	.	1	1	.	.	.	.	62
5	3	8	3	3	3	1	4	1	6	.	.	1	.	.	.	3	.	1	.	1	.	.	.	.	.	60
2	2	2	3	.	5	3	2	2	3	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	41
3	1	2	2	4	2	3	1	.	2	2	.	2	6	.	.	.	.	.	.	.	.	.	.	.	.	37
.	1	3	2	.	.	2	4	3	2	.	1	2	1	.	.	.	2	.	1	.	.	1	.	1	.	31
2	1	1	2	.	1	2	5	2	1	1	1	2	1	1	2	.	.	.	.	.	.	.	.	.	.	26
.	.	.	2	1	.	2	.	.	1	.	1	.	.	1	.	2	1	.	1	.	1	.	.	.	.	14
.	1	.	.	.	.	1	1	1	2	1	.	1	.	.	.	.	.	.	.	.	1	.	.	.	.	9
.	.	.	1	.	.	.	.	.	.	1	3	1	.	2	1	1	.	.	.	.	.	.	.	.	.	10
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28	26	32	27	14	20	22	22	13	21	10	9	13	8	7	3	8	3	2	2	2	4	1	1	1		=614

APPEN

Correlation Surface: Transverse

Firth of Forth,

Transverse diameter in tenths of millimetres.

Dorso-ventral diameter in millimetres. $M = 64.785 \pm 0.325$; $\sigma = 7.097 \pm 0.230$.

	57	58	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87
45	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
46	.	.	1	1	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
47	1	1	.	.	.	.	.	1	.	1	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.
48	1	1	.	.	1	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
49	.	.	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
50	.	.	1	.	2	2	.	.	1	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.
51	.	1	.	1	.	1	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
52	.	.	.	.	1	.	.	.	.	1	1	.	.	.	.	1	.	.	1	.	.	.	.	.	.	.	.	.	.
53	.	.	.	.	.	.	.	.	1	1	2	.	1	.	1	.	.	.	.	1	.	.	.	.	.	.	.	.	.
54	.	.	.	.	.	.	.	.	.	.	.	2	.	1	4	.	1	.	1	1	.	.	.	.	.	.	.	.	.
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56	.	.	.	.	.	.	1	.	.	.	2	.	.	.	.	2	.	.	1	.	.	2	.	.	.	.	1	.	1
57	.	.	.	.	.	.	.	.	.	.	.	1	.	1	1	.	2	.	.	1	1	.	1	.	1	.	1	.	.
58	.	.	.	.	.	.	1	.	.	.	.	.	.	.	1	.	2	2	1	2	4	3	1	1	.	.	.	1	.
59	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	2	1	2	.	.	2	.	1	2
60	.	.	.	.	.	.	.	.	.	.	.	.	1	1	.	.	.	1	1	2	.	.	1	1	4	.	.	1	2
61	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	.	2	.	3	1	2	1	.	2	3
62	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	2	.	2	2	.	3	1	.	.	1	2	3
63	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	1	1	1	2	1	1	1	2	2	3
64	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	2	.	5	.	2	3	1	2	1	.
65	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	1	.	.	.	2	.	1	1	1	2	1	5
66	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	2	1	3	.	1	.	1	3
67	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	1	2	.	.	2	1	.
68	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	2	1	.
69	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	1	1	2
70	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	1	.	1	1	.
71	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	1	.	.
72	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	1
73	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
74	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1
75	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
76	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
77	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
78	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
79	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
80	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	2	4	3	3	4	3	4	1	2	3	5	4	2	4	10	4	9	8	8	16	13	16	16	14	16	8	17	16	26

DIX F.

Diameter and Dorso-ventral Diameter.

Lower Valve.

$M = 87.594 \pm .325$; $\sigma = 10.87 \pm .231$.

88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	110	111	112	115		
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	3
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	5
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	4
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	7
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	4
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	5
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	7
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	10
.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	11
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	10
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	10
.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	20
1	1	.	1	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	15
.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	16
1	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	19
3	.	.	1	1	2	.	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	26
2	3	1	1	.	.	.	1	2	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	27
.	1	1	2	.	1	1	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	24
3	3	.	1	3	.	1	.	1	1	.	.	1	.	.	.	2	.	.	.	.	.	.	.	.	.	31
2	2	.	2	2	.	1	1	1	.	.	1	.	1	.	1	.	.	.	.	.	.	.	.	.	.	26
2	2	2	1	1	1	.	.	1	.	1	.	1	1	.	.	.	.	.	.	.	.	.	.	.	.	20
1	3	2	3	.	1	1	2	.	2	3	1	1	1	.	.	.	.	.	.	.	.	1	.	.	.	26
1	2	2	2	1	.	1	3	1	3	2	.	3	.	1	.	.	.	1	.	.	.	.	.	.	.	28
2	1	3	2	.	3	2	2	2	.	1	1	1	1	1	.	2	.	1	.	.	.	.	.	.	.	29
.	.	4	2	.	1	.	1	.	.	4	6	4	.	1	1	2	.	.	.	.	1	.	.	.	.	29
2	1	1	1	1	4	1	3	3	1	1	6	1	.	1	1	.	1	.	1	.	1	.	.	.	.	33
.	1	.	.	.	1	2	.	1	3	.	2	1	2	1	1	.	.	.	.	1	.	.	.	.	.	16
.	.	1	.	.	.	3	2	.	.	.	1	2	1	1	1	.	4	.	.	2	.	.	.	.	.	19
.	.	.	1	.	.	.	1	.	.	.	2	.	.	2	.	.	.	.	.	.	.	.	.	1	.	7
.	1	.	.	.	.	.	.	.	.	1	.	1	.	.	.	.	.	.	2	.	.	.	1	.	.	6
.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	1	.	1	.	3
.	1	.	.	.	1	.	.	.	.	.	.	1	.	.	.	1	.	1	.	.	.	.	.	.	.	5
.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1	.	.	.	.	.	.	.	.	.	.	2
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	1	.	2
20	24	19	20	9	15	13	19	12	10	13	20	19	7	11	6	7	5	3	4	3	2	2	1	3		=508

APPENDIX G.

Correlation Surface: Rays and A.-p. Diameter.

IRISH SEA, LOWER VALVE.

No. of Rays, $M = 18.101 \pm .029$; $\sigma = 1.072 \pm .021$.

Antero-posterior diameter in millimetres. $M = 66.497 \pm 0.173$; $\sigma = 6.352 \pm 0.122$.

	15	16	17	18	19	20	21	22		
25	.	.	1	.	.	.	.	.	.	1
37	.	.	.	1	.	.	.	.	.	1
38	.	.	.	1	.	.	.	.	.	1
40	.	.	1	.	.	.	.	.	.	1
41	.	.	.	.	.	1	.	.	.	1
42	.	.	1	1	.	.	.	.	.	2
44	.	.	.	1	.	.	.	.	.	1
45	.	.	.	.	1	.	.	.	.	1
46	.	.	.	.	.	.	1	.	.	1
50	.	.	.	.	.	2	.	.	.	2
51	.	.	1	.	.	.	.	.	.	1
52	.	.	1	.	.	.	1	.	.	2
53	.	.	.	1	2	1	.	.	.	4
54	.	.	1	3	.	.	.	.	.	4
55	.	1	.	.	.	.	.	.	.	1
56	.	.	2	2	1	1	.	.	.	6
57	1	.	.	2	1	2	.	.	.	6
58	.	.	4	3	3	.	.	.	.	10
59	.	1	2	9	2	1	.	.	.	15
60	.	.	3	8	3	3	.	.	.	17
61	1	1	4	5	8	2	.	.	.	21
62	.	2	6	14	9	3	1	1	.	36
63	.	1	8	8	9	4	.	.	.	30
64	.	2	8	9	14	3	.	.	.	36
65	.	2	12	7	14	1	.	.	.	36
66	.	1	9	11	10	.	1	.	.	32
67	.	3	16	21	14	5	.	.	.	59
68	.	2	10	18	15	3	2	.	.	50
69	.	1	10	20	7	1	.	.	.	39
70	.	2	6	18	13	5	.	.	.	44
71	.	2	10	16	6	.	.	.	.	34
72	1	1	13	11	7	2	.	.	.	35
73	.	1	4	7	3	2	1	.	.	18
74	.	.	8	6	3	1	.	.	.	19
75	.	1	4	5	5	1	1	.	.	17
76	.	2	2	5	2	1	.	.	.	12
77	.	1	3	3	2	.	.	.	.	9
78	.	.	1	.	2	.	.	.	.	3
79	.	.	1	1	1	.	.	.	.	3
80	.	.	.	2	1	.	.	.	.	3
	3	27	152	219	158	45	8	1	= 614	

APPENDIX H.

Correlation Surface: Rays and Transverse Diameter.

IRISH SEA, LOWER VALVE.

Transverse diameter in millimetres. $M = 96.571 \pm 0.273$; $\sigma = 10.015 \pm 0.193$.

	15	16	17	18	19	20	21	22		
40	.	.	1	.	.	.	.	.	.	1
61	.	.	.	1	.	1	.	.	.	2
63	.	.	1	.	.	.	.	.	.	1
65	.	.	.	.	.	1	.	.	.	1
66	.	.	.	1	.	.	1	.	.	2
67	.	.	.	1	.	.	.	.	.	1
68	.	.	.	.	.	1	.	.	.	1
69	.	.	1	.	.	.	.	.	.	1
71	.	.	.	1	.	.	.	.	.	1
72	.	.	1	.	.	.	.	.	.	1
73	.	.	1	.	.	1	.	.	.	2
76	.	.	3	.	1	.	.	.	.	4
77	.	.	.	3	.	1	.	.	.	4
78	.	.	.	2	1	.	1	.	.	4
79	.	1	.	.	2	.	.	.	.	3
80	.	.	1	2	1	.	.	.	.	4
81	.	.	1	.	.	.	.	.	.	1
82	.	.	.	.	1	.	.	.	.	1
83	.	.	1	1	2	2	.	.	.	6
84	.	.	.	4	2	.	.	.	.	6
85	.	1	1	5	2	1	.	.	.	10
86	.	2	4	3	2	.	.	.	.	11
87	.	2	7	6	6	2	.	1	.	24
88	.	.	7	8	5	3	.	.	.	23
89	.	.	2	6	6	1	.	.	.	15
90	.	1	9	15	2	2	.	.	.	29
91	2	1	3	9	9	2	.	.	.	26
92	.	.	9	6	3	.	.	.	.	18
93	.	2	7	9	6	1	.	.	.	25
94	.	1	6	5	15	3	1	.	.	31
95	.	.	6	8	5	3	.	.	.	22
96	.	2	3	7	5	2	.	.	.	19
97	.	.	5	7	5	.	.	.	.	17
98	.	2	4	8	10	2	1	.	.	27
99	.	1	9	5	7	2	2	.	.	26
100	.	2	8	12	9	1	.	.	.	32
101	.	1	7	6	10	3	.	.	.	27
102	.	.	5	6	3	.	.	.	.	14
103	.	1	5	9	3	2	.	.	.	20
104	.	.	3	11	3	3	1	.	.	21
105	.	1	5	10	6	.	.	.	.	22
106	.	.	4	5	2	2	.	.	.	13
107	.	1	6	11	2	.	1	.	.	21
108	.	.	3	4	2	1	.	.	.	10
109	.	1	1	.	5	2	.	.	.	9
110	.	.	2	4	7	.	.	.	.	13
111	.	1	1	6	.	.	.	.	.	8
112	.	1	3	2	1	.	.	.	.	7
113	.	.	1	2	.	.	.	.	.	3
114	.	.	.	3	5	.	.	.	.	8
115	.	.	2	1	.	.	.	.	.	3
116	1	.	1	.	.	.	.	.	.	2
117	.	.	1	.	1	.	.	.	.	2
118	.	.	1	1	.	.	.	.	.	2
119	.	1	.	1	2	.	.	.	.	4
120	.	.	.	1	.	.	.	.	.	1
121	.	.	.	1	.	.	.	.	.	1
124	.	1	.	.	.	.	.	.	.	1
	3	27	152	219	159	45	8	1	= 614	

APPENDICES I, K.

Correlation Surface: Ears and Antero-posterior Diameter.

IRISH SEA. LENGTH OF EARS.

Antero-posterior diameter in millimetres. $M = 66.518 \pm 0.187$; $\sigma = 6.425 \pm 0.132$.	Anterior, millimetres. $M = 17.255 \pm .040$; $\sigma = 1.375 \pm$ e $.028$.																Posterior, millimetres. $M = 14.821 \pm .040$; $\sigma = 1.378 \pm .028$.														
	8	9	10	11	12	13	14	15	16	17	18	19	20	21			7	9	10	11	12	13	14	15	16	17	18	19	20		
25	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	1
37	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	1	.	.	1	.	.	.	.	.	.	.	.	.	.	.	1
38	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	1	.	1	.	.	.	.	.	.	.	.	.	.	.	.	1
40	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	1	.	.	1	.	.	.	.	.	.	.	.	.	.	.	1
41	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	1	.	.	1	.	.	.	.	.	.	.	.	.	.	.	1
42	.	.	.	.	.	1	1	.	.	.	.	.	.	.	.	2	.	.	1	1	.	.	.	.	.	.	.	.	.	.	2
44	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	1	.	.	.	1	.	.	.	.	.	.	.	.	.	.	1
45	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	1	.	.	.	1	.	.	.	.	.	.	.	.	.	.	1
46	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	1	.	1	.	.	.	.	.	.	.	.	.	.	.	.	1
50	.	.	.	.	.	.	2	.	.	.	.	.	.	.	.	2	.	.	.	2	.	.	.	.	.	.	.	.	.	.	2
52	.	.	.	.	.	.	1	1	.	.	.	.	.	.	.	2	.	.	.	2	.	.	.	.	.	.	.	.	.	.	2
53	.	.	.	.	.	.	2	1	1	.	.	.	.	.	.	4	.	.	1	.	2	1	.	.	.	.	.	.	.	.	4
54	.	.	.	.	.	.	1	.	.	2	.	.	.	.	.	3	.	.	.	.	2	1	.	.	.	.	.	.	.	.	3
55	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	1	.	.	.	.	1	.	.	.	.	.	.	.	.	.	1
56	.	.	.	.	.	.	.	2	3	.	.	.	.	.	.	5	.	.	.	.	.	3	1	1	.	.	.	.	.	.	5
57	.	.	.	.	.	.	.	3	1	2	.	.	.	.	.	6	.	.	.	.	.	2	3	1	.	.	.	.	.	.	6
58	.	.	.	.	.	1	.	2	2	3	.	.	.	.	.	8	.	.	.	.	1	5	1	1	.	.	.	.	.	.	8
59	.	.	.	.	.	.	1	2	4	6	.	.	.	.	.	13	.	.	.	.	.	3	8	2	.	.	.	.	.	.	13
60	.	.	.	.	.	.	.	1	8	2	1	.	.	.	.	12	.	.	.	.	.	3	6	3	.	.	.	.	.	.	12
61	.	.	.	.	.	.	1	4	5	8	1	.	.	.	.	19	.	.	.	.	.	3	10	6	.	.	.	.	.	.	19
62	.	.	.	.	.	.	.	1	7	18	2	.	.	.	.	28	.	.	.	.	.	6	16	5	.	.	.	.	.	.	28
63	.	.	.	.	.	.	2	4	7	8	5	.	.	.	.	26	.	.	.	.	.	6	13	7	.	.	.	.	.	.	26
64	.	.	.	.	.	.	.	.	13	14	7	.	.	.	.	34	.	.	.	.	2	3	11	15	3	.	.	.	.	.	34
65	.	.	.	.	.	.	.	.	11	10	10	1	.	.	.	32	.	.	.	.	.	.	20	7	5	.	.	.	.	.	32
66	.	.	.	.	.	.	.	3	6	12	5	1	.	.	.	27	.	.	.	.	.	1	10	9	6	1	.	.	.	.	27
67	.	.	.	.	.	.	.	.	9	27	19	3	.	.	.	58	.	.	.	.	.	4	20	19	13	2	.	.	.	.	58
68	.	.	.	.	.	.	.	.	6	11	23	3	.	.	.	43	.	.	.	.	.	1	9	21	11	1	.	.	.	.	43
69	.	.	.	.	.	.	.	.	1	6	24	1	1	.	.	33	.	.	.	.	.	.	4	15	12	2	.	.	.	.	33
70	.	.	.	.	.	.	.	.	3	9	18	4	2	.	.	36	.	.	.	.	.	.	3	19	14	.	.	.	.	.	36
71	.	.	.	.	.	.	.	.	.	3	19	8	3	.	.	33	.	.	.	.	.	.	.	8	21	4	.	.	.	.	33
72	.	.	.	.	.	.	.	1	.	5	15	7	3	1	.	32	.	.	.	.	.	.	1	9	20	2	.	.	.	.	32
73	.	.	.	.	.	.	.	.	1	2	9	3	1	.	.	16	.	.	.	.	.	.	.	4	10	2	.	.	.	.	16
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CONTRIBUTIONS FROM THE RESEARCH LABORATORY OF PHYSICAL CHEMISTRY OF THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY. — No. 1.

THE ELECTRICAL CONDUCTIVITY OF AQUEOUS SOLUTIONS AT HIGH TEMPERATURES.

I. — *DESCRIPTION OF THE APPARATUS. RESULTS WITH SODIUM AND POTASSIUM CHLORIDE UP TO 306°.*

By Arthur A. Noyes and William D. Coolidge.

Investigations on Light and Heat made or published, wholly or in part, with Appropriations from the Rumford Fund.

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I. Plan of the Investigation.

Only a few isolated experiments have been previously published on the electrical conductivity of salt solutions above 100°. Sack [*] investigated the conductivity of three copper sulphate solutions up to 120°. Maltby [†] found that upon heating up to 237° the conductivity of an aqueous potassium chloride solution steadily diminished. Hagenbach [‡] observed a maximum in the molecular conductivity of a 0.01 normal KCl solution. In all of these experiments the conductivity cell was made of glass and was necessarily very small; therefore, owing to the solubility of glass at these temperatures and to the danger of polarization of the small electrodes used, the results have little significance.

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[*] Wied. Ann., **43**, 212–224 (1891).

[†] Ztschr. phys. Chem., **18**, 155 (1895).

[‡] Drude's Ann., **5**, 276–312 (1901).

Believing that accurate conductivity measurements through a wide range of temperature extending up to or nearly to the critical point would yield results of considerable theoretical interest, we have undertaken an investigation in this direction.

The first and most difficult part of this research consisted in the construction of a conductivity cell composed internally of material unacted upon by aqueous solutions and capable of withstanding without leakage the high vapor-pressure of such solutions up to the critical temperature. After working continuously for three years upon this problem, we have succeeded in overcoming the mechanical difficulties and in constructing a platinum-lined bomb with insulated electrodes which remains perfectly tight at any rate up to 306°, which occasions only an unimportant contamination even in salt solutions as dilute as $\frac{1}{2000}$ molar, which yields conductivity measurements accurate to 0.25 per cent or less, and which at the same time makes possible specific-volume determinations, which are essential to the interpretation of the results. Now that a knowledge of the necessary mechanical devices has been acquired, the making of such a bomb is an easy task for a skilled instrument maker. We shall, therefore, in this article first describe in full detail, with the help of working drawings, the apparatus used, and especially the construction of the bomb, in order to make it readily available for investigators who desire to pursue researches of the same kind or those requiring similar apparatus (such, for example, as a calorimetric bomb).

We have thus far been able to investigate only two dissolved substances, — potassium and sodium chloride, — but have made conductivity and specific volume measurements with their solutions at various temperatures ranging from 26° to 306° and at various concentrations between 0.1 and 0.0005 molar. The work will be extended during the coming year to other substances, and if possible to the critical temperature; but as it will be carried on in part by other workers and under the auspices of the Carnegie Institution, it has seemed appropriate to publish the results already obtained, especially as these seem to justify several interesting conclusions of a general character.

In concluding these introductory statements it gives us much pleasure to acknowledge our great indebtedness to the American Academy of

Arts and Sciences, which by liberal grants made from the Rumford Fund of the Academy has given us substantial assistance in meeting the considerable expense involved in this investigation.

II. Description of the Apparatus.

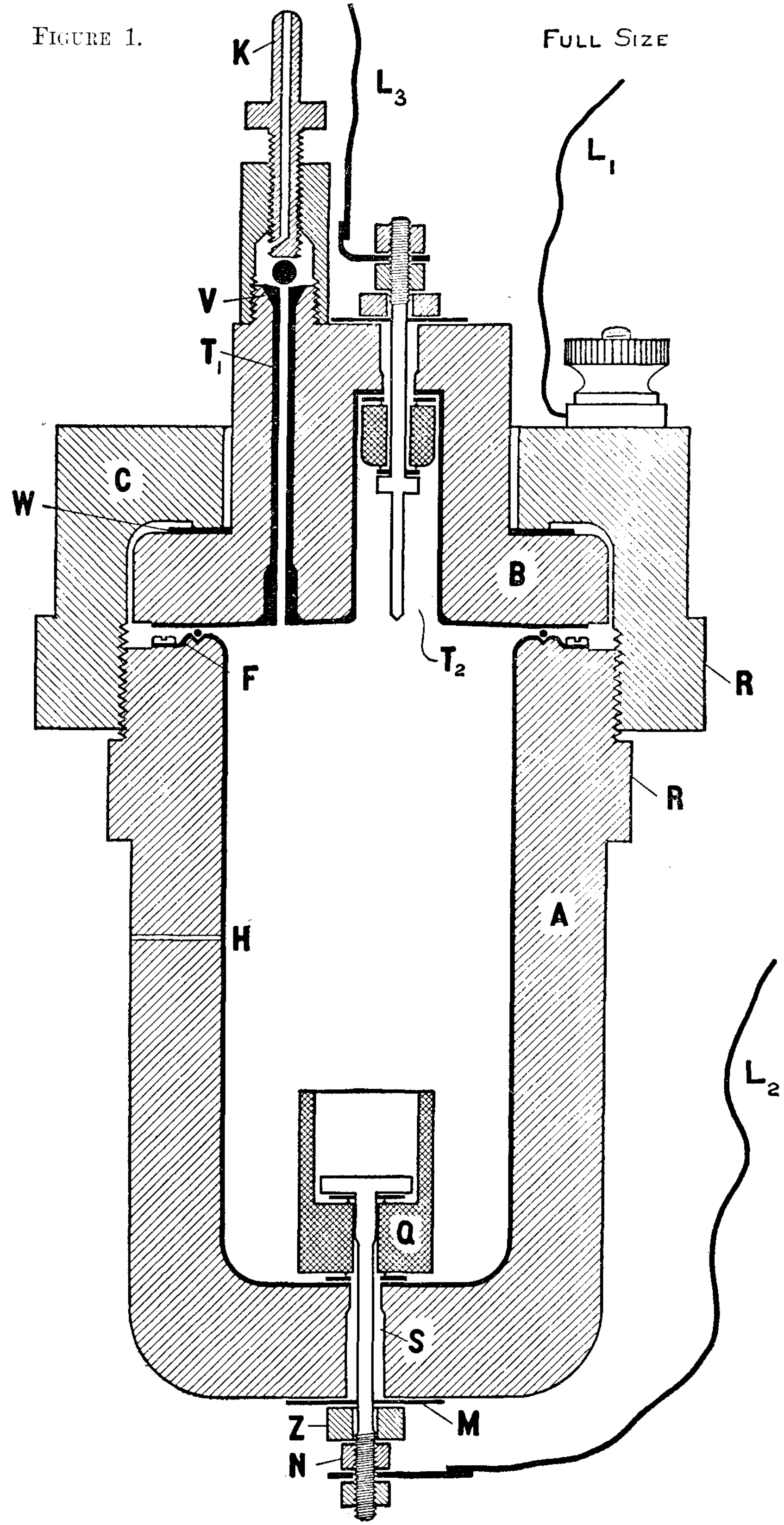
1. *The Conductivity Cell or Bomb.* — A vertical section of this is shown in full size in Fig. 1. It is a cylindrical vessel, A, with a cover, B, which is held in place by the large nut, C. A, B, and C are made of soft crucible steel. To prevent contamination, the bomb is lined throughout with sheet platinum 0.41 mm. thick. The cover joint is made tight by a little packing ring, made of pure gold wire, which fits into a shallow V-shaped groove. As may be seen in the diagram, the platinum lining, indicated by a heavy line, goes under this ring and a little distance beyond it, the outer edge being fastened to the shell by eight small steel screws, of which two are shown. The lower vessel has a capacity of about 122 ccm.

The body of the bomb serves as one electrode, connection being made with it by means of the large binding post on top of the nut C. The second electrode is brought in through the bottom of the bomb and is insulated from the latter by means of the mica washer, M, the air space, S, and the quartz-crystal piece, Q. The body of this electrode is of steel, but its upper part is covered with sheet platinum. On the bottom of the crystal piece is turned a single sharp V-shaped ridge, and this rests on a flat gold washer which is inserted between the crystal and the bottom of the bomb. Another gold washer is placed between the upper part of the electrode and a second V-shaped ridge turned on the upper face of the crystal. The nut N, fitting on the lower, threaded end of the electrode, draws the latter down, thus forcing the ridges of the crystal into the soft gold and making the joints tight. Z is a brass washer which by its greater expansion-coefficient makes up for the difference in the expansion, upon heating, of the quartz-crystal and of that part of the steel electrode which lies within. The second nut, on the lower end of the electrode, serves to bolt on a small copper tag to which the wire L_2 is silver-soldered. The quartz piece, Q, is extended in the form of a cup above the electrode, so as to increase the resistance-capacity of the cell.

In the cover, B, is a narrow cylindrical chamber provided with an "auxiliary electrode," which is insulated in just the same way as the lower electrode. The purpose of this small chamber with the auxiliary electrode is twofold: first, it serves as a safety device, showing that the

Figure 1.

Full Size


K
L3
L1
V
T1
C
W
B
F
T2
R
R
H
A
L2
Q
S
Z
M
N

bomb has not become completely full of liquid; and secondly, it furnishes a means of measuring the specific volume of the solutions. The first provision is necessary since the bomb is designed to withstand the vapor pressure, but not the fluid pressure of the liquid. A knowledge of the specific volume is required in order to calculate the equivalent from the observed conductivity. A measurement of the resistance between L_1 and L_3, together with a measurement of that between L_1 and L_2 when preceded by a calibration which may be made once for all, shows, as will be explained more fully in Section VI., at any time after the solution has expanded sufficiently to come into contact with the auxiliary electrode, just how high the liquid stands, and therefore how much vapor space remains.

The small platinum tube, T_1, serves to exhaust the air from the bomb. The method of doing this will be apparent from the diagram and the following description. The hollow screw K is connected by means of rubber tubing with a Richards water pump, and is at first raised so that air can come out under the little steel bicycle ball which rests on the upper end of the platinum tube. After the air is removed until a pressure gauge shows a pressure within of about 2 cm., and while the pump is still in operation, the part K is screwed down, thus forcing the steel ball onto its seat and closing the end of the tube.

The solution comes into contact with nothing but platinum, quartz-crystal, and gold, except at the top of the narrow tube, T_1, where it may touch the steel ball. The latter could be gold plated; but this has proved unnecessary, since there is scarcely any circulation through the narrow tube.

The lower electrode, as well as the auxiliary electrode and its surrounding tube, are well platinized. The body of the lining is not platinized, since on account of its great surface this is not necessary.

2. *The Conductivity Measuring Apparatus.* — The conductivity was measured by the ordinary Kohlrausch-Wheatstone Bridge method, using the induction coil and telephone. The slide wire was of platinum-iridium; it was one meter in length and 0.4 mm. in diameter. The resistance coils, 2000 ohms in all (or 4000 ohms in a few measurements), were of manganine. The whole conductivity apparatus was mounted on a small portable table so that it could be moved about as the bomb was changed from one heating bath to another. It was always kept at a distance from the heaters. No temperature correction needed to be applied to the resistance coils. Heavy flexible copper leads were used up to within a few centimeters of the top of the heaters, where they were

joined by means of brass connectors to the smaller copper wires, L_1, L_2, L_3, coming from the bomb. A double-throw switch served to connect the conductivity apparatus with L_1 and L_2 or with L_1 and L_3.

3. *The Heaters.* — Conductivity measurements were made at about 26°, 140°, 218°, 281°, and 306°. The first of these temperatures was attained by immersing the bomb in a bath of commercial xylene contained in a double-walled, well-jacketed, metal cylinder. This substance has the advantages that it is a good insulator, non-corrosive, and not very volatile, and that the bomb can be transferred from it directly, without cleaning, into the xylene-vapor bath by which the next higher temperature was attained. The liquid was stirred by a small propeller, and was heated electrically at will with the help of a platinum helix immersed in it.

For all the higher temperatures, vapor baths were employed, as these furnish the only safe and rapid method of heating. The temperature adjusts itself automatically, and can never rise much above the ordinary boiling point, thus giving protection against overheating and undue expansion of the liquid within the bomb, which by completely filling it might cause it to burst. Moreover, if the bomb should spring a leak, it would be dangerous in the case of a liquid bath; for the steam, escaping under such pressure, might throw some of the hot liquid onto the observer. Steam leaking out into the hot vapor, on the other hand, causes no annoyance further than that arising from the odor of the vapor and the loss of the material in the case of the expensive substances. An air bath would, of course, not be open to this objection; but the heating would be extremely slow and non-automatic.

An elevation of one of the heaters, — all of which were substantially alike, — with the bomb in place is presented in Fig. 2. The bath is made of a piece of wrought-iron pipe, A, 16 cm. in diameter and 40 cm. long, with a bottom piece welded in. Near the top two pieces of iron pipe, C, about 2 cm. in diameter and 25 cm. long are screwed in, to serve as condensers. These condenser tubes are given a slight pitch, but their outer ends should not be higher than the top of the heater. To increase their efficiency, a loose roll of iron wire gauze is put into each of them. The top of the bath, which should be turned off square in the lathe, is covered with a large watch glass, D, in which holes are drilled for the thermometer, T, and the lead-wires to the bomb. A tube of thin sheet iron, Q, about 12 cm. in diameter, with a flange at the bottom, is placed in the heater and held in the middle by projecting pins. Small holes are drilled through this tube at the bottom, and two rows of large holes at the top. The function of this tube is to prevent the bottom of the bomb from getting hotter than the top; for, if it does this by ever so little, a constant evaporation and condensation goes on in the bomb, which interferes with the readings of the auxiliary electrode and the specific-volume determinations. The inverted mica cone, N, is put in for the same purpose: it prevents the cold condensed vapor from dripping onto the top of the bomb. These arrangements also protect the bomb more effectually

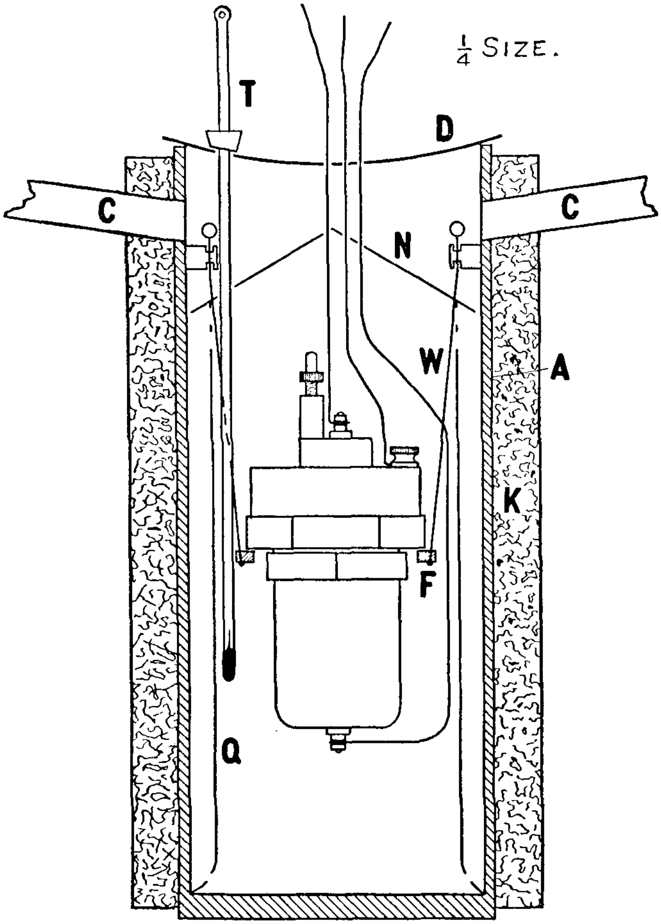


FIGURE 2.

from radiation and convection-currents from the walls of the heater. The holes in the glass cover through which the lead-wires and the thermometer enter are but little larger than these, so as to prevent loss of vapor. The thermometer is supported by means of a cork stopper which rests on the top of the watch glass. The insertion of cork stoppers in the holes is not advisable, as they cause the hot liquid to escape through their pores. The bomb is supported in the heater by means of a brass frame, F, and suspension wires, W, which hang on two steel pins screwed into the walls of the heater. At the top of each of the two suspension wires is a loop, so that by inserting a steel hook in each of these loops, the bomb is easily removed from the bath while still hot. The sides of the heater are well jacketed with asbestos. It is supported on a metal tripod by means of three steel pins, which project through the asbestos covering. It is heated by gas burners below, one sufficing after the bomb and heater have become hot.

Commercial xylene was first used for the 140° bath, but the pure metaxylene was found to give a more constant temperature and one more uniform in the upper and lower parts of the bomb: to prevent the escape of the vapor, it was necessary in this case to cause water to circulate through a jacket surrounding one of the condenser tubes. Pure naphthalene from Kahlbaum was used to give a temperature of 218°, and was found to be an ideal substance. α-bromnaphthalene was employed for the next higher temperature (281°), as it seemed to be the only available substance; it is not convenient, however, since it decomposes slowly upon boiling with formation of tar and hydrobromic acid (which attacks the outside of the bomb): it must therefore be frequently removed from the heater and redistilled. The highest temperature (306°) was maintained with benzophenone, which boils unchanged even after many days of continuous heating.

4. *Thermometers.* — The temperature of the liquid xylene bath was measured with an ordinary thermometer reading directly to tenths of a degree, and this was checked from time to time against a standard Tonnelot thermometer.

For the higher temperatures French mercurial thermometers, made by Alvergniat, with a range of 360° and graduation in degrees, were used. By the use of a little reading telescope these thermometers were read with certainty to 0.1°. They were standardized as described in Section VI of this article. The mercury column was always completely immersed in the vapor, and to take a reading the thermometer was quickly raised only enough to render the meniscus visible above the top of the heater. Repeated trials showed that the temperature of the bath throughout the space surrounding the bomb varied less than 0.1°, so that the exact position of the thermometer made no difference. Care had to be taken, however, that the mica shield above the bomb did not come in contact with the thermometer stem, thus allowing the condensed vapor coming from the shield to run down and cool the bulb. It was feared that the vapor condensing on the upper part of the thermometer itself would have the same effect; but this was proved not to be the case by fastening a small inverted watch glass about midway on the thermometer stem; this carried off the drip from the upper part of the stem, but did not affect the reading.

III. Details of the Construction of the Bomb.

The shell is made of the softest crucible-steel obtainable, because ductility is desired rather than high tensile strength.* Extra weight is not objectionable here, as it would be in the case of a calorimetric bomb; moreover, fear was entertained that a high-carbon steel might be weakened by the repeated heating and cooling to which the bomb was to be subjected. The shell was designed for approximately equal strength throughout. The existing data on the tensile-strength of steel at the high temperatures involved only suffice to show that it is considerably weaker than at ordinary temperature. Owing to this fact, and to the further fact that it was uncertain to how high a temperature the observations would ultimately be carried, a factor of safety of 20-fold was allowed, based upon the tensile strength cold and the critical pressure of water-vapor. The large nut, C, has an ordinary V-thread of 18 turns to the inch. To tighten the nut, the lower part of the bomb is held at R, which is hexagonal, by a wrench bolted to a firm table; while a second wrench, with an effective length of 46 centimeters, is placed on the hexagonal part, R′, of the nut itself. In this way sufficient pressure can be exerted on the gold packing ring to make the metal of which it is composed actually flow into the groove beneath, filling any little scratches or other depressions which may exist in the latter. Since there is a certain thickness of gold and platinum interposed between the cover and the lower part of the bomb, and since these both expand less than steel upon heating, it becomes necessary to use a compensating brass washer, W, between the nut and the cover. The proper thickness can be calculated from the known coefficients of expansion of the three metals. Care must be taken that the bearing surface of the nut, C, on the washer, W, is so large that the upward force of the steam acting on the cover does not compress the brass washer, and thus allow the cover to rise. Care must also be taken, — and this is very important, — that the distance from the centre of this bearing surface to the axis of the bomb is less than the radius of the gold packing ring; otherwise the cover might turn on the ring while the nut was being tightened, which would prevent a tight joint from being secured. Various lubricants have been tried on the screw and on the top of the brass washer; but all of these, with the exception of graphite, have proved to be worse than nothing: finely powdered graphite used dry is entirely satisfactory, however.

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* Bessemer steel would probably be somewhat safer and will be used in future work.

To facilitate the removal of the platinum lining, the inside of the steel shell was made slightly tapering (about 0.05 mm. in 10 cm.), and the little grooves left by the boring tool were carefully ground out.

In working with the bomb it proved to be necessary to drill a number of small holes through the steel shell, one of which is shown at H in Fig. 1. In the present bomb there are about 75 of these (probably half as many would have sufficed) well distributed over all its parts, A, B, and C. These holes are 0.66 mm. in diameter, — so small that they do not seriously weaken the shell, and that the platinum lining is capable of withstanding the pressure over their areas. These holes are made necessary by the fact that without them some water gets trapped between the lining and the shell, owing to slight leakage or permeation of the platinum itself when the bomb is first heated, the lining then being not in close contact with the shell at every point; and this water on subsequent heating exerts, owing to its expansion in the liquid state, an enormous pressure against the lining, causing little indentations in it and causing some water to flow back into the bomb, whereby contamination of the solution with iron is produced. The holes remedy entirely this difficulty, which otherwise becomes aggravated on each successive heating.

The lining of the lower part of the bomb, A, was made of a platinum-iridium alloy (2 per cent iridium) 0.40 mm. thick; but pure platinum would have been preferable, owing to its greater ductility, and it will be used in future work. The lining was made by Baker & Company of Newark, N. J. The flange, F, was originally made of the same material, but the closing of the bomb compressed the platinum each time under the ring so that it grew hard and thin and finally cracked at the bottom of the groove. For this reason platinum-iridium alloy containing 15 per cent of iridium had to be substituted for the flange. This is so hard that it bids fair to wear indefinitely, and yet it is not so brittle that it cannot be forced into the groove in the steel without cracking. The flange could probably just as well be welded to the platinum cup, but in our bomb it was soldered to it with pure gold. Pure gold was also used freely in making repairs on the present lining when it tore, as it frequently did at the start before the necessity of the small holes in the shell and of several other precautions was understood. In the measurements made so far the gold has exerted no deleterious influence.

The lining is made so as to fit as well as possible at the start. It is then inserted in the shell, the latter is placed in the lathe, and by means of a steel or agate burnisher the lining is spun in. This brings the lining for the most part in contact with the steel, but nothing can be done at the bottom in this way. By means of a piece of soft wood and a mallet, however, the bottom also can be made to fit closely. The flange at the top is now spun over in the lathe. The stiff alloy is easily and safely brought down into the sharp groove by proceeding as follows: One end of a short piece of brass rod 3 or 4 mm. in diameter is filed to a sharp V. Using this and a small hammer, the platinum can be driven down almost to the bottom of the groove. After every half dozen strokes the brass V will have to be sharpened. The brass is considerably softer than the alloy, and so does it no damage. Other methods have been tried, such as forcing a ring of hard-drawn copper wire down onto the platinum over the groove by means of the cover B and the large nut C. But the former method, although a little tedious, is much better. The sides of the groove in the steel make an angle of 90° with each other, and its depth is such that when a wire 0.8 mm. in diameter is laid in it, about one half of the wire lies outside the groove.

After the lining has been made to fit as closely as possible, it should be removed from the shell and heated to redness to anneal it. Even the flange had better be treated in this way, since it is hard enough even after annealing. To remove the lining after it has been fitted in in the preceding manner, the following plan was adopted: Take a stick of soft wood, perhaps 20 cm. long and 5 cm. square, and whittle one end down so that it will slip easily into the bomb. Then take a piece of cotton cloth moistened with alcohol to remove any grease, wrap it over the small end of the stick, and then with a hammer drive the latter tightly into the bomb. Now holding the bomb in the vise, grasp the projecting end of the stick firmly in the hands and twist out the lining. This method never fails, provided the steel shell was ground reasonably smooth at the start.

To remove the lining after the bomb has once been heated is a much more difficult matter. It can be accomplished without lasting injury to the platinum cup, but it necessitates a new flange. The flange is bent up into line with the cup at three points, and at these points a long piece of steel, about the size of a lead pencil and preferably elliptical in cross section, with its entering end ground down to a blunt wedge (with edges carefully rounded), is forced in between the lining and the shell. The lining along three lines parallel to the axis of the cylinder and extending nearly to the bottom is then pried in toward the centre. This loosens the cup so that it can be removed by hand. The lining can then be straightened out, the adhering iron and ferric oxide removed by the use of melted acid potassium sulphate, any needed repairs made, and a new flange put on. The removal of the lining would doubtless be made easier by making the shell much more tapering; but this is very undesirable because it greatly increases, for a given volume of the bomb, the difficulty of securing a tight joint at the cover. It is, moreover, unnecessary, because if these instructions are followed carefully there should be no need of removing the lining after it is once finally in place.

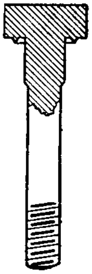

FIGURE 3.

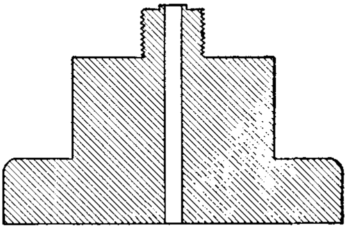

FIGURE 4.

A hole is drilled in the lining at the bottom so as to correspond with the hole in the steel shell. It is then best to close this hole temporarily with the steel piece shown in Fig. 3, using a lead washer under the V-shaped ridge for packing. Then, in order to bring the lining into perfect contact with the shell and at the same time to test it for possible faults, the lower part of the bomb is connected by means of the auxiliary cover shown in Fig. 4 with a Cailletet pump or its equivalent, — a water reservoir being interposed between the pump and bomb so as to force water instead of oil into the bomb. For this testing of the lining a pressure of 300 atmospheres has been used, the steel shell having previously been similarly tested up to 600 atmospheres pressure. The lining must be fitted as closely as possible before the hydraulic pressure is applied, since otherwise this will always result in tearing the lining. Even after expanding the lining with hydraulic pressure, there is no trouble in removing it, in case a leak develops, by the first method given above.

The next step is to fasten the edge of the flange to the shell. If this is not done, when substances like benzophenone, solid at ordinary temperature, are employed for heating the bomb, they will be drawn under the flange and into the groove in the shell, where they will solidify; upon heating the bomb the next time, the solid melts and escapes, thus relieving the pressure on the packing ring and allowing the bomb to leak: moreover, if the edge is not fastened down, there is danger of bending it when the bomb is opened and handled. To secure the flange eight small steel screws are used. The steel shell has to be recessed at this place, as shown in Fig. 1; otherwise the screw heads would interfere with the cover.

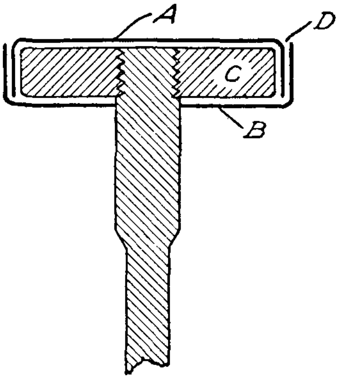

FIGURE 5.

The lower electrode is made of two steel parts, as shown in Fig. 5, the horizontal part, C, being afterwards enclosed in a platinum box, which is made as follows: The top, A, of this box is made by forcing a circular disk of pure sheet platinum (about 0.25 mm. thick) through a brass die by means of a brass punch. It is better, since it strains the platinum less, to interrupt this operation at least twice, annealing the metal each time. This box should be made to fit so tightly over C that it has to be forced on. In the same way a tight fitting bottom, B, is made for this box. A hole is drilled in the centre of this just large enough to permit the passage of the steel rod through it. It is then forced on over A. It then remains only to solder B to A with pure gold. This is easily accomplished by putting several pieces of gold on the crack, D, and directing a hot flame from the blast lamp downwards onto the box. This flame must not be too small, since the whole of the soldering must be done at once and as quickly as possible. Doing it a piece at a time involves keeping the steel rod hot for a longer time, and consequently oxidizing it more; and worse than this, the gold gets inside and alloys with the iron, bringing the latter eventually to the surface. Before soldering it is better to cover the steel rod below the box with pieces of asbestos, binding them on tightly by means of a wire, so as to diminish the oxidation.

Before adopting quartz crystal as the insulator various other substances were tried. Mica was tried first of all, using both of the methods subsequently employed by Knipp* in his work on surface tension. Our experience agreed with his, — that it is impossible to secure an absolutely tight joint with mica because of the formation of radial cracks. Nor was it an ideal substance chemically. Carnelian, flint, and agate were next tried because of their known toughness. It was with the last-named substance that we developed the method finally employed for making an absolutely tight joint; the substance itself, however, proved to be chemically unsuitable, since the hydrated silica which it contains dissolves readily in the hot water. The method which we finally employed for securing a tight joint put very little strain on the agate, so that there was no longer any reason for avoiding a substance because of its brittleness. Quartz crystal was then the natural substance to try. Japanese quartz, however, proved a failure, owing to included water or carbon dioxide, which caused it to crack upon heating; but the Arkansas quartz which we next tried was not affected by heat, and for work with neutral-salt solutions up to 306°, to which we have as yet confined ourselves, it leaves nothing to be desired. Since the thermal coefficient of expansion is so different in the directions parallel to and perpendicular to the main axis of the crystal, it seemed safer to make the axis of the cup parallel to the main axis of the crystal, and this was done.

The process employed for making the quartz cup is as follows: A crystal is selected which is perfectly clear and free from imperfections. A slice, in thickness a little greater than the height of the finished cup, is then sawed out at right angles to the main axis. For this operation a thin tinned-iron disk, whose edge is charged with diamond powder, is rotated in the lathe; and the piece, supported on a sawing table, is pressed lightly against the saw by hand, a wet sponge being held against the edge of the saw with the other hand. This operation of sawing is discussed at some length by Threlfall.† It is both easy and rapid if the saw is in good condition. Care must be taken in this and the subsequent operations that the work is not crowded too hard against the abrading surface, as this causes a local rise of temperature which may crack the crystal. A hollow drill, whose internal diameter is but little greater than the external diameter of the finished cup, is then run through the crystal piece at right angles to the sawed surfaces. Such a drill consists merely of a tinned-iron tube provided with a slit running lengthwise, and mounted so that it can be rotated in the lathe. The outer end of the tube is turned off square and is then charged with diamond powder. A small piece of wet sponge is then placed in the tube. Powdered carborundum can be used in place of diamond, and, although it is somewhat slower in starting, it appears to be equally satisfactory afterwards. The core is then taken from the drill and the ends of the cylinder are ground down flat; for the saw has left them somewhat irregular. This operation of grinding is conveniently carried out by means of a carborundum wheel rotated in the lathe, the wheel being kept wet by holding a sponge against it. Before grinding either surface, its bounding edges must be ground off (bevelled); otherwise the edges will break out irregularly. To cup out the cylinder, a hollow drill, whose external diameter is but little less than the internal diameter of the finished cup, is then run into one end to a depth almost equal to that of the desired cavity. The core which is left from this drill is too strong to be broken out without danger of injuring the outside of the cup; so another, smaller one is next run in to the same depth as the first and concentrically with it. This leaves two fragile pieces, a small rod and a thin tube, which are easily broken out. The cup is next mounted so that it can be rotated in the lathe. This is best accomplished by fastening it with stick shellac to the end of a brass rod held in the lathe chuck. To hold firmly, the crystal must be heated above the melting-point of the shellac; this can be done safely by flashing it with a gas flame. The inside of the cup is then ground to its final diameter and the bottom made flat by using carborundum powder upon the end of a brass rod which is a little less in diameter than the cavity and whose end is squared off, the rod being best held in the hand. The small hole is drilled through the bottom of the cup by the aid of a small diamond set in the end of a steel or brass rod. The diamond must, of course, be a little larger than the rod to give clearance for the latter. To start the hole the T-rest is used, but afterwards the rod is supported only by the hand. The tool must be withdrawn and moistened very frequently. The hole may be run halfway through from either end. It is afterwards expanded to its proper size by the aid of a small brass or steel rod and some carborundum. To turn the little V-shaped ridges on the ends of the cup, a diamond is set in the end of a steel rod just as for the boring tool, except that here a larger rod may be used to advantage. The tool is held in the hand and supported on the T-rest just as the ordinary hand tool is used on metal. The outside of the cup is first turned true with the diamond, and the ridges are ground out by holding a flat piece of carborundum (the face of a carborundum wheel answers nicely) against it as it revolves. To support the cup while work is being done on the lower end it is best to fasten in the chuck a piece of brass rod somewhat smaller than the internal diameter of the cup, square off its end, turn a little groove in it which will correspond to the ridge at the bottom of the cup, and then shellac the cup on, so that the ridge comes in the groove. This mode of support ensures getting the ridges, as they should be, in parallel planes and centrally located with reference to the axis of the cup. The operation of polishing is best carried out by means of different grades of corundum powder, using finally oxide of tin. These are applied wet on the end of a soft piece of wood.

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* Phys. Rev., **11**, 129–154 (1900).

† On Laboratory Arts, pp. 187–189.

In making such a cup an ordinary mechanician, after a little practice on the different operations, will spend perhaps twelve or fifteen hours.

In this connection it may be mentioned that some ready formed cups, made of fused silica, have lately been furnished us by Siebert & Kühn of Cassel, Germany. All that remains to be done to these cups is to drill the small hole in the bottom and turn the ends. They have not yet been tried, but unless they are more acted upon by hot water than those of quartz-crystal, which seems unlikely, they will be preferable for several reasons. For, in the first place, the making of such a cup from crystal is a somewhat tedious operation, and, in the second place, a very rapid cooling of the bomb has to be avoided, as the crystal would otherwise crack. Even with the fused-silica cups it will doubtless be necessary to cool the bomb slowly to avoid loosening the joint, but the cup would not be lost even if the bomb happened to be cooled suddenly. Another advantage of the fused silica is that, owing to its small expansion coefficient, there would be a much smaller change in the cell constant with the temperature (see Section VIII).

The thickness of the brass compensating washer (Z, Fig. 1) can be calculated from the known coefficients of expansion of the quartz-crystal and of the brass and steel used. That used in our bomb was 5.1 mm. in thickness. Of the two gold washers the upper one is made to fit tightly on the electrode rod, while the hole in the lower one is made to correspond with that in the bottom of the bomb. To keep the lower gold washer from touching the electrode rod, and to keep the latter from touching the steel shell, the following device was employed: The middle part of the steel rod is made about 0.5 mm. smaller in diameter than the hole in the crystal. Three thin strips of mica, each about 2 cm. long and 2 mm. wide, are inserted in the space left between the electrode rod and the crystal, so that the ends of these mica pieces project perhaps 1 cm. below the cup. The mica strips are cut so wide that they have to be pushed into place. They serve to hold the rod in the crystal and keep the lower gold washer in place. The cup can now be grasped by its edge with a pair of tweezers and the electrode rod pushed through the hole in the bottom of the bomb. It is then bolted down. It is next tested to make sure that there is no short circuit between the electrode and the bomb; and finally, to make sure that the joint is tight, the bomb is connected once more to the pump.

If, after the bomb has been in use for some time, it is necessary to remove the lower electrode, it may be done in the following way: The nut, N, cannot be unscrewed, but enough of it can be removed with saw and file so that the remainder will slip through the hole in the brass washer. A light direct blow with the hammer on the end of the electrode rod is then always sufficient to start it out. The brass remaining in the threads of the electrode rod is easily removed with any pointed tool, and the electrode is ready to use again.

The cover, B, is made slightly concave to allow the air bubbles, which might otherwise collect under it, to escape into the electrode chamber above. To line the cover a round disk of platinum-iridium alloy containing 15 per cent iridium is taken, and the two tubes, T_1 and T_2, are soldered to this with pure gold. This alloy is used rather than pure platinum on account of its greater hardness, which prevents the gold ring from cutting into it.

In the development of the bomb, the tube T_2 has probably caused more trouble than any other part. This is due in part to the fact that at high temperatures the pressure is sufficient to force water through the lining at any unsupported spot. If the tube was made of heavy metal, and especially when it was made of the 2 per cent alloy, it was itself capable, owing to its small diameter, of withstanding the pressure without expanding enough to come into perfect contact with the steel at all points; as a result, the bomb would leak at such points. Or, owing to the greater difficulty in mechanically expanding the small tube to meet the shell, the fit would be so poor at the start that the hydraulic pressure would tear it. Our earlier work here was done with the 2 per cent alloy before we fully realized the great difference in ductility between this and pure platinum. Because of its extreme ductility gold was then tried. This worked beautifully at first, but finally failed because the 22-carat gold solder employed in making the tube disintegrated under the action of the hot water. Recourse was then had to pure platinum, which completely solved the difficulty. The platinum tube is first expanded by driving in some plugs of cotton with the help of a hammer and a brass rod almost as large as the inside of the tube. Seamless tubing might be advantageously used here, but we used a tube made of sheet platinum soldered with pure gold, and this proved to be entirely satisfactory.

The small tube, T_1, is conveniently made by rolling up tightly some thin sheet platinum and then flowing gold in to fill the spaces between the convolutions. This gold is fed in from the outside, while the whole tube is kept hot in a large blast-lamp flame. Care must be taken not to use too much gold; otherwise a drop may form inside the tube, and its removal by drilling is extremely difficult. Owing to capillary forces, no gold will go to the space inside until the smaller spaces between the convolutions are all filled, so that there is no danger so long as too much gold is not employed. In this, as in all other operations when gold is used in soldering platinum, the piece should be kept hot no longer than is absolutely necessary, because the gold rapidly alloys with the platinum, and the resulting alloy is more crystalline in structure than either of the constituents and has not their ductility. To make the joint between T_1 and the lining of the cover stronger, the tube is reinforced above this point, as shown by the drawing. This was necessary in our earlier apparatus before the lining was screwed down to the cover, but is probably not necessary in the later form.

As the upper end of the tube T_1 is to act as a valve seat, and as therefore there will be a good deal of downward pressure at this point, the tube has to be well expanded into the conical cavity in the steel at V (Fig. 1). Because of this, and of the further fact that the valve seat should be as soft as possible, it is better to make the upper end of T_1 of solid gold, boring it out later. This is done as follows: The tube is first packed full of asbestos, to prevent gold from getting into it. A band of thin platinum foil is next wound tightly around the upper end and bound on by means of a platinum wire. This band is then pushed partly off of the end of the tube, so as to make a small projecting tube; and pure gold is melted into this until it is full. The platinum foil on the outside of the gold is now filed off. Both tubes are now attached to the cover lining and inserted in place in the cover. Holding the lower end of T_1 on an anvil, the soft gold, projecting perhaps 3 mm. above the steel at V, is compressed with a riveting hammer. The asbestos is now drawn out of the tube, and a hole is drilled down through the gold to meet the hole in the platinum.

The valve seat at the start is of the form shown in Fig. 1, and the slightest pressure on the steel bicycle-ball serves to force the ball down into the soft gold tube enough to make the joint tight. But after this process has been repeated a few times the bearing surface of the ball on the gold becomes so large that the pressure which can be obtained by tightening the screw, K, with the fingers is not sufficient to make the joint tight. The valve seat can be easily brought back to its original condition, however, by filling the depression, which the ball has made, with soft dental gold and opening the hole again with a scratch awl or a drill. The filling is most conveniently accomplished with a little "moss fibre" gold, using the regular dental tool. Of course the top should always be left concave, so that the ball will of itself roll to the center. Time is saved in the end by putting the valve seat in order each time; for, if this is done, there will never be a leak at this point. It is also better to use a fresh steel ball each time. The screw, K, should fit well, but still turn easily with the fingers; for, if there were much friction here, it would be impossible to tell how much of the force employed in screwing it down was being communicated to the ball and how much was wasted in friction in the screw. Smearing a little vaseline on the screw prevents air from leaking in while the bomb is being evacuated.

The edge of the lining is fastened to the cover by eight steel screws, in the same way as the flange of the lining of the body of the bomb is secured, the only difference being that it is not necessary at the top to recess the edge of the platinum. Care must, of course, be taken that the screws in the top do not come opposite those in the lower part when the top is put on. Two reference marks enable the experimenter to bring the cover always into the same position with respect to the bottom.

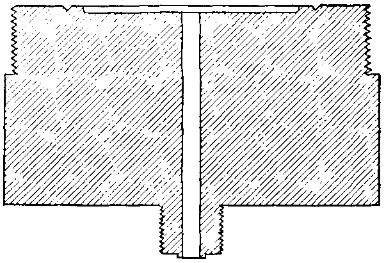

FIGURE 6.

The cover lining is forced into place and tested, just as was the lining of the lower part of the bomb, by means of the Cailletet pump, making the pump connection with a metal piece like that shown in Fig. 6, which takes the place of the lower part of the bomb.

The construction of the auxiliary electrode is similar to that of the lower one: it will be evident from Fig. 7. The part, *a*, has to be made just as small as is consistent with making the joint tight on the end of the crystal. The platinum covering consists of a little platinum box similar to that used on the lower electrode, a short piece of tubing, *b*, and a piece of platinum wire, *c*.

The gold packing rings are made as follows: A piece of gold wire about 3 mm. shorter than the circumference of the finished ring is cut off, the ends filed square and then soldered together with coin gold. The wire is then annealed and placed on a cast-iron spreader. By pushing a tapered brass plug into this spreader, it is expanded and the ring stretched. By placing a reference mark on the tapered plug, the ring can be stretched to just the same size every time. It, of course, comes off perfectly round, and after annealing is ready for use. Each ring is used but once; but after a number have been used they are remelted and made into fresh wire.

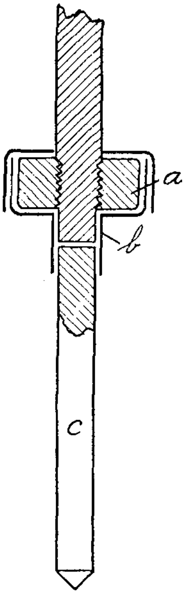

FIGURE 7.

The total weight of platinum used in our bomb is about 185 grams. This, together with the fine construction work involved, makes the initial cost considerable, but the platinum, of course, retains the larger part of its value.

Before adopting the sheet-platinum lining, attempts were made to produce a satisfactory platinum plate on the inner surface of the bomb by the electrolytic process described by Langbein*; but although a firmly adhering deposit was obtained, it was found not to be dense enough to protect the surface beneath from attack and the liquid from consequent contamination.

IV. PROCEDURE FOR THE CONDUCTIVITY MEASUREMENTS.

In making a set of conductivity determinations, the valve seat at the top of the tube T_1 (Fig. 1) is first put in order by putting in a gold filling, as has already been described in Section III. Then any loose particles of graphite or dirt adhering to the flange and cover in the neighborhood of the gold packing ring are removed with absorbent cotton saturated with benzene, and the screw thread on the lower part is cleaned in the same manner. Both the upper and lower parts are now rinsed thoroughly with good water, using the fine stream from a wash bottle to remove more effectually any loose particles of graphite which may have got into the bomb upon previously opening it. By this means, too, water can be forced through the fine tube T_1. If the solution to be investigated is a dilute one, the rinsing must be very thorough. Finally, the bomb is rinsed out with some of the solution, and as much as possible of this is then shaken out. The bomb is then ready for use.

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* Langbein's Electro-Deposition of Metals, 378 (1902).

Suppose now it is desired to make a series of measurements at the temperatures up to 281°. An amount of solution which will almost, but not quite, fill the bomb at this temperature is measured in from a pipette previously graduated to contain this amount, as will be described in Section V. A gold packing ring is annealed and cleaned by heating it in the flame of a burner, and it is placed in the groove. The cover is then put in place, taking care not to disturb the ring. The thread in the large nut is next washed out with benzene, to get rid of any solid substance which may have condensed there in a previous heating. With a piece of cloth or absorbent cotton, a lot of finely powdered graphite is then rubbed into the thread and onto the surface which bears on the brass compensating washer. The nut is now screwed on by hand, care being taken not to disturb the cover ; otherwise it might be raised slightly, so that particles of graphite would enter the bomb. The apparatus is next transported carefully to the large wrench, and the nut is tightened up. The air pressure is then reduced to about 2 cm. by connecting the small tube with a Richards water pump, the valve is screwed down, the lead-wires bolted on, and the bomb is ready for the measurements.

The conductivity is first measured at 26°. To hasten the equalization of the temperatures, the cold bomb was usually introduced after bringing the bath to about 30°. The other vapor baths are heated up meanwhile.

The bomb is then immersed in the 140° bath, whereupon the conductivity increases very rapidly. The minimum in the telephone is at first greatly disturbed by the boiling of the solution, which takes place strongly at the lower electrode, owing to the fact that this is at the start, because of its position, the hottest part of the bomb. But as the temperature of the solution approaches that of the bath the disturbance decreases, and finally ceases altogether. When the temperature has become almost constant, which is indicated by the constancy of the conductivity, the bomb is removed from the bath, shaken, and returned as quickly as possible. To shake it while hot, a piece of asbestos cloth, with a piece of woollen cloth outside, is used. If the shaking is omitted, the measured conductivity may be too high by as much as 0.5 per cent. This was found to be due to the following facts: At 140° there is still a considerable vapor space left in the bomb, the entire cover being above the liquid surface. During the first part of the heating the xylene vapor is condensed so rapidly by the bomb that it extends up only for a little distance above the bottom of the bomb, leaving the upper part completely out of it. This causes an evaporation of pure water and a condensation of it all over the colder cover, leaving the solution too concentrated. If the bomb is shaken after reaching the temperature of the bath and quickly returned, the same action does not repeat itself, since the top is now as hot as the bottom. The bridge readings are now continued (usually for about thirty minutes) till one perfectly constant for ten to fifteen minutes is obtained.

The bomb is then transferred to the naphthalene bath. Shaking was found to have no effect at this temperature, owing, doubtless, to the fact that the liquid level has then risen almost to the cover, so that large drops cannot adhere to the latter, and to the fact that the surface tension has diminished, so that less water is held clinging to the walls of the narrow chamber in the cover.

The bomb is next brought into the 281° bath. It is now necessary to keep constant watch of the conductivity between the upper auxiliary electrode and the walls of the bomb, so as to be sure that too much solution has not been put into the bomb. The reasons for putting in solution enough at the start to so nearly fill the bomb at the highest temperature are first, to reduce the vapor space at all the temperatures as much as possible, since a correction has to be made for the amount of solvent in this space; and secondly, to see that the bomb is absolutely tight even at the highest temperature, when the solution is in contact with the upper electrode. This latter is important, since a leak, if it took place above the liquid level, would cause a loss of pure solvent and a consequent increase in the concentration of the solution.

After completing the measurements at 281°, the bomb is returned to the 218° bath, then to the 140° bath, and finally it is brought back to 26°. In going from a higher temperature to a lower much time is saved by cooling the bomb, in front of a fan outside the bath, to a temperature which is at least as low as that next desired; for while heating in a vapor bath is rapid, the cooling in it of a hotter body is very slow. During the first half of the experiment, where the bomb is introduced each time into a hotter bath, stirring inside the bomb takes place of itself, it being accomplished by the rising vapor bubbles and the rapid convection currents caused by the bottom of the bomb being so much hotter than the top during the heating. During the second half it is necessary to provide for this by shaking the bomb before putting it into each bath. How much shaking is necessary can be determined by repeating the operation and seeing whether the conductivity has been affected by it.

The advantages derived from cooling the bomb down through the same series of temperatures and again taking measurements are that these furnish a check on the accuracy of the preceding ones, and especially that they show whether or not there has been any contamination, and if so, between what temperatures it took place and to how much it amounts.

The bomb is opened as soon as the experiment is completed, since otherwise there may be trouble in getting the cover off because of the strong adhesion of the gold packing ring to the platinum lining below. This effect increases with use, since a small amount of gold from the ring adheres to the platinum each time, and subsequent rings will adhere more firmly to this gold than they would to a clean platinum surface. The effect can easily be reduced, however, as soon as it grows troublesome, by rubbing the platinum cover, where it comes in contact with the ring, with a burnisher and by marking in the groove with a lead pencil. The trace of graphite which adheres to the surface is very effective.

When the bomb is not in use, good water is left standing in the lower part. The cover is inverted and the upper chamber similarly kept filled with water. If for the next experiment a more dilute solution is to be employed, the bomb must first be heated with good water to perhaps 218° for some little time. No amount of rinsing or soaking out at ordinary temperatures will answer the purpose. There is on this account a great saving of time effected by beginning with the dilutest solution to be investigated, and afterwards measuring those more and more concentrated.

Our measurements at 306° were carried out, for the most part, after complete experiments up to 281° had been made, so that they usually consisted merely of measurements at 26°, 306°, and again at 26°.

V. Procedure for the Specific-Volume Measurements.

To determine the specific volume of a solution at any temperature, such an amount of solution is weighed into the bomb as will bring the liquid level up onto the auxiliary electrode at that temperature. This amount was determined by successive heatings with increasing volumes of solution.

A pipette of the form represented by Fig. 8 was made for each of the temperatures 218°, 281°, and 306°. The stem is graduated between *a* and *b*, and the capacity up to these points is roughly determined by weighing. The volume of the pipette is made such that for water or dilute solutions it will deliver the right amount into the bomb when filled up to the point *b*. For more concentrated solutions the expansion is less, so that more of the solution must be used. The graduations on the pipette serve only as an indication of how much solution to take. The exact amount used is obtained by weighing the pipette filled and then again after discharging. During the weighing the tip is covered with a small test tube, *c*, which is held on by the rubber band, *d*.

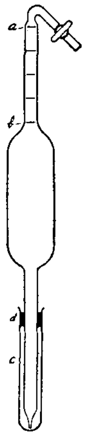

Figure 8.

The bomb is first dried out by rinsing it with alcohol and ether. The residue left by the ether upon evaporation is sufficient to affect the conductivity of the diluter solutions employed, so that no attempt was made to determine the conductivity of such solutions at the same time as their specific volume. The solution is boiled to expel the air. This increases the concentration slightly; but this is of no consequence if, as was usually the case, the experiment was made solely to determine the specific volume. If it was also to serve for conductivity measurements, the solution was boiled gently in a tall platinum vessel which was weighed with its contents before and after boiling. This gave the loss of solvent during the operation. Knowing this and the amount of solution originally present in the cylinder, the final concentration is easily calculated. After weighing the solution in from the pipette, the bomb is closed, and the air pressure reduced to 2 cm. Upon heating, careful watch is kept of the readings with the auxiliary electrode to make sure that too much solution has not been put in. After the conductivity of the upper chamber has been constant long enough to show conclusively that the temperature has become stationary and that the bomb is absolutely tight (even the slightest leak being of course indicated by a constant decrease in the conductivity of the upper chamber), the conductivities between the walls of the bomb and both the upper and lower electrodes are measured carefully, and the temperature of the bath is observed. The experiment is then complete.

The ratio of these two conductivities is calculated, and the corresponding volume is obtained by interpolation from a plot obtained as described in Section VI. This volume, corrected for the expansion of the bomb and then divided by the weight of solution in the bomb, gives the specific volume of that solution at the temperature in question, and this quantity divided by the specific volume of the solution at 4° gives the volume of that quantity of the solution that would at 4° occupy one cubic centimeter, this quantity being most convenient in subsequent computations.

VI. STANDARDIZATION OF THE APPARATUS.

Thermometers. — These were calibrated by the method recommended by Crafts,* first for irregularities of bore and then at the fixed points 0°, 100°, 218°, and 306°. For the last two temperatures the vapors of boiling naphthalene and of benzophenone were used. These substances were obtained from Kahlbaum, and were purified in the manner suggested by Crafts, until their melting points came within his limits. The form and dimensions of the vapor bath used in establishing the 218° and 306° points were essentially those given by him. To reduce the temperatures lying between the fixed points to the gas scale, his table of corrections for French glass was also used, our thermometers being of the same make as those used by him. At the beginning every temperature measurement was followed by a lag ice-reading; but this was found to be unnecessary, since the thermometers showed no lag. The ice-reading was, however, taken frequently, to make sure that the zero did not change from another cause, — evaporation of mercury from the bulb below and condensation in the chamber above. This effect was not apparent even after long use at temperatures up to 280°; but above this the zero would fall perhaps 0.1° from two or three days' use. When in use at 306°, instead of taking an ice-reading the thermometer was first placed in the benzophenone heater and then in the calibrating apparatus containing perfectly pure benzophenone. The difference in reading (usually amounting to 0.1–0.2°) was deducted from the true boiling-point of benzophenone at the observed barometric pressure.

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* Am. Chem. Jour., **5**, 307–338 (1883–84).

Two thermometers were calibrated in this way, and in actual use their corrected readings were always found to agree satisfactorily with each other.

Slide-Wire Bridge and Resistance Coils. — The slide wire was calibrated by the method of Strouhal and Barus.* The resistance coils were calibrated by comparison with a standard bridge of this Institute.

The Cell-Constant. — In order to reduce the observed to specific conductivities, the "cell-constant" was determined in the usual manner, by measuring in the bomb solutions of known conductivity. For this purpose the measurements at 26° of the solutions of both potassium and sodium chlorides, which were afterward studied at higher temperatures, were employed, the mean of the most reliable of them being taken. (See Section XI.)

The Volume of the Solution in the Bomb and the Corresponding Cell-Constant Ratio. — It was stated above that the volume of the solution at any time in the bomb was determined by measuring the ratio of the conductivities between the walls of the bomb and the lower and upper electrodes respectively. This ratio will hereafter be called the cell-constant ratio. Its value is, of course, independent of the nature of the solution in the bomb, and is determined fully by its height in the narrow chamber, and therefore by its volume. To find the values corresponding to different volumes, we proceeded as follows: The bomb is first dried by rinsing it with alcohol and ether. Some 0.02 molar potassium chloride solution † is then boiled to free it from air, and right after cooling, enough of this to fill the bottom part of the bomb to within 1–2 mm. of the flange is weighed in from a pipette. The mouth of the pipette is kept under the surface to diminish the absorption of air. The cover is next put on and screwed down, care being taken not to tip the bomb enough to get any of the solution up onto the mouth of the tube T_1. By means of the water pump the air pressure in the bomb is reduced to about 2 cm., and the valve is then closed. If the air is not removed from the solution at the start, it comes out rapidly upon reducing the pressure and spatters some of the solution up into the tube T_1, thus allowing it to be swept out by the air current.

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* Wied. Ann., **10**, 326 (1880). See also Kohlrausch and Holborn, Leitvermögen der Elektrolyte, 45 (1898).

† The reasons for taking this solution instead of pure water are that it makes the conductivity at the upper electrode high enough to give a good minimum, and that the solution is so strong that contamination could not possibly make any trouble.

The lead wires are now bolted on and the bomb is placed in the liquid xylene bath, serving ordinarily for the 26° measurements, and the temperature of the latter is raised by means of the heating coil. The liquid level in the bomb is at the start about 3 mm. below the point of the auxiliary electrode, so that the resistance of the upper cell is shown by the conductivity apparatus to be infinite; but upon heating, the level rises and finally touches the electrode, whereupon the resistance suddenly sinks to perhaps 1000 ohms. The temperature of the bath (perhaps about 130°) is now held constant until the solution in the bomb has also attained it, as will be indicated by the resistance of the lower and, far more sensitively, by that of the upper cell becoming constant. Both these resistances are then noted, and the temperature is measured.

The temperature is now raised by steps of three or four degrees until that ratio of the cell constants is reached which corresponds to the bomb being almost completely full. This limiting ratio can be determined cold at any time by measuring the resistance of the lower cell and then inverting the bomb and measuring that of the upper cell. Finally, the cell-constant ratios are plotted as abscissae and the corresponding volumes as ordinates, whereby a straight line is obtained.

The computation of the volumes is made with the help of the following data. Zepernick and Tammann* have found that equal volumes of a 0.52 normal potassium chloride solution and of water at 0° upon heating from that temperature to 140° become different from each other by only 0.1 per cent. It is therefore perfectly safe to assume that the expansion of the 0.02 normal potassium chloride solution used by us is the same as that of pure water. From Hirn's† results the specific volume of water at the temperature in question, but under a pressure of 14.8 atmospheres, may be obtained. At 135°, the mean temperature of the calibration experiments, the vapor pressure is 3.1 atmospheres. Hirn's result should then be reduced to this pressure. The coefficient of compressibility of water has been investigated by Pagliani and Vicentini‡ up to 100°. Plotting values and extrapolating gives 0.000048 for the coefficient at 135°, or for the fractional decrease in volume per atmosphere pressure. Hirn's value should then be increased by $0.000048 \times (14.8 - 3.1) \times 100 = 0.056$ per cent. Multiplying the value so obtained by the weight of solution employed and by the specific gravity of the cold solution referred to water at the same temperature, the volume corresponding to the observed cell-constant ratio is obtained.

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* Ztschr. phys. Chem., **16**, 665 (1895).

† G. A. Hirn, Ann. chim. phys. (4), **10**, 32 (1867). His series of observations covers the range of temperature up to 180°. Between 110° and 143° his values differ from those found by Zepernick and Tammann by only 0.02 per cent.

‡ Landolt und Börnstein, Tabellen, 96 (1894).

VII. Preparation of the Substances and Solutions.

The sodium chloride used was purified by precipitation with hydrochloric acid gas. It was then washed with hydrochloric acid, dried, and finally ignited until decrepitation ceased.

The potassium chloride was precipitated twice with hydrochloric acid gas, crystallized from hot water, dried, and finally ignited.

Solutions were made up, by weighing out the salts, so as to be almost exactly 0.1 and 0.01 normal at 4°. The solutions of smaller concentration were made by diluting the 0.01 normal one with the help of two graduated flasks.

The equivalent weights used are as follows: K = 39.14, Na = 23.05, Cl = 35.46. All weighings were reduced to a vacuum.

The water used throughout this investigation was prepared by redistilling ordinary distilled water, to which alkaline permanganate solution was added, from a steam-jacketed copper still with a tin condenser. The first quarter of the distillate was rejected, and the following portions were condensed hot (between 60° and 90°). The water had a specific conductivity of $(0.7 \text{ to } 1.0) \times 10^{-6}$ reciprocal ohms.

VIII. Discussion of the Systematic Errors and their Correction.

Errors Affecting the Specific-Volume Values. — 1. In calculating the specific-volume, the volume of the bomb was directly determined at about 135°, as described in Section VI, and the expansion of the metal from this point to the temperatures of the experiments was corrected for. Andrews,* working with "soft" cast steel, which corresponds to the material from which the bomb was constructed, found the mean coefficient of cubical expansion between 100° and 300° to be 0.0000450; and this value was adopted for the corrections. The difference between his steel and that used in the bomb can hardly be great enough to cause an appreciable difference in the coefficient of expansion, since his values for two steels as different as Bessemer steel with 0.15 per cent combined carbon and cast steel with 0.45 per cent differ by only 6 per cent; and an error of even 6 per cent in the coefficient of expansion would produce a maximum error, even at 306°, of only 0.05 per cent in the specific volume.

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* Proc. Roy. Soc., **43**, 299 (1887).

2. The quartz-crystal cup expands upon heating, thus diminishing the volume of the bomb occupied by the solution. The correction for this, even at 306°, amounts to only —0.03 per cent.

3. The bomb expands owing to the pressure within. At 306°, where this correction is greatest, the vapor pressure plus the air pressure may be estimated at 100 atmospheres. Assuming that the modulus of elasticity of the steel is 17372 kgm. per sqmm., which is the value found by Pisato* at 300°, the volume correction due to this cause is +0.025 per cent. This is opposite in sign and essentially equal to the preceding correction: they therefore eliminate each other.

4. The volume of the tube T_1 is only 0.07 ccm. or 0.06 per cent of the whole volume of the bomb. It is therefore so small that no irregularities in the extent to which it is filled with solution could much affect the result.

5. The volume of the bomb depends somewhat on the extent to which the large nut is tightened up and the gold packing-ring compressed. Four of the gold rings which had been used were chosen at random, and the mean thickness of each was calculated from measurements made at eight equidistant points with a micrometer caliper. The average deviation from the mean thickness of these rings was such as to affect the volume of the bomb by only 0.02 per cent. So this source of error can be unhesitatingly disregarded, especially as each final specific-volume value is the mean of the values obtained from several independent runs.

6. The bomb is never completely filled with liquid, the vapor space amounting, on an average, to about 1 ccm. or 0.8 per cent of the total volume of the bomb (about 124 ccm.). A certain fraction of the water is therefore vaporized, and the specific volume appears too small by a corresponding amount. The specific volume of the vapor is not yet known above 200°. By extrapolation, however, from the values up to 200°, the specific volume of the vapor at 218° is found to be seventy-five times that of the liquid. From this it follows that at 218° the correction is only $\frac{1}{124} \times \frac{1}{75}$, or about 0.01 per cent.

Such a calculation is not possible at the higher temperatures, 281° and 306°; but that no considerable error arises from this source was shown by direct experiments. Namely, when two or more specific-volume determinations were made, the amount of solution taken in the different experiments was purposely varied, so that the vapor space should vary from about 1.8 ccm. to 1 ccm. If, now, a considerable amount of the water were present in the vapor space, the specific-volume values obtained would, of course, be larger the smaller that space. As a matter of fact, however, the values obtained with the 1 ccm. vapor space were as often smaller as they were larger than those obtained with the 1.8 ccm. vapor space. In other words, no difference greater than the variable experimental error was observed. The error due to this source is therefore probably less than 0.1 per cent.

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* Nuovo Cimento (3), **4**, 152 (1878).

7. The temperature measurements may be regarded as accurate to within 0.2° ; and this of itself introduces an uncertainty of only 0.07 per cent in the worst case, that of the 306° values. That the bomb and its contents actually attained the temperature of the surrounding vapor is shown by the fact that the extremely sensitive reading of the upper electrode remained constant indefinitely after it had once become so ; and by the fact that there could not be a continuous loss of heat of appreciable magnitude from the bomb to the surroundings, since upon the sides the bomb was protected against radiation and cold convection-currents by the iron shield with the vapor outside, and since above there was always a layer of vapor 10 cm. in height, and since the dropping back of condensed liquid on to the bomb was prevented by the mica shield ; moreover, the copper lead-wires were only 1.2 mm. in diameter and passed through the upper layer of vapor before emerging.

8. Another possible source of error might be the gathering of vapor bubbles on the under surface of the cover, whereby the apparent volume of the liquid would be increased. That this did not occur was shown by removing the bomb from the heater, shaking vigorously, immediately replacing it, and taking conductivity readings as soon as the temperature had again become constant, whereby the same readings were obtained as before the shaking.

9. The air was not entirely removed from the bomb at the start, and, as the solution expands and the temperature rises, the air pressure increases. Assuming that the preliminary boiling had removed all of the air from the solution in the beginning, and that there is no solubility of the air in the liquid at the high temperatures, its pressure can be calculated by the gas laws. At the temperatures of 218, 281, and 306°, it would thus amount to about 1, 2, and 2.5 atmospheres respectively. The effect of these air pressures on the specific-volume values cannot be calculated, since the compressibility at these temperatures is not known ; but for these small pressures it is undoubtedly less than the errors of observation.

10. If the vapor above the solution had an appreciable conductivity it would make the conductivity between the upper electrode and the bomb appear too great. But this is not the case, as is shown by the fact that unless the liquid is in contact with the electrode there is no measurable conductivity here, even at 306° with a 0.1 normal KCl solution. We can at present only assign an upper limit to the conductivity of the vapor: but it certainly does not exceed $\frac{1}{200000}$ part of that of the solution.

Errors Affecting the Conductivity Values. — 1. All the errors in the values of the specific-volume have an effect of the same magnitude upon those of the equivalent conductivity, with the important exception of that due to the amount of solvent in the vapor space at the two highest temperatures, 281° and 306°. No error arises from this last source by reason of the fact that, owing to the increase in concentration of the solution, the specific conductivity increases in the same proportion as the volume diminishes; for at 281° and 306° (but not at 140° and 218°) the quantity of liquid in the bomb was the same in the two series of measurements.

2. The cell-constant might be expected to vary with the height of the liquid level in the bomb; but direct experiment showed that for the range of the liquid level in our measurements the effect of this was less than the error of observation. The smallest amount of solution employed in any of the experiments was first introduced into the bomb and the conductivity measured at 26°. Then more of the same solution was introduced until the liquid was in contact with the whole cover; but the resistance was not measurably changed.

3. The cell-constant changes with the temperature owing to two causes: first, the expansion of the quartz-crystal cup; and, secondly, that of the bomb itself. The correction for this is calculated as follows: The resistance inside the bomb may be considered as made up of two parts, that (a) inside the crystal cup, and that (b) between the mouth of the cup and the platinum lining. The ratio of a to b can be roughly determined by putting any solution into the bomb, measuring the resistance, and then introducing a third electrode in the form of a platinum disk placed over the mouth of the cup, and measuring the resistance between this new electrode and the electrode at the bottom of the cup. This ratio, so determined, is not strictly identical with the actual one, since the lines of current-flow inside the quartz cup have been somewhat changed in direction by the interposition of the new electrode; but for the present purpose the approximation suffices. The ratio $\frac{a}{a+b}$

was thus found for Cell II to be $\frac{615}{803}$. Le Chatelier * has studied the expansion of quartz both parallel and at right angles to the main axis up to 1000°; and, with the help of his values, the resistance of one of the cups was calculated to decrease by 0.69 per cent of its value upon heating from 26° to 306°. Multiplying this by $\frac{a}{a+b}$ gives 0.53 per cent for the decrease in the resistance of the bomb due to the expansion of the cup. On the other hand, b will increase at a rate roughly proportional to the linear expansion of the bomb, or between 26° and 306° by $0.000015 \times (306 - 26)$, or 0.42 per cent. Multiplying this by $\frac{b}{a+b}$ gives 0.10 per cent for the increase in resistance of the bomb due to the expansion of the shell. The total decrease in the resistance of the bomb is therefore 0.43 per cent. The change in the cell-constant is similarly calculated for the other temperatures. These corrections are given in Table IV and have always been applied to the results.

4. The effect of the pressure on the cell-constant is entirely negligible. For at 306° the radius is increased by the pressure 0.01 per cent, and this affects the cell-constant by only 0.002 per cent.

5. The resistance of the lead-wires has to be deducted from the measured resistance of the bomb; and since a portion of the leads is subjected to the temperature of the bath, this correction is different for different temperatures. This resistance may be considered as made up of three parts: R_1, the constant resistance of the heavy leads; R_2, the resistance of the small leading-in wires, L_1 and L_2; and R_3, that of the steel electrode rod. R_1 and R_2 were measured at room temperature. For the other temperatures R_2 was calculated from its value at room temperature. R_3 was calculated from its dimensions and the specific resistance of steel. The maximum value (at 306°) of the total resistance of the lead-wires was 0.061 ohms.

6. In the case of the more dilute solutions it was necessary to correct for the conductivity of the water used. To do this, some water prepared in the same way and of the same conductivity cold as that used for making up the solutions was put into the bomb, and just such a set of experiments was made with it as had been made with the solutions. Then for any temperature the conductivity of the water, measured at that same temperature and under the same conditions, was deducted

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* Compt. rend, **108**, 1046 (1889).

from that of the solution. This at the same time corrects for contamination, since, with a dilute, neutral-salt solution, there is no apparent reason why the contamination should not be the same as for water. For the dilutest solution used, 0.0005 normal, the maximum correction (at 306°) amounts to 1.9 per cent. See also Section XI.

7. In the conductivity experiments, the vapor space at 140° and 218° was considerable, so that at these temperatures a correction has to be applied for the vaporized solvent, since the solution is more concentrated than it would otherwise be. This correction was calculated from the known volume of the vapor in the bomb and its specific volume, using for the latter the data of Zeuner * which go up to 200°, and extrapolating for the 218° value. The correction amounts to +0.05 per cent at 140° and +0.18 per cent at 218°. As explained above, it is not required in the case of the 281° and 306° values.

8. The temperature measurement at 26° is certainly more accurate than the work requires. Above this, the temperature reading is probably correct to 0.2°. Most of the uncertainty in the equivalent conductivity values introduced by this possible error finds expression in the specific-volume values, and this has already been considered. Besides this there is the much smaller effect on the observed resistance of the bomb. The total uncertainty in the equivalent conductivity arises from both these sources: that due to 0.2° is in the worst case (at 218°) 0.09 per cent, and where, as has usually been the case, several experiments are made and the mean taken, this effect tends to be eliminated.

IX. EXPERIMENTAL DATA RELATING TO THE SPECIFIC-VOLUME DETERMINATIONS.

All of the measurements have been included in Table I, with the exception of two, which, though agreeing well with the others, were known to be less reliable.

The first and second columns are self-explanatory.

The third column gives the concentration of the solution at 4°, expressed in equivalents per liter.

The fourth column gives the corrected temperature of the experiment.

The fifth column gives the number of grams of solution which were weighed into the dry bomb at the start.

The sixth column gives the volume expressed in cubic centimeters, which, at the temperature (135°) at which the bomb was calibrated, corre-

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* Landolt und Börnstein, Tabellen, 63 (1894).

TABLE I.

The Specific-Volume Data.

Date.	Salt.	Equivalents per Liter.	Temperature.	Weight of Solution.	Volume Uncorrected.	Specific Volume.	Sp. Vol. Ratio, $218^\circ/4^\circ$.
Mar. 31, 1902	NaCl	0.002	216.8	104.16	122.75	1.1831	1.1851
Apr. 2, 1902	"	"	217.2	104.58	123.58	1.1863	1.1878
Apr. 3, 1902	"	"	217.9	103.44	122.18	1.1859	1.1861
						Mean . . .	1.1863
May 2, 1902	NaCl	0.1	218.0	103.52	121.79	1.1862	1.1862
							$281^\circ/4^\circ$.
Apr. 14, 1902	NaCl	0.002	281.0	92.11	122.08	1.3343	1.3343
Apr. 14, 1902	"	"	281.1	92.97	123.51	1.3374	1.3372
Apr. 18, 1902	"	"	281.1	92.65	122.94	1.3359	1.3356
						Mean . . .	1.3357
Apr. 29, 1902	NaCl	0.1	281.2	92.83	122.07	1.3237	1.3287
May 1, 1902	"	"	280.7	93.27	122.37	1.3207	1.3275
June 19, 1902	"	"	281.8	93.34	122.85	1.3248	1.3272
						Mean . . .	1.3278
							$306^\circ/4^\circ$.
Mar. 11, 1903	NaCl	0.002	306.3	85.40	121.82	1.4373	1.4360
Mar. 27, 1903	"	"	305.8	85.35	121.30	1.4318	1.4327
						Mean . . .	1.4343
Feb. 20, 1903	NaCl	0.01	306.7	85.69	122.14	1.4362	1.4337
Jan. 17, 1903	NaCl	0.1	305.2	86.39	120.94	1.4106	1.4201
Mar. 31, 1903	"	"	305.2	87.01	121.92	1.4117	1.4212
						Mean . . .	1.4207
Feb. 17, 1903	KCl	0.01	304.7	85.61	121.24	1.4270	1.4335
Feb. 18, 1903	"	"	306.0	85.76	122.01	1.4335	1.4342
						Mean . . .	1.4338
Jan. 30, 1903	KCl	0.1	304.9	86.75	121.56	1.4119	1.4233
Feb. 10, 1903	"	"	306.3	86.78	122.05	1.4171	1.4223
Feb. 16, 1903	"	"	305.3	86.41	121.29	1.4143	1.4239
*Feb. 16, 1903	"	"	305.3	86.41	121.24	1.4137	1.4233
						Mean . . .	1.4232

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* Same solution as in preceding experiment, after cooling, shaking, and reheating.

sponds to the observed cell-constant ratio. This volume was obtained by interpolation from a plot made as described in Section VI. The actual volume occupied by the solution at the higher temperature is greater than this by an amount equal to the expansion of the bomb upon heating from 135° to that temperature. As there stated, the temperature-coefficient of volume expansion of the steel shell of the bomb is assumed to be 0.000045 per degree.

The seventh column gives the specific volume of the solution at the temperature of observation. It is obtained by dividing the values of the preceding column, after correcting them for the expansion of the bomb as just described, by the weight of solution given in the fifth column.

The last column gives the ratio of the specific volume at the round temperatures 218°, 281°, and 306°, to that of the same solution at 4°. Thus, this ratio shows the volume occupied by that quantity of solution which at 4° has a volume of 1 ccm. The values are obtained from those of the preceding column by reducing them to these temperatures by means of the temperature-coefficient obtained from our specific-volume values, and then dividing the results by the specific volumes of the solutions at 4°. These specific volumes are as follows: 0.9958 for 0.1 normal, and 0.9996 for 0.01 normal sodium chloride; and 0.9954 for 0.1 normal, and 0.9995 for 0.01 normal potassium chloride.*

X. THE RESULTS OF THE SPECIFIC-VOLUME DETERMINATIONS.

The final results are brought together in the following table. The value at 140° is that found by Hirn † for pure water reduced from the higher pressure which he employed to the vapor-pressure.

The results with the 0.002 normal solution may be regarded as completely identical with those that would be obtained with pure water; for this solution contains only about 0.01 per cent of salt; and, moreover, the experiments themselves show that there is no difference between the specific-volume ratio of the 0.002 and 0.01 normal solutions, and that the difference between the latter and that of the 0.1 normal solution is somewhat less than one per cent, which indicates that the order of magnitude of the difference between pure water and the 0.002 normal solution is

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* These values were computed from the densities given by Kohlrausch and Hallwachs (Wied. Ann., **50**, 122, 1893) for NaCl at 18°, and from that given by Kohlrausch (Leitvermögen der Elektrolyte, 76) for a normal KCl solution at 18°, under the assumptions that the change in density is proportional to the concentration and that the expansion is the same between 4° and 18° for these solutions as for water.

† Hirn, Ann. chim. phys. (4) **10**, 32 (1867).

TABLE II.

Ratio of the Specific Volume at Various Temperatures to that at 4°.

Substance.	Equivalent Concentration at 4°.	Specific-Volume Ratio.				
		26°.	140°.	218°.	281°.	306°.
NaCl	0.002	1.0032	1.0803	1.1862	1.3357	1.4343
"	0.01	"	"	. . . .	. . . .	1.4337
"	0.1	"	"	1.1861	1.3278	1.4207
KCl	0.01	"	"	. . . .	. . . .	1.4338
"	0.1	"	"	. . . .	. . . .	1.4232

0.02 per cent. The specific volume of water is therefore 1.186 at 218°, 1.336 at 281°, and 1.434 at 306°. It is, according to our estimate of the possible errors, almost certain that these values are not in error by as much as 0.3 per cent, and it is probable that the error does not exceed half this amount. Previous determinations * of the specific volume of any considerable degree of accuracy have extended only up to 180°.

Attention may also be called to the facts that the 0.1 normal solutions between 218° and 306° expand appreciably (0.8 — 1.0 per cent) less than pure water, but that the difference (0.2 per cent) between the solutions of the two salts scarcely exceeds the experimental error.

XI. Experimental Data relating to the Conductivity Determinations.

The Cell-Constant. — This was calculated from the conductivity measurements at 26°, using for the specific conductivities of the 0.1 and 0.01 normal potassium chloride solutions the standard values of Kohlrausch, Holborn, and Diesselhorst,† and for the other solutions the values at 18° of Kohlrausch and Maltby,‡ and the temperature-coefficients of Déguisne. § The quartz-crystal cup which was used for the first half of the measurements (Cell I) was accidentally broken, and a new one had to be substituted for the rest of the work. After making three experiments with the new cup, the platinum lining of the lower part of the bomb had to be removed and repaired, and this operation changed the cell-constant. The term Cell II$_a$ will be used to characterize the bomb as it was in these first three experiments with the new cup, and the term Cell II as it was in all subsequent work. With these exceptions, the cell-constant calculated from measurements made at widely different periods did not vary throughout the work. Even when the electrode was removed in case of a leak, and then replaced, it did not make any measurable difference, as was, indeed, to be expected, since the value of the cell-constant is so largely determined by the dimensions of the quartz cup. The values of the cell-constants, with the solutions from which they were derived, are as follows:

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* Hirn, loc. cit. Waterston (Phil. Mag. (4) **26**, 116–134, 1863) has, to be sure, made rough measurements from 210 to 320°; but this was done in a glass apparatus which was greatly attacked.

† Wied. Ann., **64**, 440 and 451 (1898).

‡ Wissensch. Abhandlungen phys.-techn. Reichsanstalt, **3**, 210 (1900).

§ Dissertation, Strassburg (1895); Kohlrausch and Holborn, Leitvermögen der Elektrolyte, 199.

TABLE III.

CELL-CONSTANTS.

Cell I.			Cell II.		
Substance.	Equivalents per liter.	Cell-Constant.	Substance.	Equivalents per liter.	Cell-Constant.
KCl	0.1	0.8294	KCl	0.1	0.9853
NaCl	0.1	0.8288	"	0.01	0.9845
"	0.01	0.8280	"	0.002	0.9850
KCl	0.002	0.8280	NaCl	0.002	0.9840
NaCl	0.002	0.8317	Mean of first two . .		0.9849
Mean of first three . .		0.8287	Cell II$_a$.		
			NaCl	0.1	0.9949

The original data from which these were calculated are all given in Table VI. Each cell-constant value is the mean of all of the values calculated from all of the experiments on the solution in question at 26°.

The values derived from the 0.002 normal solutions are not included in the means, because, owing to the higher dilution, they are probably not so reliable as the others. They are given here, especially to show that our conductivity measurements were not affected either by polarization or by any unsymmetry in the telephone; for had this been the case, our cell-constant values calculated from these solutions would not have agreed with those derived from the 0.1 normal solutions.

As explained in Section VIII, the cell-constant changes with the temperature; the percentage corrections to be applied at the different temperatures of the experiments to the 26° values are given below in the table.

TABLE IV.

Cell-Constant Corrections.

	140°.	218°.	281°.	306°.
Cell I	−0.13	−0.22	−0.34	. . . .
Cell II_a Cell II	. . . .	. . . .	. . . .	−0.43

The Water Correction. — The conductivity of the water used at the various temperatures of the experiments was subtracted from the measured conductivity of the solution. Two experiments, the data of which are given in Table V, served as a basis for the correction. For a fuller discussion of this correction, see Section VIII.

The last column gives the calculated corrections to be applied at the various temperatures to the observed resistances in the case of a 0.002 normal sodium chloride solution. It is given here, especially to show the order of magnitude of these corrections. They were similarly calculated for the other solutions.

The Conductivity Data. — The first column in Table VI contains the corrected temperatures, given in the order followed in the experiment.

The second column gives the concentrations, corrected in the case of the 140° and 218° values for the solvent in the vapor space. The correction is made as explained in Section VIII, and amounts to +0.05 per cent at 140° and +0.18 per cent at 218°.

The third column contains the observed resistances of the bomb, expressed in ohms, after correcting for errors in the resistance coils and slide wire, and deducting the resistance of the lead-wires.

The last column gives the equivalent conductivities, obtained by dividing the cell-constant for the given temperature by one one-

TABLE V.

WATER. CELL I. (MAY 14, 1902.)

Temperature.	Resistance.	Correction for 0.002 Normal NaCl Solution.
26	1180000	+0.28%
140	280000	+0.33%
218	163000	+0.41%
282	137000	+0.45%
218	155000	+0.43%
140	220000	+0.42%
26	800000	+0.41%
WATER. CELL II. (FEB. 28, 1903.)		
26	980000	+0.38%
306	140860	+0.53%
26	600000	+0.62%

thousandth part of the concentration (given in the second column) and by the resistance (given in the third column) after correcting this for the water.

XII. SUMMARY OF THE CONDUCTIVITY VALUES.

All of the equivalent conductivity values of Table VI have been brought together in Table VII, after reducing them by means of the temperature-coefficient obtained from our values, to round temperatures. The different solutions always differed slightly from the round concentrations given in the table, but the difference is too small to affect the equivalent conductivities in the last place given. The first column gives the date of the experiment; the second, the equivalent conductivity "Initial" measured at 140° immediately after the measurements at 26°; and the third, the equivalent conductivity "Final" measured upon returning to 140° after making measurements at the higher temperatures. The following columns for the higher temperatures are similar to the

TABLE VI.

THE CONDUCTIVITY DATA.

Temperature.	Equivalents per Liter.	Resistance.	Equiv. Conductivity.
NaCl: 0 10275 normal at 4°. — Cell I. (April 29, 1902.)			
25.91	0.10242	74.55	108.54
281.2	0.07733	15.823	675.0
139.7	0.09520	21.568	403.1
217.8	0.08680	16.278	585.2
25.91	0.10242	74.52	108.59
NaCl: 0.10148 normal at 4°. — Cell I. (May 3, 1902.)			
25.91	0.10114	75.35	108.75
217.9	0.08556	16.524	584.9
139.9	0.09395	21.787	404.4
NaCl: 0.09990 normal at 4°. — Cell I. (June 18, 1902.)			
25.91	0.09957	76.44	108.88
140.3	0.09248	22.018	406.5
NaCl: 0.09990 normal at 4°. — Cell I. (June 19, 1902)			
25.91	0.09957	76.28	109.10
140.4	0.09248	21.926	408.2
281.8	0.07509	16.226	678.0
218.6	0.08428	16.701	587.4
141.3	0.09241	21.965	407.8
25.91	0.09957	76.23	109.19
NaCl: 0.009990 normal at 4°. — Cell I. (May 15, 1902.)			
25.92	0.009967	687.62	120.92
138.9	0.009274	194.35	459.2
218.9	0.008438	141.54	692.3
(May 16, 1902)			
25.91	0.009967	688.10	120.83
139.1	0.009271	194.09	460.0
218.4	0.008440	141.30	693.4
(Feb. 21, 1903.)			
306.4	0.006950	156.50	901.7

Temperature.	Equivalents per Liter.	Resistance.	Equiv. Conductivity.
NaCl: 0.10311 normal at 4°. — Cell I. (April 30, 1902.)			
25.91	0.10278	74.25	108 60
138.6	0.09565	21.550	401.6
217.8	0.08713	16.217	585.2
NaCl: 0.10153 normal at 4°. — Cell I. (May 1, 1902.)			
25.91	0.10118	75.12	109.03
280.7	0.07649	15.905	678.9
218.8	0.08563	16.418	588.2
139.2	0.09415	21.810	403.1
25.91	0.10118	75.12	109.03
NaCl: 0.09990 normal at 4°. — Cell II_a. (Jan. 15, 1903)			
25.94	0.09958	91.90	108.71
305.7	0.07039	20 347	691.7
25.94	0.09958	91.72	108.93
(Jan. 17, 1903.)			
25.94	0.09958	91.72	108.93
(Jan. 17, 1903.)			
305.2	0.07051	20.296	692.3
25.94	0.09958	91.72	108.93
NaCl: 0.009990 normal at 4°. — Cell I. (May 20, 1902.)			
25.91	0.009967	687.99	120.85
139.4	0.009267	193.20	462.3
218.4	0.008410	141.70	691.4
283.6	0.007441	131.62	843.3
218.4	0.008440	141.86	691.4
140.0	0.009261	193.59	461.7
25 91	0.009967	688.28	120.80
NaCl: 0.009977 normal at 4°. — Cell II. (Feb. 20, 1903.)			
306.7	0.006944	156.37	903.3
26.00	0.009945	803.6	123.24

TABLE VI. — *Continued.*

Temperature.	Equivalents per Liter.	Resistance.	Equiv. Conductivity.
NaCl: 0.0019980 normal at 4°. — Cell I. (May 10, 1902.)			
25.91	0.0019914	3322.2	124.91
140.0	0.0018500	923.8	482.7
218.3	0.0016862	669.30	729.8
281.5	0.0014945	608.90	903.6
218.3	0.0016862	669.04	730.1
140.2	0.0018501	921.0	483.8
25.91	0.0019914	3310.8	125.18
NaCl: 0.0020183 normal at 4°. — Cell I. (May 8, 1902.)			
25.91	0.0020116	3293.0	124.76
281.4	0.0015094	604.05	901.9
25.91	0.0020116	3277.5	125.18
NaCl: 0.00049950 normal at 4°. — Cell 1. (June 23, 1902.)			
25.91	0.0004979	12945.	127.18
140.7	0.0004624	3563.7	496.6
219.0	0.0004212	2565.3	755.0
281.6	0.0003733	2333.4	935.0
219.2	0.0004210	2570.6	753.3
142.0	0.0004617	3537.8	499.3
25.91	0.0004979	12835.	127.67
(June 26, 1902.)			
25.91	0.0004979	12921.	127.42
281 3	0 0003736	2333.2	934.8
219.6	0.0004208	2564.9	755.4
139.3	0.0004630	3586.2	491.3
25.91	0.0004979	12782.	128.15
KCl: 0.10014 normal at 4°. — Cell I. (Aug. 28, 1902.)			
25.91	0.09972	63.17	131.56
141.3	0.09256	19.760	452.5
221.2	0.08404	15.360	640.6
282.4	0.07422	15.223	730.9
221.2	0.08404	15.341	641.4
141.3	0.09256	19.701	453.9
KCl: 0.10014 normal at 4°. — Cell I. (Sept. 2, 1902.)			
25.91	0.09982	63.30	131.15
141.3	0.09254	19.783	452.1
218.2	0.08445	15.406	635.5
141.9	0.09248	19.735	453.5
KCl: 0.09992 normal at 4°. — Cell II. (Feb. 10, 1903.)			
306.3	0.07015	19.068	733.4
25.94	0.09960	75.30	131.34
KCl: 0.01004 normal at 4°. — Cell II. (Feb. 17, 1903.)			
304.7	0.007006	148.58	942.3
(Feb. 18, 1903.)			
306.0	0.006977	148.94	943.9
25.96	0.009972	675.0	146.31

Temperature.	Equivalents per Liter.	Resistance.	Equiv. Conductivity.
NaCl: 0.0019978 normal at 4°. — Cell I. (May 9, 1902.)			
25.91	0.0019913	3326.4	124.76
281.4	0.0014942	608.27	904.8
25.91	0.0019913	3314.3	125.05
138.9	0.0018518	927.3	497.9
217.7	0.0016851	669.60	729.8
25.91	0.0019913	3313.0	125.09
NaCl: 0.0019954 normal at 4°. — Cell II. (March 3, 1903.)			
26.00	0.0019890	3926.0	125.65
307.2	0.0013861	720.5	977.1
26.00	0.0019890	3909.4	125.87
NaCl: 0.00049950 normal at 4°. — Cell I. (June 25, 1902.)			
25.91	0.0004979	12932.	127.31
141.3	0.0004621	3553.8	498.3
219.3	0.0004213	2573.3	752.5
281.6	0.0003733	2333.5	935.5
219.5	0.0004213	2566.0	754.0
142.3	0.0004616	3493.7	505.6
25.91	0.0004979	12614.	129.84
NaCl: 0.00049920 normal at 4°. — Cell II. (March 18, 1903.)			
26.00	0.00049874	15305.	127.05
306.8	0.00034719	2732.1	1013.9
26.00	0.00049874	15163.	127.02
KCl: 0.10014 normal at 4°. — Cell 1. (Aug. 29, 1902.)			
25.91	0.09972	63.30	131.29
141.7	0.09252	19.724	453.5
(Sept. 27, 1902.)			
25.91	0.09972	63.04	131.83
281.2	0.07443	15.160	731.9
KCl: 0.09992 normal at 4°. — Cell II$_a$. (Jan. 30, 1903.)			
304.9	0.07046	19.094	729.1
KCl: 0.09992 normal at 4°. — Cell II. (Feb. 13, 1903.)			
25.94	0.09960	75.30	131.34
(Feb. 16, 1903.)			
305.3	0.07036	19.043	732.9
KCl: 0.01004 normal at 4°. — Cell II. (Feb. 19, 1903.)			
25.96	0.009972	681.8	144.87
(March 28, 1903.)			
26.00	0.009972	685.1	144.18

TABLE VI. — *Continued.*

KCl: 0.0020007 normal at 4°. — Cell I. (Aug. 20, 1902.)				KCl: 0.0020007 normal at 4°. — Cell I. Aug. 25, 1902.			
Temperature.	Equivalents per Liter.	Resistance.	Equiv. Conductivity.	Temperature.	Equivalents per Liter.	Resistance.	Equiv. Conductivity.
25.91	0.0019943	2790.4	148.56	25.91	0.0019943	2785.6	148.82
140.3	0.0018527	828.0	537.9	140.1	0.0018530	824.8	539.9
218.4	0.0016885	618.9	788.3	219.0	0.0016869	616.1	792.6
282.0	0.0014942	574.0	959.1	281.5	0.0014963	573.6	958.4
218.5	0.0016885	619.3	787.7	219.3	0.0016861	617.2	791.5
140.4	0.0018527	824.7	539.9	140.5	0.0018522	820.8	542.6
25.91	0.0019943	2778.5	149.04	25.91	0.0019943	2726.8	151.87
KCl: 0.0019974 normal at 4°. — Cell II. (March 2, 1903.)				KCl: 0.00049990 normal at 4°. — Cell II. (March 20, 1903.)			
26.00	0.0019910	3308.6	149.02	26.00	0.00049830	12981.	150.32
306.6	0.0013900	685.8	1023.7	306.1	0.00034844	2611.3	1058.0
26.00	0.0019910	3298.0	149.18	26.00	0.00049830	12763.	151.70

second and third. All of the conductivity values are expressed in reciprocal ohms.

The agreement of the conductivity determinations made at different times, and often with different solutions, will be seen from this table. A comparison of the initial and final values in the separate experiments shows also the degree of contamination during the heating.

Table VIII contains a summary of the results given in Table VII. In deriving the means from the latter table, both the "initial" and "final" values have been included for the 0.1 and 0.01 normal solutions; but for the diluter solutions only the "initial" values have been considered; for, owing to contamination, the others are not equally reliable. The 18° values are those of Kohlrausch and Maltby.* The values given in parentheses for zero concentration were obtained by extrapolation as described in Section XIII.

In order to compare the conductivity values at different temperatures, it is desirable to correct those directly measured for the change in concentration produced by the expansion when a given solution is heated. The values in Table VIII, which, owing to this expansion, refer at different temperatures to somewhat different concentrations, as is there indicated, have been reduced to the nearest round concentrations by a graphic interpolation with the help of the linear function $\Lambda = \Lambda_0 - KC^{\frac{1}{2}}$ (see Section XIII). The so-reduced values are presented in Table IX.

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* Wissensch. Abhandl. phys.-techn. Reichsanstalt, **3**, 210 (1900).

TABLE VII.

Summary of the Equivalent Conductivity Values.

Date.	26° Init.	26° Fin.	140° Init.	140° Fin.	218° Init.	218° Fin.	281°.	306°.
$NaCl$: 0.1 normal at 4°.								
Apr. 29, 1902	108.77	108.82	. . .	403.9	. . .	585.6	674.8	. . .
" 30, 1902	108.83	. . .	405.2	. . .	585.6	. . .	. . .	. . .
May 1, 1902	109.26	109.26	. . .	405.1	. . .	586.6	679.2	. . .
" 3, 1902	108.98	. . .	. . .	404.6	585.1	. . .	. . .	. . .
June 18, 1902	109.11	. . .	405.8	. . .	. . .	. . .	. . .	. . .
" 19, 1902	109.33	109.42	407.2	404.5	. . .	586.2	677.2	. . .
Jan. 15, 1903	108.87	109.09	. . .	. . .	. . .	. . .	. . .	691.7
" 17, 1903	109.09	. . .	. . .	. . .	. . .	. . .	. . .	. . .
" 17, 1903	. . .	109.09	. . .	. . .	. . .	. . .	. . .	692.3
$NaCl$: 0.01 normal at 4°.								
May 15, 1903	121.16	. . .	462.5	. . .	690.0	. . .	. . .	. . .
" 16, 1902	121.10	. . .	462.7	. . .	692.4	. . .	. . .	. . .
" 20, 1902	121.12	121.07	461.1	461.7	690.4	690.4	837.3	. . .
Feb. 20, 1903	. . .	. . .	. . .	. . .	. . .	. . .	. . .	901.6
" 21, 1903	. . .	. . .	. . .	. . .	. . .	. . .	. . .	900.7
$NaCl$: 0.002 normal at 4°.								
May 10, 1902	125.20	125.47	482.7	483.2	728.9	729.2	902.3	. . .
" 8, 1902	125.05	125.47	. . .	. . .	. . .	. . .	900.9	. . .
" 9, 1902	125.05	125.38	. . .	483.4	. . .	730.7	903.8	. . .
Mar. 3, 1903	125.65	125.87	. . .	. . .	. . .	. . .	. . .	974.0
$NaCl$: 0.0005 normal at 4°.								
June 23, 1902	127.48	127.97	494.3	492.7	751.7	749.3	933.7	. . .
" 25, 1902	127.61	130.14	494.0	498.0	748.2	749.0	933.7	. . .
" 26, 1902	127.72	128.45	. . .	493.6	. . .	750.1	933.9	. . .
Mar. 18, 1903	127.05	127.02	. . .	. . .	. . .	. . .	. . .	1011.5
KCl: 0.1 normal at 4°.								
Aug. 28, 1902	131.81	. . .	449.1	450.5	634.0	634.8	730.9	. . .
" 29, 1902	131.54	. . .	449.1	. . .	. . .	. . .	. . .	. . .
Sept. 2, 1902	131.40	. . .	448.2	448.6	635.1	. . .	. . .	. . .
" 27, 1902	132.08	. . .	. . .	. . .	. . .	. . .	731.9	. . .
Jan. 30, 1903	. . .	. . .	. . .	. . .	. . .	. . .	. . .	727.9
Feb. 10, 1903	131.51	. . .	. . .	. . .	. . .	. . .	. . .	733.7
" 13, 1903	131.51	. . .	. . .	. . .	. . .	. . .	. . .	. . .
" 16, 1903	. . .	. . .	. . .	. . .	. . .	. . .	. . .	732.1
KCl: 0.01 normal at 4°.								
Feb. 17, 1903	. . .	. . .	. . .	. . .	. . .	. . .	. . .	942.3
" 18, 1903	. . .	146.42	. . .	. . .	. . .	. . .	. . .	943.9
" 19, 1903	144.87	. . .	. . .	. . .	. . .	. . .	. . .	. . .
Mar. 28, 1903	144.18	. . .	. . .	. . .	. . .	. . .	. . .	. . .
KCl: 0.002 normal at 4°.								
Aug. 20, 1902	148.87	149.35	536.9	538.6	787.1	786.2	956.6	. . .
" 25, 1902	149.13	152.18	539.6	540.9	789.6	787.6	957.2	. . .
Mar. 2, 1903	149.02	149.18	. . .	. . .	. . .	. . .	. . .	1022.3
KCl: 0.0005 normal at 4°.								
Mar. 20, 1902	150.32	151.70	. . .	. . .	. . .	. . .	. . .	1057.7

TABLE VIII.

Final Values for the Equivalent Conductivity of Sodium and Potassium Chlorides.

Temperature.	NaCl		KCl	
	Equivalents per Liter.	Equivalent Conductivity.	Equivalents per Liter.	Equivalent Conductivity.
18°	0	(110.3)	0	(131.4)
	0.0005	107.18	0.0005	128.11
	0.002	105.55	0.002	126.31
	0.01	101.95	0.01	122.43
	0.1	92.02	0.1	112.03
140°	0	(512.)	0	(572)
	0 000463	494.1	. . . . .	. . . .
	0.00185	482.7	0.00185	538.2
	0.00926	463.1	. . . . .	. . . .
	0.0952	405.2	0.093	449.1
218°	0	(782.)	0	(845.)
	0.000421	749.9	. . . . .	. . . .
	0.00169	728.9	0.00169	788.3
	0.00844	690.9	. . . . .	. . . .
	0.0868	585.8	0.0845	634.6
281°	0	(984.)	0	(1041.)
	0.000374	933.8	. . . . .	. . . .
	0.00149	902.3	0.00150	956.9
	0.00749	837.3	. . . . .	. . . .
	0.0774	677.1	0 0745	731.4
306°	0	(1078.)	0	(1125.)
	0.000348	1011.5	0.000349	1057.7
	0.00139	974.0	0.00139	1022.3
	0.00696	901.1	0.00699	943.1
	0.0705	692.0	0.0705	731.2

TABLE IX.

THE EQUIVALENT CONDUCTIVITY AT ROUND CONCENTRATIONS.

Substance.	Equivalents per Liter.	18°.	140°.	218°.	281°.	306°.
NaCl	0	(110.3)	512	782	984	1078
"	0.0005	107.18	493	747	926	1004
"	0.002	105.55	482	726	893	960
"	0.01	101.95	461	686	830	878
"	0.1	92.02	403	577	656	643
KCl	0	(131.4)	572	845	1041	1125
"	0.0005	128.11	. . .	. . .	. . .	1051
"	0.002	126.31	538	786	950	1007
"	0.01	122.43	. . .	. . .	. . .	922
"	0.1	112.03	447	620	699	686

XIII. THE CHANGE OF CONDUCTIVITY WITH THE CONCENTRATION.

It is a well-known fact that the Mass-Action Law does not express even approximately the change with the concentration of the dissociation of salts and strong acids and bases, when this, in accordance with the Ionic Theory, is calculated from the conductivity ratio Λ/Λ_0. This has led to the proposal of numerous other functions,* some of them derived inductively and others through hypothetical considerations, which have for their purpose an accurate representation of the experimental conductivity values and the dissociation values deduced therefrom. The extended discussion of the matter has not yet led to any conclusion as far as the theoretical explanation of the phenomenon is concerned. There have, however, been discovered some simple empirical formulas which at ordinary temperatures express the observed results satisfactorily.

Those which contain only a single arbitrary constant have the following form when expressed in terms of the equivalent conductivity (Λ) at any concentration C and the limiting conductivity Λ_0 at zero concentration:

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* Compare Kohlrausch, Wied. Ann., **26**, 200 (1885); **50**, 394 (1893); MacGregory, ibid., **51**, 133 (1894); Barmwater, Ztschr. phys Chem., **28**, 134, 428 (1899); Sabat, ibid., **41**, 224 (1902); Muller, Compt. rend., **128**, 505 (1899); Kohlrausch, Sitzungsber. preus. Akad., **44**, 1002 (1900); Rudolphi. Ztschr. phys. Chem., **17**, 385 (1895) van't Hoff, ibid., **18**, 300 (1895); Kohlrausch, ibid., **18**, 662 (1895); Storch, ibid., **19**, 13 (1896); Bancroft, ibid., **31**, 188 (1899); Jahn, ibid., **37**, 499 (1901); **41**, 265, 288 (1902); Nernst, ibid., **38**, 493 (1901).

$$\frac{\Lambda_0 - \Lambda}{C^{\frac{1}{3}}} = K \text{ (Kohlrausch)},$$

$$\frac{\Lambda_0 - \Lambda}{\Lambda^{\frac{1}{3}} C^{\frac{1}{3}}} = K \text{ (Barmwater)},$$

$$\frac{\Lambda_0 - \Lambda}{\Lambda^{\frac{3}{2}} C^{\frac{1}{2}}} = K \text{ (van't Hoff)},$$

$$\frac{\Lambda_0 - \Lambda}{\Lambda^{2} C^{\frac{1}{2}}} = K \text{ (Rudolphi)}.$$

It seemed therefore to be of especial interest to test the applicability of these formulas at the widely different temperatures employed in our experiments. In making such a test, it must be borne in mind that the results will be in a high degree dependent on the values of Λ_0 employed, since in dilute solutions $\Lambda_0 - \Lambda$ is a relatively small quantity; yet in several instances authors have not given sufficient consideration to this matter. The most satisfactory method of procedure seems to us to be the elimination of the Λ_0 value, which cannot be determined with sufficient accuracy by extrapolation, by writing the functions in the following form:

$$\Lambda = \Lambda_0 - K C^{\frac{1}{3}} \text{ (Kohlrausch)},$$

$$\Lambda = \Lambda_0 - K \Lambda^{\frac{1}{3}} C^{\frac{1}{3}} \text{ (Barmwater)},$$

$$\Lambda = \Lambda_0 - K \Lambda^{\frac{3}{2}} C^{\frac{1}{2}} \text{ (van't Hoff)},$$

$$\Lambda = \Lambda_0 - K \Lambda^{2} C^{\frac{1}{2}} \text{ (Ostwald)},$$

and then plotting the values of Λ along one co-ordinate axis and those of the C-Λ function constituting the last term (that is, $C^{\frac{1}{3}}$, $\Lambda^{\frac{1}{3}}C^{\frac{1}{3}}$, etc.) along the other axis, as is illustrated by Figure 9. If the function in question holds, the points will of course lie upon a straight line; and by comparing, in the case of the different functions, the deviations of the separate points from the best representative straight line that can be drawn, a measure of the degree of applicability of each function is obtained. All our complete series of measurements and those of Kohlrausch and Maltby on the same salts at 18° have been studied in this way, a plot on a very large scale being employed. The straight lines were drawn in every case so as to make the percentage deviations of the two conductivity values for the two more concentrated solutions and

TABLE X. — Deviation of the Observed Conductivity Values from Those Calculated by Various Empirical Formulas.

Salt and Temperature.	Milli-Equivalents per Liter.	Equivalent Conductivity.	Percentage Deviation of Observed from Calculated Conductivity Values.			
			$C^{\frac{1}{3}}$.	$\Lambda^{\frac{1}{3}}C^{\frac{1}{3}}$.	$\Lambda^{\frac{3}{2}}C^{\frac{1}{2}}$.	$\Lambda^{2}C^{\frac{1}{2}}$.
NaCl 18°	0.5	107.18	−0.16	−0.17	+0.03	+0.04
	2.0	105.55	+0.08	+0.11	−0.06	−0.06
	10.0	101.95	+0.09	+0.15	−0.17	−0.24
	100.0	92.02	−0.16	−0.24	+0.11	+0.18
		Mean	0.12	0.17	0.09	0.13
NaCl 140°	0.463	496.3	+0.24	+0.12	+0.28	+0.41
	1.85	483.5	−0.25	−0.21	−0.62	−0.42
	9.26	464.8	+0.02	+0.19	−0.44	−0.05
	95.2	407.3	−0.02	−0.15	+0.37	+0.05
		Mean	0.13	0.17	0.43	0.23
NaCl 218°	0.421	753.3	+0.05	−0.07	+0.28	+0.16
	1.69	732.4	−0.04	+0.11	−0.20	−0.08
	8.44	693.5	−0.18	+0.04	−0.34	+0.08
	86.8	588.3	+0.18	−0.03	+0.52	−0.03
		Mean	0.11	0.06	0.34	0.09
NaCl 281°	0.374	937.6	+0.01	−0.18	+0.11	−0.18
	1.49	906.2	−0.02	+0.17	−0.05	+0.11
	7.49	840.9	−0.60	−0.15	−1.43	+0.28
	77.4	680.1	+0.50	+0.14	+0.48	−0.46
		Mean	0.28	0.16	0.27	0.26
NaCl 306°	0.348	1013.	−0.05	−0.17	−0.14	−0.02
	1.39	975.2	+0.05	+0.14	+0.11	+0.02
	6.96	902.2	+0.34	+1.20	+1.50	+3.54
	70.5	686.0	−0.45	−1.31	−1.68	−2.84
		Mean	0.22	0.71	0.76	1.61
KCl 18°	0.5	128.11	−0.08	− .10	+0.12	+0.07
	2.0	126.31	+0.04	+ .09	−0.11	−0.05
	10.0	122.43	−0.06	+ .05	−0.40	−0.70
	100.0	112.03	+0.06	− .02	+0.58	+0.90
		Mean	0.06	0.07	0.30	0.43
KCl 306°	0.349	1059.	−0.21	−0.45	−0.22	−0.60
	1.39	1024.	+0.18	+0.41	+0.29	+0.49
	6.99	945.8	+0.05	+0.80	+0.90	+2.01
	70.5	732.1	−0.05	−0.67	−0.95	−2.44
		Mean	0.12	0.58	0.59	1.38

also of those for the two more dilute solutions opposite and nearly equal. The results are presented in detail and in summarized form in Tables X and XI. The concentration is expressed in milli-equivalents per liter. The numbers in the last four columns show the percentage difference between the observed conductivity and that required by the assumed linear function, the nature of whose argument ($C^{\frac{1}{3}}$, $C^{\frac{1}{3}}\Lambda^{\frac{1}{3}}$, etc.) is shown by the headings.

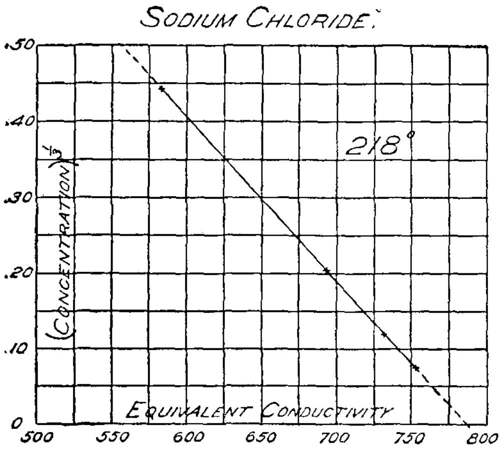

Figure 9.

TABLE XI.

Summary of the Mean Deviations.

Salt.	Temp.	$C^{\frac{1}{3}}$	$\Lambda^{\frac{1}{3}}\,C^{\frac{1}{3}}$	$\Lambda^{\frac{3}{2}}\,C^{\frac{1}{2}}$	$\Lambda^{2}\,C^{\frac{1}{2}}$
NaCl	18°	0.12	0.17	0.09	0.13
"	140°	0.13	0.17	0.43	0.23
"	218°	0.11	0.06	0.34	0.09
"	281°	0.28	0 16	0.27	0.26
"	306°	0.22	0.71	0.76	1.61
KCl	18°	0.06	0.07	0.30	0.43
"	306°	0.12	0.58	0.59	1.38
Mean		0.15	0.27	0.40	0.59

An examination of Table XI leads to the following conclusions: All four formulas express fairly well the results with sodium chloride at 18°; but the formulas of van't Hoff and of Rudolphi do not apply well to those with potassium chloride at 18°. The Kohlrausch formula expresses the results for both salts at all temperatures without great error,* and the same is true of the Barmwater formula, except at the highest temperature, where the deviations with both salts are large. The van't Hoff and Rudolphi formulas do not accord at all with the observed values at 306°, the deviations in the case of the latter formula being especially large, while at the lower temperatures of 140, 218, and 281° the van't Hoff formula is far less satisfactory than those of Kohlrausch and Barmwater. On the whole, therefore, the simple Kohlrausch formula furnishes the best representation of the results, and the Barmwater the next best, — facts which are directly indicated by the final means at the foot of the last table. Whether within the range of concentration in question (0.0005 — 0.1 normal) the deviations corresponding to the former are really less than the experimental errors cannot be decided with certainty: the greatest deviation (see Table X) from the very accurate values of Kohlrausch and Maltby at 18° is 0.13 per cent † in the case of sodium chloride, and 0.06 per cent in that of potassium chloride; the greatest deviations at the highest temperatures are 0.55 per cent at 281° and 0.40 per cent at 306° in the case of sodium chloride, and only 0.20 per cent at 306° in that of potassium chloride. It seems improbable that the experimental errors are as large as these deviations in the case of the sodium chloride; but it is perhaps not impossible.

It may be of interest to state also the percentage deviations of our straight line corresponding to the Kohlrausch function from the points representing the conductivities of sodium and potassium chloride at 18° in the still more dilute solutions investigated by Kohlrausch and Maltby. These deviations are —0.53 and —0.42 per cent, respectively, in case of the 0.0001 normal solutions, and —0.36 and —0.25 per cent, respectively, in that of the 0.0002 normal solutions. It is to be noted with reference to the significance of this disagreement, that the conductivity of the water used for the 0.0001 normal solutions formed from 7 to 10 per cent of that of the salt, and that the results were corrected for it under the assumption that it was uninfluenced by the addition of the salt. Kohlrausch and Maltby, however, consider it almost impossible that from this source an error of the magnitude of these deviations can arise.

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* Compare also Fig. 9, which is an accurate plot of the values for sodium chloride at 218°.

† Assuming the first two as well as the last two deviations to have been completely equalized, which could have been done by slightly displacing the representative straight line.

The at least approximate validity of the simple Kohlrausch equation under such widely different conditions, is a fact that must receive attention in any theoretical explanation of the phenomenon. The fact seems somewhat remarkable when it is considered how great the change is in the state of the solvent, which has been raised from near its melting point to not very far below its critical point, and when it is considered that the dissociation has decreased in the 0.1 normal solutions from about 84 per cent at 18° to 60 per cent at 306° (see Section XV). The equation cannot, however, retain its validity as the dissociation tendency approaches zero; for then with increasing concentration the calculated values of Λ would soon become negative: it must, if it is to apply generally, be modified by multiplying the cube-root of the concentration by some function of Λ which does not vary much as long as Λ/Λ_0 is large, in a manner similar to that which has been proposed by Barmwater.

The fact that the van't Hoff equation does not express satisfactorily the results with many salts even at 18° (see KCl in Table X) has led to the suggestion by Storch and Bancroft that a general expression of the form $\Lambda_0 - \Lambda = K\Lambda^n C^{n-1}$ be employed, the exponent n being different with each salt. Our results show that in order to attain agreement it would be necessary to vary the value of n also with the temperature. Thus it was found that by putting $n = 1.6$, the results with sodium chloride at 306° are expressed with a mean deviation of only 0.15 per cent, but the use of this same exponent with the results at 18° gives rise to a mean deviation of 0.69 per cent, while as shown in the above tables, the van't Hoff function, with $n = 1.5$, applies well at 18°, but fails at 306°. The fact that even at the highest temperature the exponent required has risen only to 1.6 shows that the results do not correspond much more closely with the Mass-Action Law, which requires the exponent 2, than they do at the ordinary temperature.

In view of the foregoing considerations there is at present no more reliable means of deriving the conductivity Λ_0 for zero concentration from our results at the higher temperatures than by the application of the Kohlrausch equation. We have therefore determined from our plots the intercept of the straight line representing this equation with the axis along which the conductivity values are laid off; and it is these values of Λ_0 which are recorded within parentheses in Table VIII. For the sake of uniformity, the Λ_0 values at 18° were derived in the same way from the data of Kohlrausch and Maltby; they are about 1.2 per cent larger than those deduced by these investigators.

XIV. Change of the Conductivity with the Temperature.

In order to show more clearly the effect of temperature on the conductivity, the values of Table IX are represented graphically in Figures 10 and 11. (The dotted curve for the 0.5 normal sodium chloride solution is based on values extrapolated from 0.1 normal by means of the Kohlrausch linear equation, and it has been drawn only to indicate roughly the general character of the curve at a higher concentration.)

A consideration of these plots and of the data themselves leads to the somewhat striking conclusion that throughout this wide range of temperature of nearly 300°, the conductivity extrapolated for zero concentration, and therefore the migration velocity of the ions, increases in the case of sodium chloride at a rate closely proportional to the increase of temperature. In the case of potassium chloride this principle expresses the results without considerable error up to 218°, but at the higher temperatures the conductivity increases somewhat more slowly than it requires.

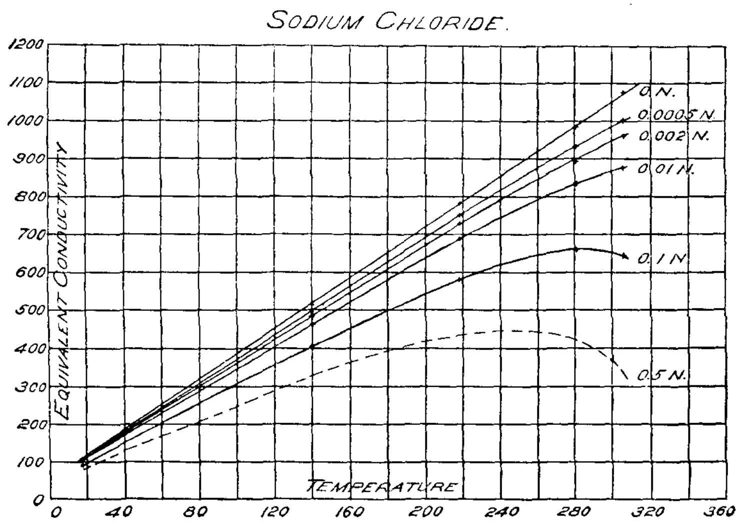

Figure 10.

To show the extent of the deviations, the conductivity has been expressed by a linear equation of the form: $(\Lambda_0)_t = (\Lambda_0)_{18} + a\,(t - 18)$. The value (3.34) adopted for a in the case of the sodium chloride was

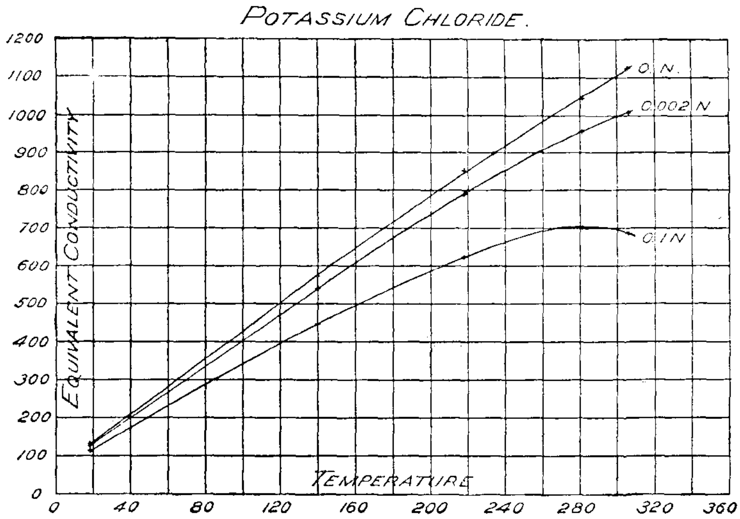

FIGURE 11.

the mean of that derived by using the values of Λ_0 at 18 and 281° and at 18 and 306°. The value of 3.57 in the case of potassium chloride was calculated from the values of Λ_0 at 18° and 218°. The values of Λ_0 calculated from these equations are given in the following table beside those previously found.

TABLE XII.

THE Λ_0 VALUES AS A LINEAR TEMPERATURE FUNCTION.

Temperature.	Sodium Chloride.			Potassium Chloride.		
	Found.	Calculated.	Per cent. Diff.	Found.	Calculated.	Per cent. Diff.
18°	110	. . .	. . .	131	. . .	. . .
140°	512	517	−1.0	572	566	+0.7
218°	782	778	+0.5	845	845	. . .
281°	984	988	−0.4	1041	1070	−2.7
306°	1078	1072	+0.6	1125	1159	−2.9

It will be seen that the differences in the case of sodium chloride are in opposite directions at 140 and 218°; they are therefore doubtless in large part, if not entirely, due to experimental errors; and therefore, within the limits of these, the conductivity at infinite dilution is a linear function of the temperature in the case of this salt.[*] With potassium chloride the deviation at 140°, though probably real, since it falls upon the smooth curve drawn through all the values, amounts to only 0.7 per cent. At the higher temperatures the deviations are seen to be much larger, though still of secondary magnitude. This approximate proportionality between the increase of migration velocity and that of the temperature has, up to 100°, already been observed by Schaller.[†]

The values of the percentage temperature-coefficients referred to the value at 18° $\left(\frac{100\,\Delta\Lambda_0}{(\Lambda_0)_{18}\,\Delta t}\right)$ are 2.74 for potassium chloride (between 18 and 218°) and 3.05 for sodium chloride. These are much higher than those observed on dilute solutions (0.01 — 0.001 normal), by Kohlrausch between 18 and 26° (2.21 and 2.38), by Déguisne between 18 and 34° (2.28 and 2.40), by Arrhenius between 18 and 52° (2.33 and 2.53), and by Krannhals between 18 and 99.4° (2.40 and 2.62)[‡]. This is due in part to the facts that the values of some of these investigators were not corrected for the expansion of the solutions upon heating and that they refer to solutions not infinitely dilute;[§] but it also undoubtedly arises in part from the fact that the temperature function is not perfectly linear until temperatures considerably above 18° are reached.

The ratio of the limiting conductivity of potassium chloride to that of sodium chloride is 1.19 at 18°, 1.13 at 140°, 1.08 at 218°, 1.06 at 281°, and 1.04 at 306°. The migration-velocities of the potassium and sodium ions are therefore slowly approaching equality.

The conductivity curves (see Figs. 10 and 11) at the higher concentrations recede more and more from that for zero concentration as the temperature rises, owing to a decreasing dissociation tendency (see

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[*] This is also clearly shown by the values of a calculated for the four successive temperature-intervals 18 — 140, 140 — 218, 218 — 281, and 281 — 306°; these are 3.30, 3.47, 3.21, and 3.75 (this last value being much in error because the interval is small).

[†] Ztschr. phys. Chem., **25**, 512, 523 (1898.)

[‡] These data are taken from Kohlrausch and Holborn's Leitvermögen der Elektrolyte, 197-199.

[§] Thus, the temperature-coefficient between 18° and 140° of our 0.01 normal NaCl solutions is 2.90, while that at infinite dilution is 3.05.

Section XV). This even gives rise to a maximum value in the case of the 0.1 normal solution of both salts at a temperature of about 280°. With more concentrated solutions this effect would undoubtedly be much more pronounced, as is indicated by the dotted curve in Fig. 10.

XV. CHANGE OF THE DISSOCIATION WITH THE CONCENTRATION AND TEMPERATURE.

It has already been shown in Section XIII that the conductivity Λ changes with the concentration C at all temperatures closely in accordance with the equation $\Lambda_0 - \Lambda = K\, C^{\frac{1}{3}}$. Expressed in terms of the dissociation, $x = \frac{\Lambda}{\Lambda_0}$, this becomes $\frac{1-x}{C^{\frac{1}{3}}} = K$; that is, the fraction of the salt undissociated is directly proportional to the cube-root of the concentration, or the concentration of the undissociated molecules, $(1-x)\,C$, is directly proportional to the $\frac{4}{3}$ power of the total concentration.

The change of the degree of dissociation with the temperature is shown for both salts in Table XIII, and for sodium chloride in Fig. 12.

TABLE XIII.

DISSOCIATION.

Substance.	Equivalents per Liter.	18°	140°	218°	281	306°
NaCl	0	100.0	100.0	100.0	100.0	100.0
"	0.0005	97.3	96.6	95.8	94.3	93.2
"	0.002	95.7	94.5	93.0	91.0	89.1
"	0.01	92.5	90.3	87.9	84.5	81.4
"	0.1	83.4	79.0	74.0	66.8	59.8
KCl	0	100.0	100.0	100.0	100.0	100.0
"	0.0005	97.5	. . .	. . .	. . .	93.4
"	0.002	96.1	94.1	92.7	91.3	89.5
"	0.01	93.2	. . .	. . .	. . .	82.0
"	0.1	85.2	78.3	73.3	67.2	61.0

It will be seen that, especially in the 0.1 normal solution, the dissociation has decreased very greatly at the higher temperatures, namely, from 83–85 per cent at 18° to 60–61 per cent at 306°; and that the decrease is becoming extremely rapid at those temperatures, the course of the curve indicating that the dissociation is very small in the neighborhood of the critical temperature (about 360°).

Table XIII also shows that the dissociation values for the two chlorides are nearly identical at all temperatures and concentrations, the extreme variation being about 2 per cent in the 0.1 normal solution. This gives support to the idea that the decrease of conductivity and of the calculated dissociation is due to a physical cause (probably in some way to the electrical charges on the ions) and not to specific chemical affinity. The theoretical discussion of the results is, however, best postponed until we have made more extended series of measurements.

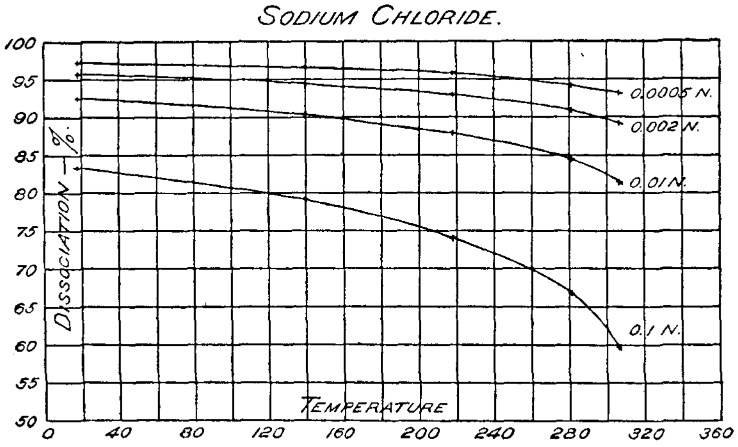

FIGURE 12.

The values of the constant K in the equation $(1 - x) = KC^{\frac{1}{3}}$ are tabulated below, as these give a measure of the dissociation-tendency independent of the concentration. They were obtained from the plots described in Section XIII by taking the ratio $\left(\frac{\Delta \Lambda}{\Delta C^{\frac{1}{3}}}\right)$ of the intercepts on the two axes, of the best representative straight line, and dividing this ratio by Λ_0.

TABLE XIV.

VALUES OF THE DISSOCIATION CONSTANT $(1 - x) / C^{\frac{1}{3}}$.

Salt	18°	140°	218°	281°	306°
NaCl	0.366	0.448	0.573	0.745	0.877
KCl	0.321	0.468	0.577	0.713	0.853

XVI. Summary.

In this article has been described the construction of a platinum-lined bomb of 124 ccm. capacity with electrodes insulated by quartz-crystal cylinders, by means of which the conductivity and specific volume of aqueous salt solutions can be determined with an accuracy of 0.2 or 0.3 per cent, at least up to a temperature of 306°. The methods employed for overcoming the serious mechanical difficulties may be of assistance in other investigations requiring similar apparatus; for instance, in calorimetric bomb work.

Measurements of the conductivity and specific volume have been made upon solutions of sodium and potassium chlorides at concentrations varying from 0.0005 to 0.1 normal, at the temperatures 140, 218, 281, and 306°.

The results obtained with these salts show that even at the highest temperature the dissociation calculated from the conductivity ratio does not change with the concentration in much closer accord with the requirements of the Mass-Action Law than at the ordinary temperature. The various empirical functions which have been proposed for the expression of the change of conductivity with the concentration were tested as to their applicability to the results at these widely different temperatures by a graphical method by which the effect of the uncertainty in the conductivity values (Λ_0) for zero concentration was eliminated. It was found that that given by Kohlrausch, $\Lambda_0 - \Lambda = KC^{\frac{1}{3}}$, was most satisfactory, the deviations being scarcely greater than the possible experimental error. Except at the highest temperature, the function of Barmwater, $\Lambda_0 - \Lambda = K\, \Lambda^{\frac{1}{3}}\, C^{\frac{1}{3}}$, also gave good results. To make applicable the general function $\Lambda_0 - \Lambda = K\, \Lambda^n\, C^{n-1}$ (of which the van't Hoff function $\Lambda_0 - \Lambda = K\, \Lambda^{\frac{3}{2}}\, C^{\frac{1}{2}}$ is a special case), it is necessary to vary the exponent n not only with the nature of the salt, but also with the temperature.

The conductivity values extrapolated for infinite dilution, and therefore the migration-velocities of the ions, were found to be an approximately linear function of the temperature, throughout the whole range of temperature in the case of sodium chloride, and up to 218° in the case of potassium chloride, the deviations being moreover not very large (3.5 per cent) even at the highest temperatures with the latter salt. The temperature-coefficients referred to the values at 18° are 3.05 per cent for sodium chloride between 18 and 306°, and 2.74 per cent for potassium chloride between 18 and 218°.

The ratio of the conductivities of potassium and sodium chlorides at infinite dilution decreases from 1.19 at 18° to 1.04 at 306°, showing that the migration velocities of the sodium and potassium ions are slowly approaching equality.

The degrees of dissociation of the two salts are nearly identical (extreme variation about 2 per cent) at all temperatures and concentrations. The dissociation in 0.1 normal solution has approximately the following values: 84 per cent at 18°, 79 per cent at 140°, 74 per cent at 218°, 67 per cent at 281°, and 60 per cent at 306°. It is decreasing with great rapidity at the higher temperatures. Its change with the concentration is at all temperatures accurately expressed by the equation $(1 - x) / C^{\frac{1}{3}} = K$.

The conductivity of the vapor over a 0.1 normal potassium chloride solution at 306° is too small to be observed with the present apparatus: its specific conductivity is certainly less than $\frac{1}{200000}$ part of that of the liquid.

The specific volume of the 0.002 normal solutions, which can be regarded as identical with that of pure water, was found to be 1.186 at 218°, 1.336 at 281°, and 1.434 at 306°. The expansions of the two 0.1 normal solutions are substantially identical, but somewhat less than that of water, as is shown by the fact that the ratio of their specific volumes at 306° and 4° is 1.422, instead of 1.434.

This investigation is being continued with the co-operation of others. During the coming year other di-ionic salts and some tri-ionic salts will be investigated in the same manner. A separate research will deal with weak acids and their neutral salts with the aim of studying hydrolysis and the dissociation-constant of water. Another line of work to be carried out with the bomb will consist in the determination of the dielectric constant of water up to 306°. Then the attempt will be made to extend all these measurements to the critical temperature.

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CONTRIBUTIONS FROM THE ZOÖLOGICAL LABORATORY OF THE MUSEUM OF COMPARATIVE ZOÖLOGY AT HARVARD COLLEGE. E. L. MARK, DIRECTOR. — No. 146.

THE LAWS OF HEREDITY OF GALTON AND MENDEL, AND SOME LAWS GOVERNING RACE IMPROVEMENT BY SELECTION.

By W. E. Castle.

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I. The "Law of Ancestral Heredity."

In the year 1889, the eminent English statistician, Francis Galton, attempted to give precise mathematical expression to the well-known fact that the child resembles in varying degree its ancestors near and remote. From a study of family statistics of stature, he found that children resemble their parents, on the average, more closely than their grandparents, and the latter more closely than their great-grandparents, and so on to ancestors still more remote. He tentatively advanced the hypothesis that the resemblance to each earlier generation of ancestors is just half that to the next later.

Galton subsequently tested this hypothesis in the case of a domesticated animal, by applying it to an extensive series of records of the inheritance of black spots in Basset hounds. Satisfied with the result, Galton (**'97**, p. 502) then formulated as follows the general "Law of Ancestral Heredity": — "The two parents contribute between them, on the average one-half, or (0.5) of the total heritage of the offspring; the four grandparents, one-quarter, or $(0.5)^2$; the eight great-grandparents, one-eighth, or $(0.5)^3$, and so on. Thus the sum of the ancestral contributions is expressed by the series $[(0.5) + (0.5)^2 + (0.5)^3$, etc.], which, being equal to 1, accounts for the whole heritage." Galton found that, allowance being made for male prepotency, the theoretical values calculated in accordance with this "law" conform very closely to the values actually observed in the series of generations of Basset hounds. He, therefore, put his law forward as a general law of ancestral heredity.

But subsequent examination by Pearson ('98) of the material studied by Galton, and of other material similar in nature, has failed to substantiate Galton's conclusion, except in a much modified form. In the most recent statement of his views, Pearson (:03) holds with Galton that the best prediction as to the character of the offspring must be based upon the character of the ancestors, and that the influence of the various ancestors diminishes as they become more remote. He believes that "the contributions of the ancestry follow a geometrical series, although not that originally proposed by Mr. Galton." From a study of the inheritance of eye-color in man and coat-color in thoroughbred horses, he concludes that "as far as the available data at present go, inheritance coefficients for ascending ancestry are within the limits of observational error represented by a geometrical series and by the same series." This series, he observes, approximates those designated I and II below: —

	Pearson's Series I.	Pearson's Series II.	Galton's Series.
Parental influence	.49	.50	.50
Grandparental influence	.32	.33	.25
Great-grandparental influence . . .	.20	.22	.125
Great-great-grandparental influence .	.13	.15	.0625

Comparing Pearson's series with that of Galton, we see that the parental influence is reckoned as substantially the same by both Galton and Pearson, but that Pearson assigns a much greater influence to the more remote ancestors than does Galton.

It should be observed that the "available data" upon which principally Pearson bases his conclusions consist of two cases of pigment inheritance, one in man, the other in the horse. A third well-known series of this sort has not been utilized by Pearson, though our information about it is much more complete and precise than that about either of the other two. I refer to the statistics about color inheritance in mice recorded by von Guaita ('98, :00), of which an analysis has been made by Davenport (:00). In this series the inheritance of color follows closely neither the law of Galton nor the series suggested by Pearson.

The lack of agreement in this case with Galton's law has been pointed out by Davenport for certain of the color categories. He concludes that in the case of gray alone does the color inheritance among von Guaita's mice conform closely with Galton's law. But in reality, even in the case of gray, close agreement does not occur; Davenport's conclusion that it does occur results from the inclusion by him in a single color category of two sorts of mice which are clearly quite distinct, namely, (1) mice gray all over like the wild house-mice, and (2) gray mice with white markings. Even when these two categories are combined, Davenport's figures show close agreement between the observed and calculated numbers in two only of the five filial generations with which he deals, namely, in the third and sixth generations (of von Guaita's nomenclature), in which he finds observed and calculated to agree perfectly. But in the three remaining generations he finds observed and calculated percentages to be related as follows: —

Generation	II.	IV.	V.
Observed	100%	58%	48%
Calculated	0%	48%	60%

Davenport, moreover, has excluded from the category of "albinos" white mice which possess the dancing character. But this is manifestly an error, for the dancing character has nothing to do with coat-color, and is inherited quite independently of it. Davenport's classification, accordingly, makes the category of albinos appear smaller than it really is. If we include all albinos (whether dancers or not) in one category, and make separate classes for gray, gray-white, black, and black-white mice, the relations between the observed and calculated numbers in each generation are found to be as indicated in Table I, Davenport's method of calculation being followed.

An examination of this table shows no close agreement between calculated and observed conditions throughout any single category or any single generation, although the totals turn out better than the predictions for the generations considered separately. In Generation II the discrepancies are glaring. In the column, white, the grand totals alone agree closely, yet this agreement is clearly without significance; it is a chance agreement in the totals of two series divergent throughout.

The observed numbers, it is evident, agree no better with one of Pearson's series than with that of Galton. The discrepancies noted

TABLE I.

Test of Galton's Law by the Statistics of Von Guaita.

B, black; *B-W*, black-white; *G*, gray; *G-W*, gray-white; *W*, white.

Generation.		*G.*	*G-W.*	*B.*	*B-W.*	*W.*	Total.
II.	Calc.	0			14	14	
	Obs.	28			0	0	28
III.	Calc.	22	0	0	11	11	
	Obs.	17	8	4	1	14	44
IV.	Calc.	14	1	0	3.8	12.1	
	Obs.	16	5	4	1	5	31
V.	Calc.	23	45	13.7	16	15.5	
	Obs.	3	55	12	24	19	113
VI.	Calc.	10.5	31.2	18.4	6	20.4	
	Obs.	2	22	9	17	36	86
VII.	Calc.	1.2	1	0.7	0.2	3.7	
	Obs.	0	4	0	0	3	7
Total	Calc.	70.7	78	32.8	51	76.7	
	Obs.	66	94	29	43	77	309
Ratio, Obs. : Calc.		93.3%	120.5%	88.4%	84.3%	100+ %	

between observed and calculated will remain and even be accentuated if we replace Galton's series with one of those suggested by Pearson. For the result will be unchanged in Generation II, but the calculated numbers will in most cases diverge still more from the observed ones, in the later generations, because Pearson attaches more weight to the remoter ancestors than does Galton.

It is evident, then, that some fundamental defect exists in the "law of ancestral heredity," as stated by either Galton or Pearson. It fails in the case just examined not only to account for the observed result, but even to enable one to predict that result with any degree of accuracy, and that too in the very category of cases which it was originally formulated to cover, namely in color inheritance among mammals. Galton himself ('97, p. 403) recognized the existence of such a defect, though he considered it, for practical purposes, of little consequence. Stated in his own words it is as follows: —

"The chief line of descent," it is generally believed, "runs from germ to germ and not from person to person." Yet "the person may be accepted on the whole as a fair representative of the germ, and, being so, the statistical laws which apply to persons would apply to the germs also, though with less precision in individual cases." Failure of Galton's law in the case of von Guaita's statistics is due to the falsity of the assumption here made by Galton that the person is "a fair representative of the germ." In all cases of alternative inheritance the person (or soma) represents only a *part* of the ripe germs produced by the individual, in some cases it may even represent none of them. Hence any theory of heredity which bases its predictions as to the character of the offspring solely upon the character of the soma of the ancestors, is clearly inapplicable to cases of alternative inheritance. The presumption is against its application to any other class of cases until that applicability has been demonstrated.

II. MENDEL'S LAW OF HEREDITY.

Certain facts of alternative inheritance were clearly stated and accounted for many years ago by Gregor Mendel ('66). He thus not only formulated laws of alternative inheritance, whose correctness has been fully confirmed by a number of independent observations, but he also laid the foundation for a general theory of heredity. In the history of the study of heredity his discovery is the most fundamental and far-reaching. Its importance is not lessened by the fact that it was long unrecognized. Only under the fertilizing influence of Weismann's ideas was the rediscovery of Mendel's law accomplished independently by de Vries (:00), Correns (:00), Tschermak (:00), and others. To its further development no one has contributed more than Bateson (:02).

Where Galton's law gives us at best rough approximations based upon averages of heterogeneous material, and with no attempt at an explanation of the results, Mendel's law enables us to make predictions for specific cases as to both the character and the numerical proportions of the offspring to be expected, and furnishes us at the same time with a rational explanation of the outcome. It thus meets the two-fold requirements of a scientific theory, a statement of phenomena and an explanation of them; the "law of ancestral heredity" attempts only the first of these two things, and even here fails lamentably. It will thus be seen that the claims of Mendel's law are much greater than those of Galton's law. If it fails, its failure is as much more signal.

The same test may be applied to Mendel's law as to Galton's. Can we, on the basis of Mendel's law, make predictions concerning the various generations of von Guaita's mice with greater accuracy than has been found possible under Galton's law? Before we can frame an answer to this question, we must know precisely what the Mendelian predictions are.

Mendelian predictions are based, not on the *somatic character of the parents*, but on the character of the *germ-cells* formed by the parents. The simplest way of determining the character of the germ-cells formed by an animal or plant is by experimental breeding tests. In cases where this is not practicable, one can often predict with equal confidence from a knowledge concerning the grandparents, not as to their *somatic* character, but as to the character of their *germ-cells* as evidenced by the nature of the offspring produced by them. Stated in the terminology of present-day biology, the principles which underlie the Mendelian predictions are these: —

1. Every gamete (egg or spermatozoön) bears the determinants of a complete set of somatic characters of the species. Accordingly when two gametes (an egg and a spermatozoön) have met in fertilization, there are present in the fertilized egg the representatives of *two sets* of somatic characters, which may or may not be the same. If they are the same for a given character, as, for example, coat-color in mammals, the individual which develops from the egg must inevitably have that same character. Thus when gametes formed by one white mouse meet in fertilization gametes formed by another white mouse, the offspring are invariably white. Similarly when a wild gray mouse is bred to another wild gray mouse the offspring are invariably gray. And when a pure-bred spotted black-white mouse is bred to a mouse like itself, the offspring are all spotted black-white.

2. But when the two gametes uniting bear each what represents a *different* somatic character, only one of these characters may be manifested by the individual (or zygote) formed. Thus, when wild gray mice are mated with white mice, only gray offspring are produced. The gray character is, in Mendel's terminology, *dominant*, the white character

recessive. Or, when wild gray rats are mated with black-white rats, only gray rats are produced. The wild gray character is, accordingly, dominant not only over white, but also over black-white.

3. Sometimes the zygote formed by the union of two unlike gametes (heterozygote, Bateson, :0) develops the character of neither parent in its purity. It may have a character intermediate between those of its parents, or something entirely different from either. Thus when black-white mice are mated with white mice, the offspring are gray like the wild house mouse.

4. Whatever the somatic character of the zygote is, the germ-cells which it forms will be, in respect to any particular character, like those which united to produce it, — half like the maternal and half like the paternal gamete. Thus, a gray mouse obtained by crossing a wild gray mouse with a white one forms in equal numbers gametes which bear the gray character and those which bear the white character. This is conclusively shown by two simple breeding tests: 1) when a cross-bred (or hybrid) gray mouse is bred to a white mouse, half the offspring are hybrid grays, half are white. This is precisely the result we should expect if the cross-bred gray mouse forms, in equal numbers, as we have supposed, gametes which bear the gray and those which bear the white character. For

The gray mouse will produce gametes	G and W
The white mouse, gametes	W and W
And the possible combinations of these 2 sets are their product	$2\,GW + 2\,WW$

But, as we have already stated, when a zygote contains *both* the gray character and the white character, only the former will be visible. This may be indicated by placing the (invisible) W within a parenthesis. Further, in the expression $2\,WW$ one of the identical letters may be dropped as superfluous. Our formula, representing the outcome of the breeding test described, then reads $2\ G\ (W) + 2\ W$, and signifies that two in every four of the offspring produced will be gray hybrids, and the remaining two white. 2) When two cross-bred (or hybrid) gray mice are bred together, the offspring consist of gray mice and white mice in the ratio of three gray to one white. Moreover, breeding tests show that of the three gray mice thus obtained one is pure, that is, will form only gametes bearing the gray character, while two are hybrid, that is, will form gametes some of which bear the gray character, others the white character. This is precisely the result expected under our hypothesis that each hybrid individual forms gametes G and W in equal numbers.

For the possible combinations of two sets of gametes each G and W are represented by their product $GG + 2GW + WW$, or simplified as already explained, $G + 2G(W) + W$.

The principle illustrated by these examples is, as pointed out by Bateson (:02), the most fundamental and far-reaching of the Mendelian ideas. It is known as the law of segregation, or "splitting" (de Vries, :00) of the parental characters at gamete formation, or as the "principle of gametic purity" (Bateson, :02). Dominance is purely a secondary matter; it may or may not occur along with segregation, though the latter can be more easily demonstrated in cases where it is associated with the former. The principle of gametic purity just stated rests upon the assumption that gamete-formation is the reverse of fertilization. In fertilization, gametes A and B unite to form a zygote AB; when this zygote in turn forms gametes, they will be again A and B. From a knowledge of the *somatic form* alone of pure As and Bs, one can make no trustworthy prediction as to the form of AB. — Here is the fundamental error of the "law of ancestral heredity" as stated by Galton ('97) or Pearson (:03). — AB may have invariably the somatic form of A or of B (cases of simple dominance, as of gray over white in mice); or it may have *sometimes* the form of A, sometimes that of B (cases of alternative dominance — see Tschermak (:02) —); or, finally, the somatic form of AB may be different from both that of A and that of B (cases like that of the gray hybrid formed by the cross of black-white with white mice). But, no matter what the somatic form of AB is, we may with confidence predict that its gametes will be essentially pure As and pure Bs, and the two will be produced in proportions approximately equal. This is the Mendelian expectation in all cases of alternative inheritance. Whether it applies to other cases also, and if so to what extent, is not yet known. For the present we may confine our attention to the case which afforded a basis for the "law of ancestral heredity," namely alternative color-inheritance among mammals.

In Table II are given the Mendelian predictions for the inheritance of complete albinism in the various generations and matings of von Guaita's mice. These predictions are based upon the fact repeatedly observed that complete albinism behaves as a *recessive* character in heredity with reference to a pigmented character of any sort (gray, black, or spotted). Predictions are not made for the other color categories separately, because their relations to each other are not entirely clear from von Guaita's experiments. It seems probable, however, that they bear one toward another relations of alternative dominance. This

TABLE II.

Test of Mendel's Law by von Guaita's Statistics.

Abbreviations as in Table I.

Generation	Pair.	Total Young.	Mendel's Law, Calc. No. *W*.	Observed No. *W*.	Galton's Law, Calc. No. *W*.
II.		28	0	0	14
III.		44	11	14	11
IV.	(1)	4	4	4	2 5
	(2)	16	0	0	6
	(3)	2	0	0	0.25
	(4)	7	0	0	2.62
	(5)	2	1	1	0.75
	Total	31	5	5	12.12
V.	(1)	16	0	0	1
	(2)	5	1.2+	2	1
	(3)	32	8	7	6
	(4)	13	0	0	1.6
	(5)	44	11	9	5.5
	(6)	3	0.7+	1	0.4
	Total	113	21	19	15.5
VI.	(1)	32	8	10	2
	(2)	6	6	6	4
	(3)	43	21.5	20	14
	(4)	2	0 ?	0	0.2
	(5)	3	0 ?	0	0.2
	Total	86	35.5	36	20.4
VII.	(1)	4	0	0	1.4
	(2)	3	3	3	2.3
	Total	7	3	3	3.7
Total . . .		309	75 5	77	76.7

matter is now undergoing experimental tests which, when complete, may enable us to make predictions for these color categories not less precise than those given for white.

In the last column of the table are given for comparison the predictions based on Galton's law for the corresponding generations and pairs. If we were to consider the grand totals only, we might conclude that the Galtonian predictions are quite as good as the Mendelian, but if we examine item by item the two series from which these totals are made up, we see that there is no comparison in point of accuracy between the two sets of predictions. The Mendelian predictions are very close to the observed numbers throughout the table, generation by generation and pair by pair. In all cases except four the predictions are either perfect or within one of perfection, and in one only of these cases is the error greater than two. This one case is the total for generation III where the observed number is fourteen, the expectation eleven. Rarely do the Galtonian predictions come within one, or anywhere near one, of prefection. They demand the occurrence of white individuals in every generation and among the offspring of nearly every pair in the series, whereas white individuals are entirely wanting, and according to Mendel's law are not to be expected, among the offspring of *all* pairs in the second generation, and of eight other pairs in later generations of the series. The test is conclusive in favor of Mendel's law and against the "law of ancestral heredity," in the special case of albinism in mice. Elsewhere Castle and Allen (:03) have shown that among organisms in general albinism probably follows the same (Mendelian) law of inheritance.

Numerous other cases of Mendelian inheritance covering a wide range of characters are recorded in recent papers by de Vries (:01–03), Correns (:01, :03), Tschermak (:01, :01^a, :02), Bateson and Saunders (:02), Webber (:00), Spillman (:02, :02^a), Hurst (:02, :03), and others. These cases show that the Mendelian laws are widely applicable. They are not laws of hybridization merely, as Vernon (:03) and some others assume, but are general laws of alternative inheritance.

III. Yule on Galton's Law and Mendel's Law.

Bateson (:02) has taken the very reasonable position that Mendel's law and the law of " ancestral heredity " cannot both be applicable to the same classes of cases. But Yule (:02) sees no incompatibility between the two, and this view Pearson (:03) endorses. Yule says (p. 226), "Mendel's Laws, so far from being in any way inconsistent with the Law of Ancestral Heredity, lead then directly to a special case of that law, for the *dominant* attribute at least. For the *recessive* attribute it does not hold." Let us see how Yule reaches this curious conclusion, that certain Mendelian predictions are only a special category of the more general predictions of the law of ancestral heredity.

After a statement of the Galton-Pearson law, whereby it is limited to no particular series, geometrical or otherwise, but is made to include any set of empirical averages of the characters of the ancestors, which can be made the basis of predictions, he proceeds as follows: "The first question to be asked is one that does not seem to have occurred to any of Mendel's followers, viz.: what, exactly, happens if the two races *A* [dominants] and *a* [recessives] are left to themselves to inter-cross freely *as if they were one race?*" In answer to this question, Yule draws the correct conclusion that the first cross-bred (or hybrid) generation will consist exclusively of dominants, but that all subsequent generations will consist of dominant and recessive individuals in the proportions, 3 dominant: 1 recessive, [provided no selection is practised and all individuals are equally fertile]. Yule next inquires, if I understand him rightly, what will be the effect of eliminating in each generation *all the recessive individuals.* Starting with 300 dominant individuals, which are in the Mendelian proportions, 100 pure: 200 hybrid, he finds that the successive generations will contain the following proportions of dominant individuals: —

.83333
.85000
.85294
.85345
.85354

He considers it useless to carry the series farther, as it "tends toward the limiting value .85355339 . . ." Now, what, in plain unmathematical language, does this mean? It means that when a dominant form has once been crossed with a recessive (as a pigmented animal, for example, with an albino), the stock of the former is forever contaminated, and cannot be freed entirely from the albino character by mere elimination of white individuals, however long the process is continued. Ever afterward the cross-bred dominant stock will produce on the average at least fourteen or fifteen white individuals in every hundred born. This conclusion is absurd, as every breeder knows. There is certainly something wrong with Yule's figures, for they do not accord with observation. In reality, an error lies in the very first step of his calculation, which invalidates all that follows. He says, "The 100 pure individuals will give rise to dominant forms in the proportion of 50 pure to 50 hybrids." On the contrary, "pure" dominants bred *inter se* will produce only pure dominant offspring; but if they mate at random with any individuals of the entire 300, there are only two chances out of three that they will mate with *hybrid* dominants, which mating alone could yield "dominant forms in the proportion 50 pure to 50 hybrids." Yule accordingly estimates too low the proportion of dominant individuals in the various generations.

IV. Race Improvement by Selection of Desirable or by Elimination of Undesirable Individuals.

On the hypotheses, which I understand Yule to adopt, of random mating and equal fertility on the part of all individuals, 300 dominant forms, of which 100 are pure and 200 hybrid, will produce more than 88 per cent of dominant individuals, instead of 83 per cent as estimated by Yule. For if we suppose each class to consist of males and females in equal numbers, the chances are just twice as great that an individual will mate with a hybrid dominant, *A* (*B*), as that it will mate with a pure dominant, *A*. Or, to put the matter in another way, there are, for each individual of the entire 300, 50 possible *A* mates, and 100 possible *A* (*B*) mates. This makes the entire number of different matings possible: —

	A.	*A* (*B*).	*B*.
5,000 *A* × *A*, yielding offspring	5,000		
20,000 *A* × *A* (*B*), yielding offspring . . .	10,000 +	10,000	
20,000 *A* (*B*) × *A* (*B*), yielding offspring .	5,000 +	10,000 +	5,000
Total	20,000 +	20,000 +	5,000

or 4 *A* : 4 *A* (*B*) : 1 *B*. It will be observed that $\frac{8}{9}$, or 88.8 per cent, of the offspring have the dominant form, being either *A* or *A* (*B*) in character. Eliminating the one recessive individual, *B*, in each nine offspring, the parents of the next generation will consist of 4 *A*s (pure dominants) and 4 *A* (*B*)s (hybrid dominants); that is, of equal numbers of individuals *A* and *A* (*B*). The possible matings * in this case will be: —

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* To simplify the calculation, it is well to remember that the numerical proportions of the various matings possible within a population are expressed by the square of that population. Knowing the nature and numerical proportions of the possible matings, one can quickly calculate the numerical proportions of the offspring. Thus, in a population consisting of equal numbers of individuals *A* and *A* (*B*), the possible matings are expressed by the square of *A* + *A* (*B*), or

	A.		$A(B)$.		B.
2 $A \times A$, yielding offspring	2				
4 $A \times A(B)$, yielding offspring	2	+	2		
2 $A(B) \times A(B)$, yielding offspring	$\frac{1}{2}$	+	1	+	$\frac{1}{2}$
Total	$4\frac{1}{2}$	+	3	+	$\frac{1}{2}$

or 9 A : 6 $A(B)$: 1 B. The offspring in this generation are $\frac{15}{16}$, or 93.7 per cent, of the dominant form. Calculating in a similar way for the next four generations, we find that the proportion of dominant individuals steadily increases. The complete series for generations 1–8 following the cross between a pure A and a pure B is shown in Table III.

TABLE III.

Results of Selection for the Dominant Character A in the various Generations following a Cross between a Pure A and a Pure B.

Generation.	Parents.	Offspring.	Per cent A or $A(B)$.
1	$A + B$	$A(B)$	100
2	$A(B)$	$A + 2A(B) + B$	75
3	$A + 2A(B)$	$4A + 4A(B) + B$	88.8
4	$A + A(B)$	$9A + 6A(B) + B$	93.7
5	$3A + 2A(B)$	$16A + 8A(B) + B$	96
6	$2A + A(B)$	$25A + 10A(B) + B$	97.2
7	$5A + 2A(B)$	$36A + 12A(B) + B$	98
8	$3A + A(B)$	$49A + 14A(B) + B$	98.4

Inspection of the table will allow one to continue it to any desired extent.* Compare Diagram on p. 239, D.

$A^2 + 2A \cdot A(B) + \overline{A(B)}^2$. Treating the progeny of each mating as equal to four, we have

	A.		$A(B)$.		B.
1 mating $A \times A$, yielding offspring	4				
2 matings $A \times A(B)$, yielding offspring . . .	4	+	4		
1 mating $A(B) \times A(B)$, yielding offspring .	1	+	2	+	1
Total offspring	$9A$	+	$6A(B)$	+	B.

[*] The percentage of dominant forms in the various generations may be quickly calculated by observing that it equals the series

$$\frac{1}{1} + \frac{(2)^2 - 1}{(2)^2} + \frac{(3)^2 - 1}{(3)^2} + \frac{(4)^2 - 1}{(4)^2} + \frac{(5)^2 - 1}{(5)^2} \text{ etc.}$$

From the foregoing considerations, we see that it is entirely possible for a breeder, under the conditions stated, practically to eliminate an undesirable recessive character, in a very few generations, *merely by not breeding from individuals which manifest that character.* This accords with experience. There is, however, a much quicker and surer way of accomplishing the desired result, namely, by selection of *pure* dominants only for breeding purposes. If a dominant individual, when bred to a recessive mate, has produced among two or more offspring no recessive individual, it is probable that the dominant is *pure*, and if mated to a similar individual will produce no recessive offspring in subsequent generations.

By means of a few preliminary breeding tests of individual animals or plants the breeder is thus enabled to establish a race of pure dominants as early as the second generation following a cross with recessives. A race of recessives which will breed true, may of course be established at any time by mating two recessive individuals. If the Galton-Pearson law were correct, neither of these things would be possible.

Suppose that the breeder, as is often the case, does not care to take the trouble to establish a perfectly pure race, being anxious to market large numbers of individuals as soon as possible. By merely weeding out the undesirable recessive individuals his race will steadily improve, as indicated by Table III and the diagram on p. 239, *D.* In the second generation following a cross between pure dominant and pure recessive individuals it will, as we have seen, consist of 75 per cent dominant individuals; in the next generation it will consist of 88.8 per cent dominants, and so on.

If the breeder eliminates recessives but once, namely, in the second generation following the cross, the series will be as follows: —

1st generation	100 %	dominants
2d generation	75	“
3d generation	88.8	“
4th generation	88.8	“

etc., *ad infinitum.* If he eliminates recessives twice only, namely, in the second and third generations, the race will thereafter continue to contain 93.7 per cent dominant individuals, as follows: —

1st generation	100 %	dominants
2d generation	75	“
3d generation	88.8	“
4th generation	93.7	“
5th generation	93.7	“

etc., *ad infinitum.* Similarly, if recessives are eliminated three times only, the race will be stable at 96 per cent dominants; and if four times, at 97.1 per cent dominants. In general, *as soon as selection is arrested the race remains stable at the degree of purity then attained,* provided of course that one form is as fertile as the other, and subject to no greater mortality.

Such is the law governing the transmission of a dimorphic condition within a race, or, to give the matter a practical bearing, we may call it the *law governing race improvement,* in cases of alternative inheritance, in which one of a pair of characters is uniformly dominant over the other. In cases in which dominance alternates between the two characters A and B (and such cases are probably commoner than is generally suspected) the process of race improvement by elimination of undesirable individuals progresses at first somewhat more slowly, but ultimately even more rapidly than in the case already discussed. A cross between A and B will, when dominance is alternative, yield offspring 50 per cent $A(B)$, 50 per cent $B(A)$. Selecting for A, that is, breeding only from $A(B)$s, the next generation will consist of equal numbers of forms

TABLE IV.

Results of Selection for the Character A in the various Generations following a Cross between a Pure A and a Pure B.

Generation.	When Dominance is Alternative between A and B.			When A is uniformly Dominant over B.
	Parents.	Offspring.	Per cent A or $A(B)$	Per cent A or $A(B)$.
1	$A + B$	$A(B) + B(A)$	50	100
2	$A(B)$	$A + A(B) + B(A) + B$	50	75
3	$A + A(B)$	$9A + 3A(B) + 3B(A) + B$	75	88.8
4	$3A + A(B)$	$49A + 7A(B) + 7B(A) + B$	87.5	93.7
5	$7A + A(B)$	$(15)^2 A + 15A(B) + 15B(A) + B$	93.7	96
6	$15A + A(B)$	$(31)^2 A + 31A(B) + 31B(A) + B$	96.8	97.2
7	$31A + A(B)$	$(63)^2 A + 63A(B) + 63B(A) + B$	98.4	98
8	$63A + A(B)$	$(127)^2 A + 127A(B) + 127B(A) + B$	99.2	98.4

A, $A(B)$, $B(A)$, and B respectively, or once more 50 per cent individuals A in appearance. See Table IV. Selecting again for A, the parents for generation 3 will consist of equal numbers of individuals A and $A(B)$ in character. Continuing the calculation in this way, we get the series of generations indicated in Table IV, and expressed graphically in the diagram on p. 239, A. For convenience in comparison, there are also given in the last column of Table IV the percentages of A and $A(B)$ individuals to be expected when A is uniformly dominant over B. Compare Table III. Inspection of Table IV will allow one to continue it to any desired extent.

TABLE V.

Chances in 100 of isolating a Pure A by Random Selection from Individuals manifesting that Character in the various Generations following a Cross between a Pure A and a Pure B.

Generation.	When Dominance is Alternative between A and B.[*]	When A is uniformly Dominant over B.[†]
1	0	0
2	50	33.3
3	75	50
4	87.5	60
5	93.7	66.6
6	96.8	71.4
7	98.4	75
8	99.2	77.7

The same law governs arrest of selection in cases of alternative dominance, as in cases of uniform dominance of A over B. As soon as selection by elimination ceases, the race continues in the condition at that time attained, provided forms A and B are equally fertile and subject to the same mortality.

On the other hand, if the breeder has the patience to make individual breeding tests, and then to select for *pure* individuals on the basis

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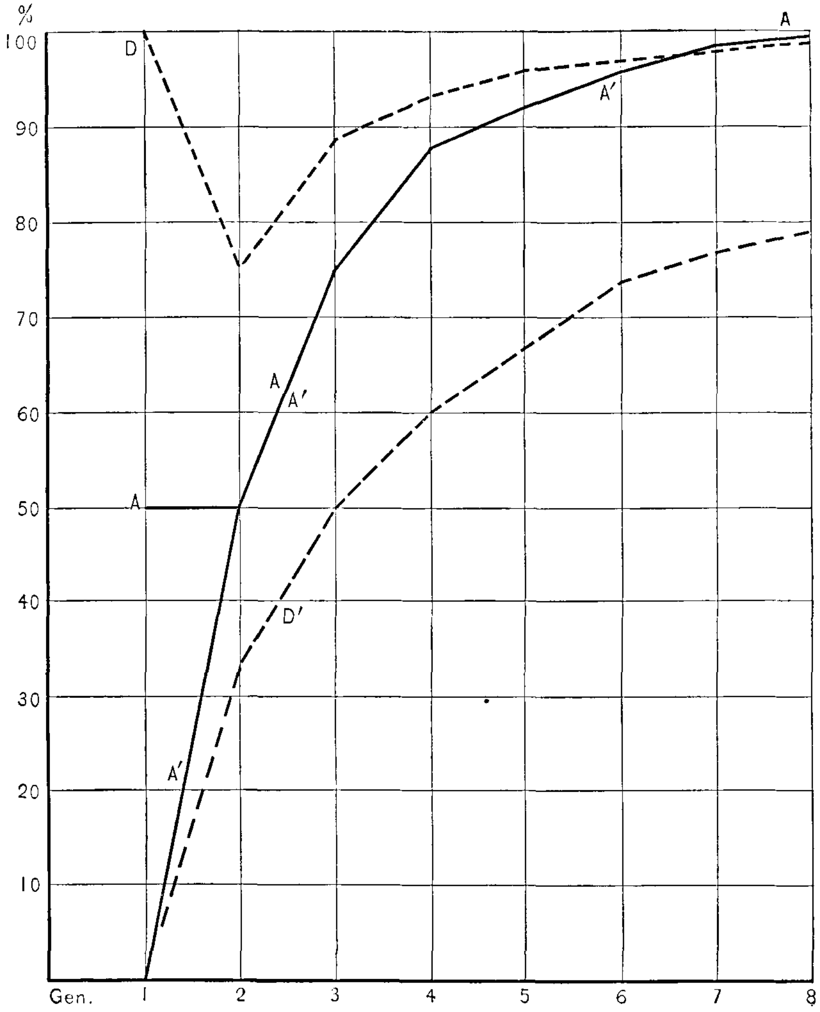
[*] These percentages equal the series 0. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{15}{16}$, $\frac{31}{32}$, etc.

[†] These percentages equal the series 0, $\frac{1}{3}$, $\frac{2}{4}$, $\frac{3}{5}$, $\frac{4}{6}$, $\frac{5}{7}$, etc.

Diagram showing the Progress of Selection in Cases of Alternative Inheritance.



A, Rate of Race Improvement by Elimination, Dominance being Alternative.
D, Same, Dominance being Uniform.
A′, Chances in 100 of obtaining a Pure Individual, Dominance being Alternative.
D′, Same, Dominance being Uniform.

of the tests made, immediate success in obtaining a pure race is assured, whether dominance be alternative or not. But if he selects for pure individuals quite at random, without breeding tests, his chances of success are considerably greater in a case of alternative dominance than in a case of uniform dominance of one character over the other, as will be clear from an examination of Table V. In the second generation following a cross between a pure *A* and a pure *B*, dominance being alternative, the chances are even that any *A* individual selected at random will breed true; and when individuals possessing the character *B* have been eliminated for three successive generations, the chances become approximately 94 in 100 that any *A* individual selected at random will breed true; whereas, when *A* uniformly dominates over *B*, the chances, in the corresponding generations, of securing a pure *A* are only 1 in 3, and 2 in 3 respectively.

A graphic presentation of these facts is made in the diagram on p. 239, *A'*, *D'*.

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY
OF HARVARD COLLEGE.

A REVISION OF THE ATOMIC WEIGHT OF IRON.

SECOND PAPER. — THE ANALYSIS OF FERROUS BROMIDE.

By Gregory Paul Baxter.

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A REVISION OF THE ATOMIC WEIGHT OF IRON.

SECOND PAPER.—THE ANALYSIS OF FERROUS BROMIDE.

BY GREGORY PAUL BAXTER.

Presented by T. W. Richards, October 14, 1903. Received October 3, 1903.

FOUR years ago a determination of the atomic weight of iron, made in this laboratory by reduction of the oxide in hydrogen, led to the value 55.883 ($O = 16.000$).* Since the atomic weight in use at that time, 56.02, differs so materially from the above value, it seemed advisable to redetermine the constant in question by a radically different method.

Bromides may be analyzed with the greatest ease and exactness, provided they can be obtained in a state of purity. Since ferric bromide is far too unstable at high temperatures to permit thorough drying of the salt, ferrous bromide was chosen as the substance for analysis. This salt rapidly oxidizes in the presence of moist air; but the ease with which ferric bromide is decomposed by heat into ferrous bromide and bromine is an advantage in the preparation of ferrous bromide, for it proved possible completely to free the latter salt from ferric impurity by subjecting it to a high temperature in an atmosphere of dry hydrobromic acid gas.

Ferrous bromide was prepared by heating metallic iron in a current of dry nitrogen and hydrobromic acid gases, at a temperature sufficiently high to sublime the resulting salt. When the salt was removed from the sublimation tube, it necessarily came in contact with moist air, and thus become covered with a thin coating of ferric salt. It was then subjected to prolonged heating in a current of dry nitrogen and hydrobromic acid gases. In order to determine whether the ferric salt was thus decomposed, samples of the product were dissolved in a freshly boiled, acid solution of ammonic sulphocyanate. If the hydrobromic acid and ferrous bromide contain no chlorine, the salt obtained in the above manner gives a barely perceptible coloration with the sulphocyanate, showing that reduction is complete. On the other hand, if the ferrous bromide was prepared from commercial bromine, it was never found possible to eliminate completely the ferric salt; although by colorimetric comparison with standard ferric solutions to which ammonic sulphocyanate had been added, it was shown that the proportion of iron in the ferric state did not amount on an average to more than two hundredths of one per cent. The greater difficulty with which ferric chloride is reduced is doubtless the cause of this difference in behavior of the two varieties of material.

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* Richards and Baxter, These Proceedings, **35**, 253.

The apparatus with which these operations were carried out was essentially that used for similar purposes in the determinations of the atomic weights of cobalt,* nickel,† and uranium ‡ in this laboratory. A mixture of air and ammonia was passed over heated rolls of copper gauze and the excess of ammonia was removed by means of sulphuric acid. The partially dried gas, after bubbling through bromine in a small flask, was conducted through hydrobromic acid in which was suspended red phosphorus to convert the bromine into hydrobromic acid. Suitable desiccating agents then dried the mixture of gases. Pure iron, contained in an unglazed porcelain boat, was placed in a porcelain tube heated by a Fletcher furnace. The mixed nitrogen and hydrobromic acid gases were passed over the hot iron, forming ferrous bromide, and since the temperature was high enough to sublime the ferrous bromide, the salt was carried along by the current of gases to be deposited in a smaller cool porcelain tube which telescoped into the larger tube. As soon as the inner end of the smaller tube was stopped up with bromide, the furnace was allowed to cool for some time, and the sublimed salt while still warm was transferred to a weighing bottle. The portions next the porcelain were never collected for analysis. The ferrous bromide, contained in a platinum boat, was then heated to about 400° C. in a current of nitrogen and hydrobromic acid gases for an hour or more. Finally, after the boat had become cool, these gases were displaced, first by nitrogen, then by dry air, and the boat was transferred to a weighing bottle without exposure to moisture, by means of the bottling apparatus so frequently used for the purpose in this laboratory. §

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* Richards and Baxter, These Proceedings, **33**, 117.

† Richards and Cushman, These Proceedings, **33**, 99.

‡ Richards and Merigold, These Proceedings, **37**, 378.

§ Richards and Parker, These Proceedings, **32**, 59.

As has been stated before, ferrous bromide prepared in this way does not contain a weighable amount of ferric salt, and is a perfectly stable substance in dry air. The pure salt varies in color from light yellow to dark brown, according to the thickness of the crystals.

In several cases the platinum boat which contained the salt during the drying became covered with a black film, which disappeared when the boat was ignited, without producing any change in weight. Since a similar difficulty had never been met before when a platinum boat was heated in hydrobromic acid, the phenomenon was at first ascribed to the presence of either ferrous or ferric salt; but the eventual conversion of the platinum into a brittle condition seemed to indicate some other cause of the difficulty. The presence of phosphorus compounds in the mixed gases was suspected, and when tests were made with ammonic molybdate, both in the salt and in hydrobromic acid which had been generated in the apparatus and collected in water, traces of phosphoric acid were found in both cases. The source of the phosphoric acid was easily discovered.

Whenever bromine is passed for a sufficiently long time into a solution of hydrobromic acid containing red phosphorus, a yellow crystalline deposit is formed upon the walls of the flask above the solution. This yellow substance resembles the pentabromide of phosphorus in appearance and gives off much hydrobromic acid when treated with water, hence it must have been the pentabromide. Since at ordinary temperatures the pentabromide is not volatile while the tribromide is volatile, probably the formation of pentabromide is due to volatilization of the tribromide from the solution and consequent conversion into pentabromide by bromine which escapes reduction in bubbling through the flask. Obviously the formation of either substance in appreciable quantity is impossible so long as the solution in the flask is very aqueous. The aqueous tension of the solution is never high, however, since the concentration of the hydrobromic acid is at a maximum, and the aqueous tension is diminished still farther as the concentration of phosphorous and phosphoric acids in the solution increases. Eventually, then, the aqueous tension of the solution should be such as to make possible the existence of the bromides of phosphorus. The attacking of the boat was evidently due to the presence of compounds of phosphorus in the mixed gases.

In order to remove the traces of bromine which escaped reduction in the first flask, containing hydrobromic acid and phosphorus, the gases, after passing through this flask, were always conducted through a short U-tube containing these same substances. The concentration of acids of phosphorus never became high in this U-tube, but it was evident that a single such tube was not enough to remove the bromides of phosphorus completely. Consequently a second U-tube, filled with beads moistened with hydrobromic acid, was inserted, and pains was taken that no very considerable concentration of acids of phosphorus should ever exist in the hydrobromic acid generating flask. After this modification of the apparatus had been made, the boat remained absolutely unchanged in appearance and weight.

These phenomena became very evident in the present work because such large quantities of hydrobromic acid were necessary for the synthesis and sublimation of ferrous bromide. Also, since the bromide was condensed from an atmosphere of the gas, the conditions were those capable of causing the retention of a maximum amount of phosphorus. In the cases of the cobaltous and nickelous bromides, on the other hand, synthesis was effected by means of *bromine* vapor, very little hydrobromic acid being used in their preparation. It was only during the drying of the salts that they came in contact with hydrobromic acid for any considerable length of time, and even then the contamination could have been only superficial. As a matter of fact, it was found impossible at the time to detect the slightest trace of phosphorus in either of the two bromides.

Considerable difficulty was experienced in obtaining ferrous bromide which would give a clear solution in water. The earlier samples contained, besides silica from the new porcelain tubes, a slight black residue which became red upon ignition, and which may have been the magnetic oxide of iron. The amount of this residue was very much diminished by a lavish expenditure of hydrobromic acid during sublimation. These residues were filtered upon tiny filter papers, ignited, and weighed. In none of the samples used for analysis did the weight of the insoluble residue amount to more than one thirtieth of a per cent of the weight of the salt. Even in the best material the insoluble residues contained traces of phosphoric acid. In Analysis 2 (p. 252) the phosphoric acid in the residue was determined by precipitation with ammonic molybdate to be 0.0001 gram. It was easily shown, however, that all the phosphoric acid was in the insoluble residue, for in Analysis 4 the filtrate from the silver bromide was evaporated to small bulk and treated with ammonic molybdate, without the formation of a visible amount of precipitate. Evidently no error was introduced into the analysis by the presence of the phosphoric acid.

The determination of the bromine in the salt offered certain difficulties owing to the fact that ferrous salts precipitate silver from its solutions under certain conditions. Obviously it was necessary to oxidize the ferrous bromide solution before addition of the silver nitrate. Hydrogen peroxide proved too slow an oxidizing agent at ordinary temperatures, potassium permanganate is too vigorous in its action, and sets free bromine from the salt. Even potassium dichromate, at concentrations such as are used in ordinary analytical work, sets free bromine rapidly. But if a very dilute solution of dichromate is used, and if the sulphuric acid is added gradually with the dichromate and only in quantities theoretically necessary, so as to keep the velocity of reaction as low as possible, and finally if the solution of dichromate and sulphuric acid is introduced beneath the surface of the solution of ferrous bromide, so that any bromine set free may be taken up by the ferrous salt before it can escape from the solution, oxidation can be accomplished without the loss of enough bromine to be detected with starch iodide paper. Furthermore complete oxidation is unnecessary, for although dilute solutions of ferrous and silver salts deposit silver upon standing, if considerable ferric salt is present no precipitation of silver takes place owing to the retarding influence of the ferric salt.

PURIFICATION OF MATERIALS.

Pure ferric oxide was prepared in an exactly similar fashion to that used in the previous investigation upon the oxide.* In order to convert this oxide into the metal, one portion was heated in a current of ammonia. But since metal reduced by ordinary ammonia, when dissolved in sulphuric acid, leaves a residue of carbon owing to the presence of amines in the ammonia, the material used in all analyses except Analysis 1 was reduced in pure electrolytic hydrogen.†

Silver was purified in the manner usually employed in this laboratory, except that the electrolytic crystals were finally fused in a current of pure hydrogen instead of in a vacuum. Even spongy silver occludes no appreciable amount of hydrogen when heated in this gas, ‡ hence no error could have resulted from this source.

Bromine was partially freed from chlorine by solution in concentrated aqueous calcic bromide made from a portion of the same sample of bromine. It was then converted into hydrobromic acid by means of washed red phosphorus and water, and the resulting hydrobromic acid, which contained a considerable excess of bromine, was freed from iodine by distilling off this bromine. After several distillations the hydrobromic acid was converted into bromine by treatment with recrystallized potassium permanganate. The resulting bromine was thus distilled a second time from a solution of a bromide of much greater purity than in the first case. Analysis of this bromine was carried out by precipitating a known weight of silver with a slight excess of ammonic bromide made from this bromine.

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* Richards and Baxter, These Proceedings, **35**, 256.

† These Proceedings, **34**, 357.

‡ Baxter, Amer. Chem. Jour., **22**, 362.

Weight of Silver in Vacuum.	Weight of fused Silver Bromide in Vacuum.	Ratio Ag : AgBr.
grams.	grams.	
4.77783	8.31754	57.4428
5.87977	10.23533	57.4459
4.82995	8.40809	57.4441
	Average	57.4443

This value is almost identical with the average of Stas's experiments, 57.4445, and shows conclusively that both bromine and silver were pure.

Method of Analysis.

The method of analysis was as follows. The salt was dissolved in water which had been slightly acidified with sulphuric acid to prevent the formation of insoluble basic ferric salt, and the trace of insoluble residue was collected upon a small filter paper and determined as previously described. The solution of ferrous bromide, which had been diluted to a volume of at least four hundred cubic centimeters by the wash waters, was then oxidized by adding slowly, through a funnel tube with a fine tip which dipped below the surface of the solution, slightly less than the calculated amount of a solution of potassic dichromate. This solution contained one and one-half grams of potassic dichromate in a liter of solution, together with a slight excess of the quantity of sulphuric acid necessary to complete the reaction. The dichromate had been recrystallized from the purest water, and the sulphuric acid had been redistilled, so that no halogens could have been contained in the oxidizing solution. In Analyses 1 and 2 an excess of the calculated quantity of pure silver was dissolved in pure nitric acid and the solution, after dilution, was added slowly to the bromide. The resulting silver bromide was filtered upon a Gooch crucible, heated to 200° in an electric oven, and weighed. The asbestos shreds which passed through the crucible, together with any accompanying silver bromide, were collected upon a small filter. This filter was then ignited, the ash treated with a drop of a mixture of pure nitric and hydrobromic acids, and finally the weight of the residue determined. After the weight of the silver bromide had been found, it was fused in a porcelain crucible and the loss in weight determined. In Analyses 5 and 6 exactly the calculated amount of silver was used in the precipitation, and portions of the clear solution above the precipitate were tested with solutions of hydrobromic acid and silver nitrate in a nephelometer kindly loaned for the purpose by Mr. R. C. Wells. In both analyses a very slight opalescence was produced to the same extent in both portions, showing that the exact amount of silver necessary to combine with the bromine had been added. The absence of any appreciable amount of precipitate in either portion is evidence that no chlorine was present in the salt; for silver chloride, owing to its greater solubility, would have made itself evident by a considerable cloudiness in the nephelometer test. In these two analyses an excess of silver nitrate was then added and the weight of the silver bromide was determined (Analyses 3 and 4).

In order that the weight of the ferrous bromide might be reduced to a vacuum standard, its specific gravity was found by determining the weight of kerosene displaced by a known quantity of salt. The kerosene was first dried by means of stick soda, and then distilled. The portion distilling between 200° and 240° was used in the determinations. Its specific gravity at 25° referred to water at 4° was found to be 0.7693.

Weight of Ferrous Bromide in Vacuum.	Weight of Kerosene in Vacuum.	Specific Gravity of Ferrous Bromide 25°/4°.
grams.	grams.	
2.9660	0.4918	4.639
2.5712	0.4260	4.643
3.3619	0.5592	4.625
Average		4.636

Accordingly, to every apparent gram of ferrous bromide a vacuum correction of 0.000118 gram was added. The vacuum correction of silver bromide was assumed to be +0.000046, and that of silver to be —0.000031. The atomic weights used in the calculations are: Oxygen, 16.000; Bromine, 79.955; and Silver, 107.93.

The weights, which were of brass, platinum plated, were carefully standardized to hundredths of a milligram.

Number of Analysis.	Weight of Ferrous Bromide in Vacuum.	Weight of Residue.	Weight of Silver Bromide in Vacuum.	Loss on Fusion.	Weight of Asbestos.	Weight of Silver in Vacuum.	Atomic Weight of Iron.
	grams.	grams.	grams.	grams.	grams.	grams.	
1	3.55996	0.00067	6.19852		0.00021		55.856
2	3.07566	0.00118	5.35453	0.00030	0.00027		55.852
3	2.96128	0.00026	5.15706	0.00020	0.00010		55.849
4	4.00816	0.00025	6.97966	0.00013	0.00030		55.862
5	2.96128	0.00026				2.96234	55.854
6	4.00816	0.00025				4.00937	55.871
	Average						55.857

Both ferric and chromic salts show a tendency to be occluded by precipitates, and it was feared that difficulty might arise from this source. The close agreement of the results obtained by weighing the silver bromide with those calculated from the weight of the silver shows that no error of the sort actually existed. The ratios of silver to silver bromide in the two analyses in which both substances were determined are 57.444 and 57.442.

One point remains to be considered, — the presence of alkaline bromides in the salt. All bromides made by sublimation in porcelain tubes have been shown to contain sodium bromide in appreciable quantities. An attempt was made to determine the amount of this impurity by reducing the ferrous bromide in hydrogen and subsequently leaching out the soluble salts remaining. This method was used in the cases of cobalt and nickel, and afforded at the same time a means of determining the per cent of metal in the salt. Unfortunately the method was not practicable in this case, for although ferrous bromide is slowly reduced in moist hydrogen, it is a very difficult matter to avoid considerable sublimation of the salt during the process, and complete reduction could be obtained only at a temperature at which all the sodic bromide sublimes out of the boat. Recourse was taken to the method of precipitating the iron from a solution of the salt with ammonia, and determining the sodium in the filtrate. The solution of sodium and ammonium bromides was evaporated to dryness, and after the ammonium salts had been volatilized by gentle ignition, the residue was taken up with water and again evaporated with a slight excess of sulphuric acid. Finally the residue was ignited in a stream of ammonia. These operations were carried out entirely in platinum vessels, and the water used was condensed in a block tin condenser and collected in platinum. The residues obtained in this way gave no test for any metal except sodium when examined spectroscopically.

Weight of Ferrous Bromide.	Weight of Sodic Sulphate.	Weight of Sodic Bromide Calculated.	Per cent of Sodic Bromide.
grams.	grams.	grams.	
4.553	0.0050	0.0073	0.160
6.311	0.0051	0.0074	0.117
Average			0.138

Although the porcelain tubes were exactly similar to those used in the sublimation of cobalt, nickel, and uranium bromides, the amount of the alkaline impurity is slightly larger than that found in those cases. Presumably this difference is due to the unglazed porcelain boats, which it was hoped would be more resistant to the action of hydrobromic acid gas, but which were attacked very badly during the sublimation. The attacking of the boat may have been due to the fact that the ferrous bromide fused in the boat while the other bromides did not fuse to a noticeable extent.

Since in the cases of cobalt and nickel, where the question of impurity introduced from the porcelain tubes was sifted to the bottom, sodium was the only metal found, it is reasonable to conclude that here also sodium was the only impurity. Sodium bromide contains a higher percentage of bromine than ferrous bromide, hence the weights of silver and silver bromide determined by analysis of the sublimed salt are slightly too large. 1.00000 gram of salt yielded 1.74155 grams of silver bromide. Of this silver bromide, 0.00252 gram correspond to 0.00138 gram of sodium bromide contained in the salt. If both these corrections are subtracted, the atomic weight of iron calculated from the quantities remaining is 55.871.

This value is slightly lower than the result of the analysis of ferric oxide, 55.883. But this was to be expected, for it was pointed out at the time the earlier results were published, that the value obtained was rather too high than too low, since the errors which would affect determinations made by reduction of the oxide, i. e., those due to the presence of non-reducible impurities in the material and to incomplete reduction, would have raised the atomic weight above its true value. As a matter of fact, it was found impossible to prove that either of these errors existed. *Occluded gases, which would have produced the opposite effect,* were conclusively proved to be absent. It is interesting to note that Analysis 6 of the earlier series, in which most material was used and which consequently should be the most accurate, gave a result, 55,870, practically identical with that of this research. On the other hand, the presence in the ferrous bromide of a trace of ferric salt would have lowered the atomic weight. The average of the two series, 55,877, must represent very closely the true atomic weight in question.

The value 55.88 for the atomic weight of iron thus receives fresh support. It is hardly conceivable that sufficient undiscovered impurity could have been present in the material used for either series of analyses to have affected the second figure of decimals. Nevertheless, further work on the subject is planned with the hope of increasing the purity of the material used for analysis.

Attention has been recently called to the possibility that the atomic weights of the magnetic metals might be affected by the attraction of the earth's magnetism. Obviously the error introduced by this weak attraction could not be large, but nevertheless the ascertaining of the order of inaccuracy is a point of some interest to the chemist.

The intensities of the earth's magnetic field in various places are well known, but the calculation of the attractive action of this field upon a given magnet depends, of course, upon the intensity and length of the magnet as well as upon the field. If the magnet is so short that its two poles are essentially the same distance from the earth's magnetic north pole, it is clear that the repelling effect of the earth upon one pole will practically counterbalance the attracting effect upon the other pole; hence the effect of the earth's magnetism will be simply directive, and will have no attractive influence. Moreover, since the pure metals used in the atomic weight investigations upon iron, cobalt, and nickel in this laboratory were incapable of retaining magnetism, the earth's field will not have even a directive influence on these substances, but will only affect the polar direction of the weak induced magnetism.

If any further argument were needed as to the infinitesimal nature of the effect of the earth's magnetism upon the atomic weights in question, it might be found in the analytical results. Compounds are well known to be usually far less magnetic than the metals which they contain *; hence the magnetic attraction would increase the weight of the metal to a greater extent than that of the compound. But the complete analysis of the cobaltous and nickelous bromides accounted for all the material taken, without any considerable surplus, even although the metal was weighed in the elementary state. Hence, one concludes that the magnetic attraction must have been unimportant. Still, in order to prove the point directly, the following experiments were carried out.

The method of determination consisted in weighing iron in a normal fashion in the earth's magnetic field, and then weighing the iron again in a non-magnetic field, formed by neutralizing the earth's magnetism by means of a temporary magnet.

The vertical component of the earth's magnetism was, of course, the one which was to be eliminated; but if the magnet is placed in the line of the dipping needle, an intensity which would neutralize one component would also neutralize the other. Hence, the position of neutrality was determined by a horizontal needle or compass.

Upon the pan of a very sensitive balance was placed a small sensitive magnetic needle. The needle was then brought into an astatic condition by bringing near it one pole of a straight electro-magnet, thirty centimeters long, which was placed in a position as nearly as possible parallel with the lines of the earth's magnetic force. This electro-magnet consisted of a core of soft iron wire covered by six layers of stout copper wire. The magnetic attraction of the earth and of the electro-magnet neutralized each other when the distance of the electro-magnet from the needle was eighteen centimeters, and the current passing through it was 0.15 ampere.

The magnet was now removed and the compass replaced by a weighing bottle containing 6.7 grams of pure iron (reduced from the oxide by ignition in a current of hydrogen), which was then carefully counter-

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* For example, see Plücker, Pogg. Ann., 1848, 1851.

balanced with weights. Upon placing the electro-magnet in its former position and passing the same current through it as before, no perceptible difference in the weight of the iron could be observed. Two repetitions of the experiment gave the same result. Evidently the weight of iron is not influenced by the earth's magnetism to an extent which would necessitate the consideration of the permanent magnetic field even in the most accurate analytical work.

About ten grams of pure ferric oxide were now counterbalanced and the electro-magnet placed in a horizontal position so that one pole was directly below the weighing bottle and five centimeters distant from it. A much stronger current (0.4 ampere) was passed through the magnet, but no perceptible change in the weight of the oxide could be observed. The same result was obtained when the oxide was replaced by about the same quantity of bromide. When the metallic iron was placed upon the balance pan, an increase in weight of 0.0035 gram was produced by the magnet under the above conditions. The empty balance was not affected in the least by the magnet. It is easy to calculate from the figures given above that the vertical pull of the magnet upon the iron in the new position must be over a thousand times as great as it was in the more distant position. Hence it is not surprising that the magnet produced no change in the first experiment, for a difference in weight of 0.000003 gram could not have been detected.

To sum up the results of the investigation: —

1. The result for the atomic weight of iron previously obtained by analysis of the oxide, 55.88 ($O = 16.000$), was confirmed.

2. The specific gravity of ferrous bromide was found to be 4.636, at 25° referred to water at 4°.

3. The effect of the earth's magnetism on the weights of small quantities of the magnetic metals was shown to be negligible in even the most precise atomic weight work, as was to have been expected.

I wish to express my gratitude to Professor T. W. Richards for many friendly suggestions in the course of the work, especially in connection with the effect of terrestrial magnetism upon the weight of the magnetic metals. I am indebted to the Cyrus M. Warren Fund for Research in Harvard University for much of the necessary apparatus, and to Dr. Wolcott Gibbs for indispensable platinum vessels.

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CONTRIBUTIONS FROM THE ZOÖLOGICAL LABORATORY OF THE MUSEUM OF COMPARATIVE ZOÖLOGY AT HARVARD COLLEGE. E. L. MARK, DIRECTOR. — No. 147.

THE COLOR CHANGES IN THE SKIN OF THE SO-CALLED FLORIDA CHAMELEON, ANOLIS CAROLINENSIS CUV.

BY FRANK C. CARLTON.

WITH A PLATE.

CONTRIBUTIONS FROM THE ZOÖLOGICAL LABORATORY OF THE MUSEUM OF COMPARATIVE ZOÖLOGY AT HARVARD COLLEGE. E. L. MARK, DIRECTOR. — No. 147.

THE COLOR CHANGES IN THE SKIN OF THE SO-CALLED FLORIDA CHAMELEON, ANOLIS CAROLINENSIS Cuv.

By Frank C. Carlton.

Presented by E. L. Mark, October 14, 1903. Received September 30, 1903.

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I. Introduction.

Although from the earliest times the color changes of the African chameleon have attracted the attention of naturalists, a wholly satisfactory explanation of these changes has not yet been worked out. This is probably due to the fact that the color changes in this lizard are very likely the most complex of any in the reptiles, and that in consequence their elucidation is proportionally more difficult. As Thilenius (**'97**) has intimated, it would seem wisest to study first some of the simpler examples of color change, and thereby obtain a clue to the explanation of those in the more complex forms, like the chameleon, rather than to take up the most complex first. In pursuance of this idea I have undertaken the study of the color changes in the skin of the so-called Florida chameleon, Anolis carolinensis Cuv. This lizard belongs to the family Iguanidae Cope (:00), and thus is not a true chameleon, although, as its popular name indicates, it possesses remarkable properties in the matter of the color changes of its skin.

A large number of living specimens of Anolis were obtained during the winter of 1902–03 from a dealer in Jacksonville, Florida. They lived well in confinement and exhibited characteristic color changes. As Lockwood ('76, p. 12) has already observed, they are as a rule dark brown during the day and pea-green at night. These extremes, with a series of transitional tints running through shades of brown and yellow to green, were the chief colors regularly noticed. Whether these changes, which were observed during the winter in the laboratory, are also characteristic of the animals in their natural haunts, I am unable to say. As far as I have been able to observe, the play of colors so conspicuous in the African chameleon is rarely if ever approached in Anolis. Lockwood ('76, p. 13), however, has described conditions which more nearly resemble those of the African chameleon than any I have seen. In this respect Anolis is much simpler, and consequently much more satisfactory for experimental work, than the true chameleon.

To insure precision in my work, I not only chose a lizard with a relatively simple color change, but I experimented as a rule only on definite portions of its skin, namely, the uniformly tinted areas which cover the sides of the animal's body. These regions showed the full range of color change from dark brown to pea-green, and proved to be convenient areas to deal with. Unless otherwise stated, what is contained in the following account refers to the skin of these regions. I shall begin with a description of the histology of the skin in the brown and in the green states, and I shall afterward take up the physiology of the changes concerned in the production of these two states.

The subject of this research was suggested to me by Dr. G. H. Parker, under whose guidance I have carried out the work.

II. The Histology of the Color Changes.

The skin of Anolis carolinensis, like that of other reptiles, consists of a relatively thin epidermis, in which a horny layer (Fig. 1, *st. crn.*) and a mucous layer (*st. muc.*) can be distinguished, and a derma (*drm.*), composed chiefly of interlacing connective-tissue bundles. Among other structures the derma contains numerous chromatophores, and other less clearly definable pigment masses. The skin is broken up into very clearly marked scutes, in which all the layers of the skin are considerably thickened, the small tracts between the scutes being the thinnest portions of the skin. The pigment is restricted to the regions under the scutes, and in this respect Anolis resembles more or less closely the chameleon as described by Brücke ('52) and Keller ('95).

So far as the color changes in Anolis are concerned, the active operations seem to be limited to the pigment of the derma, for in the epidermis I have sought in vain for any signs of alteration. In the following description I shall therefore confine my attention to the pigment-bearing organs of the derma.

Keller ('95, p. 141), in his account of the skin of the chameleon, has described five kinds of pigment bodies differing from each other essentially in the kinds of pigment that they contain. All the pigment bodies *in the skin of Anolis are easily referred to two, or at most three, of these* five kinds. The conspicuous black bodies of Anolis (Fig. 1, *mela'ph.*), well buried in the derma and sending branching processes outward toward the epidermis, correspond to the melanophores described by Keller. The material (Fig. 1, *och'ph.*) that fills in the spaces between the processes of the melanophores is bluish-green by reflected light, and corresponds both in position and character to Keller's layers of ochrophores. It is possible that occasionally a deeper reflecting layer of whitish material corresponding to what Keller calls the leucophore layer may be present, but this is certainly exceptional in Anolis, if in fact it occurs in this lizard at all. The two remaining types of pigment bodies in the chameleon, erythrophores and xanthophores, were not identified in Anolis.

Since the skin of Anolis presents two extreme conditions, dark brown and pea-green, and since the active changes by which these conditions are produced are limited to the two sets of pigment bodies in the derma, I shall restrict the remainder of my account of the skin to these bodies, beginning with the ochrophores.

The ochrophore layer (Fig. 1, *och'ph.*) consists of pigment masses arranged in several irregular rows parallel with the surface rather than in columns perpendicular to it, as in the chameleon.

The physical properties displayed by this pigment in its relation to light are of interest. When a section of skin is viewed under the microscope with transmitted light, this pigment appears yellowish with a slight tinge of green; in reflected light it has a bright greenish-blue tint, suggestive of the color assumed by the animal at night.

Since the ochrophore pigment is readily dissolved in mineral acids and is doubly refractive, as may be demonstrated under the polarizing microscope, it is in all probability an inorganic crystalline deposit. I have not been able to make out its relation to cells. The pigment occurs in blocks, and it is easy to see that nuclei are scattered irregularly between these. The nuclei are similar to those figured by Keller ('95, Taf. 4, Fig. 10) for the ochrophores of the African chameleon, and they may belong to the cells which produce what I have called the ochrophore pigment; but whether this pigment is in the cells or in intercellular spaces I have not been able to determine.

The ochrophore pigment showed no differences in the brown and the green states of the skin, so far as I could observe. The pigment masses did not change in position, the blocks were neither nearer together nor farther apart, and the physical properties remained the same.

The relation of the melanophores to the ochrophore layer is best seen in a section of brown skin parallel to the surface. In such a section (Fig. 2) the ochrophore layer (*och'ph.*) has the appearance of a more or less homogeneous mass, irregular in outline and penetrated in many places by the processes of the melanophores. These processes spread out irregularly over the distal surface of the ochrophore layer.

The melanophores (Fig. 1, *mela'ph.*) occur under the scutes, but not in the spaces between them, and are rather uniformly distributed to the number of about one hundred to a scute. The melanophores are for the most part round or oval in outline. They have a sharply marked contour, and the black pigment that they contain is in the form of small granules. Chlorine, when applied to sections by Mayer's method, destroys the color in these granules. Depigmented sections stained with Delafield's haematoxylin show that each melanophore contains a nucleus, thus demonstrating its cellular nature.

In the dark-brown skin (Fig. 1) each melanophore gives rise on its distal surface to some six or seven processes which extend distally to the region immediately under the epidermis. Near the body of the melanophore the processes show very few branches; but as they approach the epidermis they divide into smaller and smaller branches, which mingle together and form a dense interlacement; in a section perpendicular to the surface of the skin (Fig. 1) these branches seem to form an almost continuous black band. I have seen no evidence that the finer branches anastomose, for in sections of the skin parallel to its surface (Fig. 2) the processes and branches appear as a great number of dots of varying sizes without any connecting network. The processes and their branches are everywhere crowded with black pigment granules like those in the body of the melanophores.

The melanophores in the green skin (Fig. 3, *mela'ph.*) differ from those in the brown skin in that their branches seem to have disappeared, the main processes terminating in rounded ends. Hitherto it has been a question whether the branches are really drawn in like the pseudopodia of an amoeba, or whether the pigment granules simply move down the branch into the process, or into the cell, thus making it difficult to see the transparent branch. The latter condition is now held to be true for the chameleon, as was first maintained by Brücke ('**52**, p. 201). In attempting to ascertain the condition in Anolis I partially depigmented some sections of skin and stained them in Delafield's haematoxylin. In these sections I could see clearly the empty branches extending out from the region where the pigment ceased, to the base of the epidermis, and I was thus convinced that the branches are not withdrawn, but simply emptied of pigment. In the green condition of the skin the cell body of the melanophore, as might be expected, is more densely filled with pigment granules than in the brown state.

It is clear from the foregoing account that the active pigment changes accompanying the color changes in the skin of Anolis occur only in the melanophores. In this respect these changes are like those in Varanus, Uromastix, and Agame as described by Thilenius ('**97**, p. 532). In the dark brown state the pigment of the melanophores fills the processes and branches of these cells to their distal extremities, thus producing an almost continuous dark layer immediately under the epidermis and external to the ochrophore layer. It is this dark layer that gives to the animal its dark color. In the green state the dark pigment of the outer branches of the melanophores has retreated into the deeper processes, or even into the cell body beneath the ochrophore layer, thus exposing this layer to the light. As microscopic preparations show that the material of the ochrophore layer reflects a bright bluish-green light, it is probable that the pea-green color characteristic of this state is due chiefly to reflexion from this layer, though the transparent epidermis may modify more or less the color that would otherwise appear.

The outward migration of the melanophore pigment granules of Anolis in the light and their inward migration in the dark is like that recorded by almost all observers for the African chameleon. It is the reverse of the condition described for Stellio by Filippi ('**66**), for Phrynosoma by Wiedersheim (Hoffmann, '**90**, p. 1353) and for Varanus, Uromastix, and Agame by Thilenius ('**97**). In all these reptiles the melanophore pigment moves *inward* in the *light*, and *outward* in the *dark*.

III. The Physiology of the Color Changes.

1. *Introductory.* — As previously stated, Lockwood ('76) has observed — and I can abundantly confirm his observations — that Anolis carolinensis when in confinement during the winter is usually dark brown by day and pea-green at night. That this change is not simply rhythmic in character, but is dependent upon factors in the environment, is seen from the following experiments. When an Anolis that has turned green at night is placed in a dark-box so that it remains in the dark after daybreak, it retains its green color until exposed to daylight, whereupon it rapidly turns brown. If, on the other hand, a brown Anolis is exposed to illumination from a strong arc light (a gas-light is not sufficient) as evening comes on, it retains its former color and does not become green as long as it remains in a strong light. Thus the color changes are not rhythmic in correspondence with day and night, but depend upon the immediate effects of some factor in the environment, presumably light.

TABLE I.

Individual.	Time in Minutes required to change from	
	Dark Brown to Green.	Green to Dark Brown.
No. 1	26	4
" 2	22	3
" 3	36	2
" 4	23	8
" 5	17	2
Average	25	4

When a brown Anolis is put in the dark it invariably becomes green. This experiment I have repeated with perhaps one hundred animals, each one being tried several times, and I have never found an exception to this rule. The absence of light is therefore a means of inducing the green state.

When a green Anolis is put into the light it almost invariably becomes brown. I have repeated this experiment many times, and I have only rarely found individuals which retained their green color in the light for any great length of time. All such individuals, moreover, have eventually become dark brown, though in extreme cases the change has not occurred until the animal had been exposed to the light as long as three hours. Excepting these few instances, in which there has been a temporary retardation of the brown state, it may be stated that generally daylight induces the brown condition.

The changes from brown to green and from green to brown are, however, by no means reversed repetitions of each other. The rate of change differs according to its direction; in Table I are given a few characteristic rates. As this table shows, the change to brown is accomplished much more rapidly than that to green. However, a few individuals were found in which the two rates were nearly equal; thus in one instance the change to green was accomplished in 14.5 minutes, that to brown in 13.5 minutes. Such cases as these are, however, rare, the great majority following the general rule indicated in Table I.

The chief facts thus far observed may be summarized as follows: The dark brown condition of the skin is produced by the outward migration of the pigment of the melanophores, a process which takes place in the light and requires on the average about four minutes for completion. The green condition is produced by the inward migration of the melanophore pigment whereby the ochrophore layer is exposed to the light, a process which takes place in the dark and requires on the average about twenty-five minutes for completion.

2. *Brown Condition.* — I shall now endeavor to make clear the factors which are concerned with these two changes, beginning with the change from green to brown. This change is, as I have said before, almost always accomplished quickly when a green animal is exposed to daylight. Under these conditions it is conceivable that the change may be due either to the direct action of the light on the melanophores or to changes induced in these cells through the nervous system, which in its turn is stimulated by light.

My first experiments were directed to finding out whether the illumination of one part of the body had any influence on the color changes in those parts which were not illuminated. To determine this I used a dark-box about six inches long, two inches high, and five inches broad. The box was blackened inside and was provided with a movable lid. At one end of the box a small hole was made sufficiently large to admit the head of a lizard of average size. The hole was surrounded by a collar of loose black cloth, which could be made to fit snugly to the neck of the animal and thus hold it in place as well as keep light from entering the box. The lizard could be placed either with the head projecting out of the box and in daylight, while the trunk remained in the box in darkness, or the reverse.

To test the possible effects of this apparatus independent of light conditions, I placed a brown Anolis in the box with its head projecting outward; then, leaving the lid off, I put the dark-box and the lizard in another and larger dark-box, which I thereupon closed. The lizard underwent the usual change, finally becoming green. On exposing the open box and lizard to daylight, the animal changed to brown. Thus the retention of the animal in the small dark-box by the cloth collar had no observable influence on the color changes.

After this test of my apparatus I proceeded to determine whether the light which fell on one part of the animal's body could be said to have any influence on the color changes in other non-illuminated portions. Six green animals were placed in the box in turn, each with its head outside and its trunk inside. All turned brown, on the trunk as well as on the head, though in every case a trace of green, for some unknown reason, remained on the neck, and in one instance one leg remained somewhat green. Notwithstanding these slight irregularities, this experiment showed beyond a doubt that the illumination of the head not only induces a change to brown in that region, but also in the part of the animal in the dark.

Next, six green lizards were placed in the box in turn, this time with the head inside and the trunk outside. Three animals changed wholly to brown excepting for some small spots on the neck, one changed partly to brown, and two remained green. Here, again, in three instances at least, the illumination of one part led to the appropriate color change in the non-illuminated part.

It might be concluded at once from these observations that the illuminated parts do influence the non-illuminated ones, and that therefore the change to brown is under the control of nerves. But it must be remembered that even with the best management of the apparatus some light always entered the dark-box through the open space between the animal's neck and the cloth collar, and that this light might be responsible for the change in the so-called non-illuminated part. To test this I put a green animal wholly in the dark-box, while another green one was placed in the opening with its head inside the box. Notwithstanding the fact that both head and trunk of the animal in the opening changed to brown, the introduced lizard remained green. When, under similar circumstances, a brown lizard was introduced into the dark-box, it turned green. Moreover, when a brown animal was put inside the dark-box and the whole opening was left free for the entrance of light, the animal turned green and remained so, even in this diffuse light. There seems, therefore, no escape from the conclusion that the illumination of one part of the animal not only turns that part brown, but induces similar changes in the non-illuminated parts. I know of no way of explaining the induced changes except on the assumption that nerves serve as intermediate organs.

Why two of the animals in the original experiments remained green when their trunks were exposed to light, I do not know. It is possible that they were individuals whose reactions went on very slowly, and that I did not give them sufficient time; for when I tried these experiments I had not learned how slowly these changes sometimes come on. Or, since these two cases occurred among animals whose heads were in the dark, it is possible that they indicate that the condition of the head more easily impresses itself on that of the rest of the body than the reverse. I am, however, at present not in a position to decide this question.

The results of the experiments with the dark-box lead to two conclusions; first, that the nerve terminals in the skin of Anolis are sensitive to light and, secondly, that the change from brown to green may be brought about indirectly through nerves. That the nerve terminals in the skin of a vertebrate like Anolis should be sensitive to light is novel, so far as I know, and somewhat remarkable; but since a green animal with its head in the dark but its trunk in the light may change to brown all over, the evidence of this seems to me conclusive. It is, however, not without precedent, for in a recent paper Parker (:03) has shown, partly through the observations of others and partly through his own, that this property is possessed by several amphibians. From the evidence I have presented, I believe that the nerves of the skin in Anolis are sensitive to light.

If the change from green to brown is under the control of nerves, one should expect that the cutting of these nerves ought to prevent this change. Unfortunately the nerves are so small in Anolis that even those of the legs cannot be operated on with success, and hence a direct test of this proposition could not well be made. It is, however, possible to cut and destroy parts of the spinal cord. After such an operation one might expect to find the part of the body supplied by nerves from the destroyed portion of the cord incapable of changing to brown. To test this I etherized some six or seven animals and destroyed the spinal cord by pithing from the middle trunk region posteriorly. After twenty-four hours, when the shock effects of the operation had probably passed off, I experimented on these animals by exposing them while in the green state to the light. Much to my surprise, in all cases the whole body, both in the regions with and those without cord, eventually turned brown; the portion with cord turned as in a normal animal, that without cord turned with less uniformity, though in the end it was indistinguishable from the part containing cord.

These results seemed at first sight to contradict the conclusion that the change from green to brown is controlled by nerves, but in reality they showed merely that the *spinal* nerves are not directly concerned with this change. Since it seems impossible to explain the conditions without assuming some nerves to be involved, and since the only other nerves present are the sympathetic, these observations point to the sympathetic fibres as the ones controlling the change from green to brown. When it is remembered that in mammals the muscles of the integumentary blood-vessels, and of the hair and the sweat glands are controlled by the sympathetic, this conclusion, that a change in the tegumentary chromatophores of Anolis is under the control of the same nerves, seems natural enough.

It is greatly to be regretted that Anolis carolinensis is so small that operations on the sympathetic ganglia, etc., such as have been carried out so extensively on mammals, could not have been resorted to in order to test this conclusion in detail; but such operations, though tried, were in the end abandoned. It is, however, well known, chiefly through the researches of Langley, that nicotine is a powerful poison, especially for the sympathetic ganglia. I therefore hoped to get localized effect by opening the body cavity of Anolis and painting certain ganglia with a solution of this drug; but here again, because of the small size of the lizard, my efforts were without avail. However, I experimented with subcutaneous injections and observed the influence of this drug on the color changes.

As a rule I injected under the skin of the flank of the lizard about $\frac{1}{65}$ of its body weight of nicotine solution. The strengths used were 1 per cent, $\frac{1}{10}$ per cent, and $\frac{1}{100}$ per cent. The first of these caused the animal to change from brown to green, induced a pronounced muscular trembling, and resulted very shortly in death. To show that these effects were not brought about by the simple operation of injection, a like amount of pure water was injected under the skin of several lizards with the result that no color changes, muscular trembling, or other such disturbances were observed. The injection of $\frac{1}{10}$ per cent nicotine caused a change from brown to green and induced a slight muscular trembling, but the animals recovered in two or three hours in all respects except that the injection blister remained dark. With $\frac{1}{100}$ per cent nicotine no muscular trembling was observed, but the brown animals within one minute became green, and remained so for about three hours. When green animals were injected with either $\frac{1}{10}$ per cent or $\frac{1}{100}$ per cent nicotine, they remained green in the light about the same length of time as those did which were originally brown. It was remarkable that the side of the animal on which the injection was made remained green somewhat longer than the other side, and that all these animals changed to green, and remained so, in full daylight.

These results favor the conclusion already reached, that the change from green to brown is under the influence of the sympathetic nerves. Yet it might be maintained, so far as any facts thus far presented are concerned, that the action of the nicotine was not on the sympathetic system, but directly on the melanophores. That this is not so, may be shown in the following way. If the blood supply be cut off from any portion of the skin of Anolis, that part becomes green even when exposed to daylight, and will not turn brown from light stimulation until the blood is again freely admitted to the part. It follows from this, that since cutting off a piece of skin stops the flow of blood in it, one would expect such a piece to turn green, and such in fact is true. A piece of skin which by removal from the animal has become green, may, however, be made to assume the brown state by gently tapping it with a blunt instrument, provided it is kept on moist filter paper to prevent drying. The mechanical stimulation thus applied is sufficient to induce the temporary assumption of the brown condition, and demonstrates that the melanophores are not dead, but that they have simply drawn back their pigment. If now such a piece of skin is placed on filter paper moistened with nicotine solution and allowed to rest there till the solution has thoroughly permeated it, it will still, on being tapped, change to brown. Thus it is not the melanophores that have been changed, but the deeper mechanism through which the outward movement of their pigment is induced, and this, as we have already seen, is the sympathetic nervous system.

Although I was unsuccessful in my attempt to poison the sympathetic ganglia by local application, some evidence of local poisoning was obtained from the injection experiments. As I have already mentioned, the side of an animal on which the injection is made always remains green longer than the opposite side. Since the nicotine does not act directly upon the melanophores, I believe this condition can be explained only on the assumption of a stronger poisoning effect on the sympathetic of the operated side than on the other. This is the only piece of evidence I have on the local action of the drug, but so far as it goes it indicates that, as in mammals, local action of the sympathetic would be followed by local reponse in the skin.

In none of my experiments have I seen the least evidence that light has any direct influence on the melanophores on entering or passing out of the brown state. If a piece of brown skin is cut off from an animal and quickly divided in two, one piece being put in the dark, and the other exposed to daylight, both turn green at the same rate, and apparently irrespective of the light. Many experiments of this kind have been tried, but without the least evidence of the direct action of light on the melanophores.

The change to the brown state is thus not influenced by the direct action of light. It may be brought about in at least two ways: first, through the direct stimulation of the melanophores by such mechanical means as tapping; and, secondly, through the indirect action of light, which stimulates the nerve terminals in the skin, and thereby induces through the sympathetic fibres an outward migration of the melanophore pigment.

3. *Green Condition.* — The change from brown to green, as has already been stated, invariably takes place when an animal is put into the dark, and requires on an average twenty-five minutes for its completion. Since the dark is due to the absence of the stimulus, light, and is not a stimulus itself, it would seem probable that the condition brought on in the dark corresponds to the unstimulated or resting state of the cell.

Much evidence has been obtained favoring this view, but before this is presented, a word must be said about two ways by which the green state may be brought about. As already noted, any means of stopping the circulation will invariably induce the green state in the portion of the skin affected. Thus, when a ligature is tied around the leg of an animal, so as to stop the flow of the blood, the leg soon becomes green. Since the animal reacts when the leg is touched, and moves the leg almost normally in locomotion, it cannot be said that the ligature has temporarily incapacitated the nerves. When the ligature is removed, the leg quickly becomes brown. The green condition, then, may be produced by cutting off the blood supply.

Secondly, when a brown Anolis is put in the dark, the skin becomes green. The transfer to the dark is certainly not accompanied with any cessation of the circulation, and I therefore believe that the change in this instance is dependent upon the withdrawal of the stimulus transmitted over the sympathetic nerves, which, as I have already shown, can call forth the brown state.

It follows from these observations that when a piece of skin is cut from an Anolis, the cause of the change to green may be either the loss of circulation, or the loss of nerve stimulus, or both, and that in experimenting these two factors must be kept constantly in mind. It is greatly to be regretted that these factors cannot readily be separated in experiments. Because of the small size of the animals it has been found impracticable to cut nerves without interfering with blood-vessels, or to ligate blood-vessels and leave the nerves uninjured, hence some important and interesting lines of experimental work have of necessity been abandoned.

In one direction, however, a definite conclusion has been reached. As already described, a ligated leg with the nerves functional, and the whole animal in the light, turns and remains green. This observation shows that, notwithstanding the influence of the nerves, the loss of circulation, even under conditions favorable for brown, is followed by the green state. The circulation, in other words, is a more important factor than the influence of nerves, and it is my belief that the change of a piece of excised skin to green is more dependent on the loss of circulation than on the loss of nerve connections.

It is interesting to observe that a piece of excised skin assumes the green color more quickly than the animal from which it comes. The following experiment illustrates this point. A normal Anolis was placed in the dark several times and found to change from brown to green in about twenty minutes. It was then kept in the light, and its right hind leg ligated so as to stop the circulation, but not to interfere with the use of this leg in locomotion and hence to leave its efferent nerve supply uninterrupted. Under these circumstances the leg changed from brown to green in about six minutes. The ligature was then removed and the animal was allowed to assume the brown condition, whereupon it was killed by decapitation and the leg previously ligated was immediately cut off. This leg became green in between three and four minutes. These observations show that the change to green comes on more quickly when both nerve action and circulation are interrupted, than when only the circulation is interrupted, and that, therefore, the sympathetic nerves must be regarded as inhibiting slightly the change to the green state. The facts that ordinarily the brown condition is retained only in the light and that the change to the green state is somewhat inhibited when the nerves are intact, suggest that the sympathetic centres exert a tonus influence over the melanophore cells as long as the animal is exposed to daylight, and that the tonus only gradually subsides when the animal is placed in the dark.

The green state is not only produced by darkness, the withdrawal of the circulation, and possibly the cutting of nerves, but also in other ways. Most animals in narcosis from ether are green, and nicotine, as already mentioned, calls forth the green state. All animals change to green when they die. Thus the green state is in all cases the result of influences which either greatly reduce the stimuli or absolutely prevent them from reaching the melanophores. I therefore believe that the green state represents the resting condition of the melanophores — the state to which the cell returns on ceasing to receive stimuli. The brown state may be produced by direct mechanical stimulation of the melanophore cell, but I believe that usually it is the result of a continuous stimulation received from the sympathetic centres, which in turn are indirectly stimulated by light falling on some part of the animal's skin.

If this last conclusion is true, it follows that a brown animal, when placed with its head or its trunk in the dark-box, ought to show no change in color, for in each case enough of the body is exposed to light to keep up the sympathetic tonus for the whole integument. As a matter of fact such is the case; thus when each of six animals in the brown state were placed in the box, first with the head in, then with the trunk in, all remained brown.

From these various observations I conclude that the green state in Anolis represents the non-stimulated or resting state of its melanophores, and that this state is brought on by any means that wholly interrupts or greatly reduces the stimuli which naturally come to the melanophores. The change to this state is somewhat retarded by the partially inhibitory action of the sympathetic centres.

4. *Comparison with other Lizards.* — Enough has already been published on the color changes in lizards to show that these changes are not carried out in any uniform way in this group of animals. Thus in Chameleon, according to Brücke ('52), Keller ('95), and in fact all who in the last fifty years have examined the skin of this reptile carefully, the melanophores show an outward migration of pigment in the light and an inward migration in the dark. The same is true of Anolis. But in

Veranus, Uromastix, and Agame, according to Thilenius ('97), just the reverse takes place, — an inward migration in the light, and an outward one in the dark. That Thilenius's observations are probably correct is shown by the fact that similar conditions have been described by Filippi ('66) for Stellio, and by Wiedersheim (Hoffmann, '90), for Phrynosoma. Apparently lizards fall into two classes in this respect: those with an outward migration of pigment in the light, and those with an inward migration under like circumstances.

Although Anolis agrees with Chameleon in that its melanophore pigment moves outward in the light and inward in the dark, it differs fundamentally in other respects. I believe I have found satisfactory evidence to show that the outward migration of pigment in Anolis is dependent on the action of the sympathetic centres. The investigations of Brücke ('52), Bert ('75), Keller ('95), and others on the effects of nerve cutting and local stimulation by light in Chameleon have shown conclusively that the outward migration in this form is independent of nerves and due to the direct stimulation of the melanophore cells by light. On the other hand the inward migration of pigment in Anolis is a return of the cell to a resting state, dependent neither upon direct stimulation nor nerves, while in Chameleon this change has been shown beyond a doubt (Keller, '95, p. 137) to depend upon nerves. Thus, so far as the relation of the color changes to nerves is concerned, Chameleon and Anolis are the reverse of each other.

From the experiments of Brücke ('52), Bert ('75), Krukenberg ('80), and Keller ('95) on cutting the spinal cord of Chameleon, it would seem, since the skin became black posterior to the cut, that the spinal nerves control the inward migration of the pigment in this reptile. As already shown, such an operation in Anolis has little or no effect on the final change of color, and hence the sympathetic nerves must be assumed to act in Anolis.

Thus in a second fundamental particular Anolis differs from Chameleon. Anolis also differs not only from Chameleon, but also from all other lizards, so far as is known, in that the migration of the pigment in its melanophores is not directly influenced by light. It has been abundantly shown that the outward migration of the melanophore pigment in Chameleon is dependent upon the direct action of light, and Thilenius ('97, p. 539) has maintained that direct light stimulation also occurs in Varanus, Uromastix, and Agame. Nothing of this kind has been observed in Anolis. Here the inward migration might be suspected to occur under local stimulation, but it takes place in the dark as well as in the light, and, as already shown, gives not the least evidence of direct stimulation of the melanophores.

Thus in three fundamental respects the pigment changes in Anolis differ from those in Chameleon, and suggest separate lines of phylogenetic differentiation, even though all such changes are to be traced back to the responses to light of the simpler form of tegumentary chromatophores.

IV. SUMMARY.

1. The skin of Anolis carolinensis may be made to assume one or other of two extreme colors, dark-brown and pea-green.

2. The brown state for animals in confinement is taken on in daylight, and is produced by the outward migration of pigment granules from the bodies of the melanophores into their processes and ultimate branches.

3. The outward migration is accomplished in about four minutes.

4. It may be induced either by the mechanical stimulation of the skin or by the action of the sympathetic nerve centres.

5. The brown state is ordinarily maintained through a tonus established by the sympathetic nerves and dependent upon the stimulation of the nervous end-organs in the skin of Anolis by light.

6. The melanophores of Anolis are not stimulated directly by light.

7. The green state is taken on in the dark.

8. It is produced by the inward migration of the pigment granules of the melanophores, whereby the reflecting ochrophore layer becomes exposed to light.

9. The inward migration is accomplished in about twenty-five minutes.

10. It may be induced by any means which brings the melanophores into an unstimulated state: darkness, cessation of circulation, ether narcosis, nicotine poisoning, and possibly the cutting of nerves.

11. The green state represents the state of rest for the melanophore cells.

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(See Hoffmann, C. K.)

EXPLANATION OF PLATE.

The three figures are reproductions from photomicrographs taken by Mr. W. C. Greene from sections of the skin of *Anolis carolinensis* Cuv. All figures are magnified 200 diameters.

Abbreviations.

drm.	Derma.	*och'ph.*	Ochrophore.
e'drm.	Epidermis.	*st.crn.*	Horny layer.
mela'ph.	Melanophore.	*st.muc.*	Mucous layer.

Fig. 1. Section of a scute, perpendicular to its surface, from a piece of skin in the brown state. The branched processes of the melanophores are shown filled with pigment out to the epidermis.

Fig. 2. Tangential section of a scute from a piece of skin in the brown state.

Fig. 3. Section of a scute, perpendicular to its surface, from a piece of skin in the green state. The melanophore processes from which the pigment has migrated proximally may be traced, even in the photograph, almost to the epidermis.

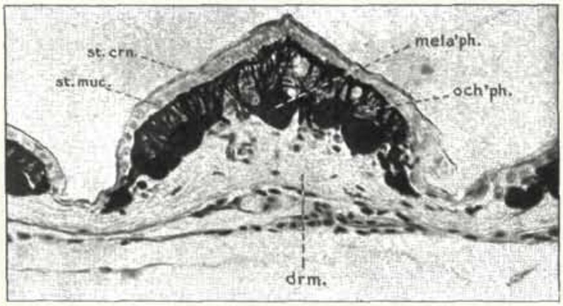
st. crn.
st. muc.
mela'ph.
och'ph.
drm.

Fig. 1.

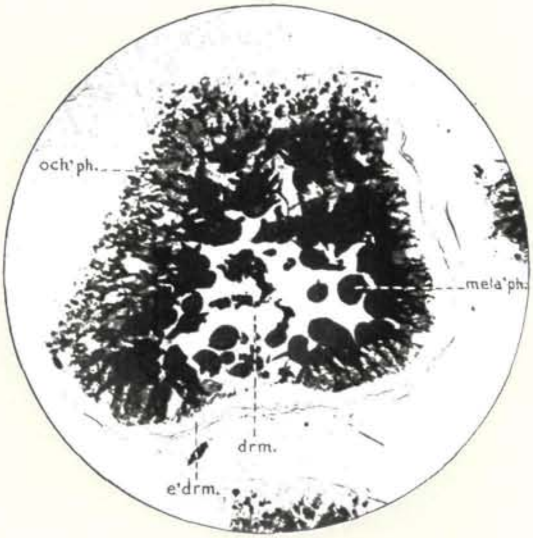
och'ph.
mela'ph.
drm.
e'drm.

Fig 2

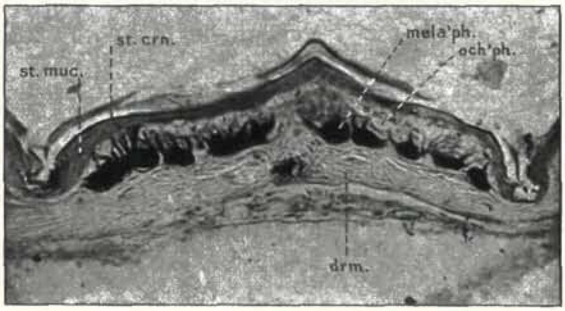
st. crn.
st. muc.
mela'ph.
och'ph.
drm.

Fig. 3.

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CONTRIBUTIONS FROM THE GRAY HERBARIUM OF HARVARD UNIVERSITY.

New Series. — No. XXVI.

A REVISION OF THE GENUS FLAVERIA.

By J. R. Johnston.

A REVISION OF THE GENUS FLAVERIA.

By J. R. Johnston.

Presented by B. L. Robinson April 8, 1903. Received October 12, 1903.

Not since 1836, when A. P. DeCandolle enumerated in the Prodromus, v. 635, only four species of Flaveria, has there been a revision of the genus. Since DeCandolle's time there have been over a dozen different plants published as new Flaverias, seven of which have proved to be good species. The need of another revision so far as the Mexican species are concerned is mentioned by Hemsley in the Biol. Cent.-Am. Bot. ii. 215 (1881–82), and the confusion in identification of certain species, together with the recent accumulation of specimens in the herbaria of this country, have emphasized its need.

The history of the genus is considerably complicated by the diverse views expressed by the early writers, who treated its species. The name Flaveria (from the Latin *flavus*, golden yellow, the plants having been used to dye yellow) was first proposed by A. L. de Jussieu (1789), in his Genera Plantarum, for two plants from Chili and Peru. His meagre descriptions and the fact that he omitted specific names for the plants, and distinguished them from each other only by the spicate inflorescence of the Peruvian plant and the glomerate capitate heads of the Chilian, have given rise to different ideas concerning the type plant as well as its name.

The reference of Jussieu to Feuille, Journ. Obs. Physiques, Mathematiques et Botaniques, iii. 18, t. 14, in speaking of the Chilian species, leaves no doubt that at least this one of the species described was the plant called "contrayerba" by the natives of Chili. Cavanilles in his Icones Plantarum, i. 2, t. 4 (1791), also referred to Feuille's figure in describing *Milleria Contrayerba*, thus making it synonymous with the Chilian plant of Jussieu, who had distinguished Flaveria from Milleria merely because of the supposed absence of ligulate flowers. These were, nevertheless, found by Cavanilles, and in consequence he returned to the name Milleria, thus reducing Flaveria to the rank of a synonym. Ruiz and Pavon, who likewise had to deal with this "contrayerba" of South America in writing their Flora Peruviana Prodrom. 114 (1794), asserted rightly that from its characters it could not belong to the genus Milleria, hence they proposed a new name Vermifuga, but it was not until later in their Systema Vegetabilium Fl. Peruv. et Chil. 216 (1798), that they added the specific name "corymbosa." Persoon in his Synopsis Plantarum, ii. 489 (1807), referred both *Milleria Contrayerba*, Cav., and *Vermifuga corymbosa*, R. & P., to *Flaveria Contrayerba*, the name which has been used up to the present time.

In the meantime, however, Gmelin in his Systema, 1549 (1796), reverted to Flaveria and published the specific binomials *F. chilensis* and *F. peruviana* for the two original plants of Jussieu. It is thus evident that *F. chilensis*, Gmel., is the first properly named species of Flaveria, and it is also clear that this was the Chilian plant of Jussieu, the "contrayerba" of the Chilians. It may be said further in regard to Gmelin's two names, that later writers have quite correctly called *F. peruviana* a synonym of *F. Contrayerba*, Pers., but that they have also with little reason considered *F. chilensis* a synonym of *F. angustifolia*, Pers., which was first described as a Milleria by Cavanilles in his Ic. Plant. iii. t. 223. This opinion seems to have been based on very slight grounds; in fact, merely upon the incomplete description of the form of inflorescence. The Chilian plant is described as having a glomerate capitate inflorescence which fits *F. angustifolia* well, but Jussieu refers the Chilian plant to that of Feuille, the illustration of which agrees fairly well with the appearance of *F. chilensis*, Gmel., and is undoubtedly the "contrayerba." Moreover, "angustifolia" is typically a Mexican plant, having never been reported, so far as I can make out, in South America.

To the Peruvian plant, however, Jussieu ascribes a spicate inflorescence which does fairly well for some species of *F. Contrayerba*, Pers., or, using the earlier name, *F. chilensis*, Gmel. Thus, it seems probable that both of Jussieu's specimens may properly be referred to *F. chilensis*, Gmel., and that *F. peruviana*, Gmel., may be considered its synonym. Although Jussieu identified his plants with *Milleria chiloensis* in Hortus Regius Parisiensis, the first instance of a specific name under Flaveria is that of *F. chilensis*, Gmel., so that that name should take precedence over all others.

F. chilensis, Gmel., is then the type plant of the genus, and *F. angustifolia*, Pers., is the second good species published in the group. A detailed study of *F. Contrayerua*, Cav. (*F. Contrayerba*, Cav.) was undertaken by Sprengel in Schrad. Journ. Bot. pt. 2, 186, t. 5 (1800).

As he concluded both that the plant he had at hand was not a Milleria, and that Flaveria was not a good genus, he proposed a new name, Brotera, so that the plant stood as *Brotera Contrayerva*, Spreng. The plant described and illustrated by Sprengel is, however, an entirely different plant from *Milleria Contrayerba*, Cav. Wildenow (1804) in Species Plantarum, iii. pt. 3, 2393, having previously used the name Brotera for a genus (Cardopatium, Juss.) published *Nauemburgia trinervata* for Sprengel's plant, not for Cavanilles's. Lagasca, however, Gen. et Sp. Nov. 33, no. 406 (1816), named a plant *Flaveria repanda*, which Sprengel in 1826, Systema Vegetabilium, iii. 500, identified with both Nauemburgia and Brotera. Sprengel also enumerated *F. Contrayerba*, Pers., *F. angustifolia*, Pers., and *F. linearis*, Lag. As late as 1832, Lessing, Synopsis Generum Compositarum, 235, maintained Nauemburgia distinct from Flaveria, and DeCandolle, Prod. v. 635 (1836), retained *Broteroa* (Brotera, Spr.) *trinervata*. As the distinction between Brotera (or its synonym Nauemburgia) and Flaveria consists merely in the presence of setae upon the receptacle, a character variable in some genera of the Compositae, it alone is not sufficient to separate the two. As other characteristics of the plants correspond very well, it has seemed best to unite the two genera in this revision.

Since the publication of the above species, as has been said, seven good species have been added to the genus, and about as many more plants have been given new names under Flaveria, which have subsequently proved identical with existing species or not to belong to the genus at all. The abundance of material at hand has afforded opportunity for better characterizing the species, for increasing the known range of some of them, and it has also furnished sufficient evidence for naming one variety and four new species of plants which have hitherto been placed with others.

It may be said that the genus groups itself fairly well into subdivisions; for example, *F. australasica* and *F. repanda* are similar in habit, and are the only two having setae upon the receptacle. Those whose heads have three bracts also form a characteristic group, which, however, passes into the group characterized by five bracts. Besides those with perennial roots which do not resemble each other at all, there are several other exceptional forms, as *F. anomala*, which has the three bracts with bulbous bases, and *F. chloraefolia*, which is the only species with conspicuously perfoliate leaves. In the subgroup, which is characterized by possessing three involucral bracts, however, there has been considerable confusion in separating the species, due to the similarity sometimes in habit and again in floral structure. *F. chilensis* is the only one of this group having a distinctive ligulate flower, the ligule being slender, upright, and short, compared with the others, which are oval, usually reflexed, and large, the specific distinctions between the latter consisting mainly in habital characters. Again, the group characterized by five involucral bracts has an element of confusion in it, due rather to the extreme variation from a shrubby form, growing on hot and dry sea beaches, to herbaceous forms found in wetter places. In one species, *F. linearis*, as heretofore understood, may be found forms possessing no regular branching at all, and forms having the dichotomy characteristic of the genus. There is also a variation from plants having the leaves mostly whorled, about .2 cm. wide and 2 to 3 cm. long, to others having the leaves prevailingly opposite, .5 to 2.5 cm. wide and 4 to 12 cm. long. Moreover, the internodes in plants of this so-called species vary from 1.5 to 5 cm. long, a difference, however, which may well be due to individual environment. Notwithstanding these differences, however, among the species, the genus, as a whole, is one easily recognized and not likely to be confounded with others.

The characteristic habitat is shown by *F. longifolia*, which grows in alkaline meadows of San Luis Potosi, Mexico. The genus, for the most part, is Mexican, though *F. australasica* is found only in Australia, and *F. repanda* and *F. chilensis* have a range from southern United States to Chili and Argentine Republic. *F. linearis*, also, grows in Yucatan, Cuba, and Florida on the sandy beaches, and *F. campestris* is confined, so far as is known, to the western central United States, Arkansas to Colorado, growing in alkaline soil. The remainder, with the exception of the Florida species *F. floridana*, occur in Mexico. *F. angustifolia* is found in rich valleys of Mexico and ascends to 2,000 or 2,500 m. altitude, and *F. chilensis* has been reported at the same height in Peru.

So far as uses are concerned, *F. chilensis*, Gmel., is the only plant in the genus which has been reported of any economic value. Feuille, Ruiz, and Pavon, all speak of its medical properties. The latter say that the natives bruise the plants in a salt brine and apply to putrid ulcers to drive out worms. Feuille states that, boiled in water, it makes a beautiful yellow stain.

In preparing this paper the library and specimens of the Gray Herbarium have been consulted, as well as material from the private herbarium of Mr. John Donnell Smith, from the herbarium of the Missouri Botanical Gardens, from the Engelmann Herbarium, and from the United States National Herbarium. To those who have so kindly given the use of these, and especially to Professor B. L. Robinson, under whose supervision the writer has been working, and to Miss M. A. Day, librarian at the Gray Herbarium, many thanks are due.

FLAVERIA. Heads small, heterogamous or homogamous, 1- to 15-flowered, rays usually only one. Involucral bracts 2 to 8, subequal, sometimes with two smaller exterior bracts. Receptacle small, naked or setose. Achenes linear-oblong, glabrous, 8- to 10-ribbed, black. Corolla ♀ ligulate, from ½ length of achene to 3 or 4 times as long, entire, emarginate or tridentate, upright or reflexed. Corolla ☿ regular, turbinate, 5-parted. Corolla tubes villous or not, the hairs consisting of 6 to 12 short thick cells. Anthers at base obtuse, entire. Apex with a conical appendage. Style of ☿ flower 2-parted, reflexed, obtusely rounded. Pappus usually none, in one species broad, scale-like, dentate, or fimbriate.

Herbs glabrous or puberulous, often yellowish or rubescent. Leaves opposite, narrow, entire or dentate. Capitula narrow, sessile or with short pedicels, borne in dense cymes or glomerules, which are pedunculate or sessile, corymbose-paniculate or solitary at tips of branches. Corolla pale to deep yellow. — Juss. Gen. Pl. 186 (1789); Gmel. Syst. 1269 (1791); Willd. Enumeratio, 941; Persoon, Synopsis Plantarum, ii. 489; Spreng. Syst. iii. 500, n. 2737; Less. Synopsis Generum Compositarum, 235; DC. Prod. v. 635; Torr. & Gray, Fl. N. Am. ii. 360; Benth. & Hook. f. Gen. Pl. ii. 407; Hemsl. Biol. Cent.-Am. Bot. ii. 215; Hoffm. in Engl. & Prantl, Nat. Pflanzenf. iv. Abt. 5, 258. *Vermifuga*, Ruiz & Pavon, Flora Peruviana Prod. 114, t. 24 (1798–1802). *Milleria*, pro parte, Cav. Ic. Pl. i. 2, t. 4 (1791). *Brotera*, Spreng. in Schrad. Journ. iv. 186, t. 5 (1800). *Nauemburgia*, Willd. Sp. Pl. iii. 2393 (1804). *Broteroa*, DC. Prod. v. 636 (1836). *Selloa*, pro parte, Nutt. Am. Journ. Sc. v. 300 (1822). *Gymnosperma*, pro parte, DC. Prod. v. 312 (1836).

§ 1. Receptacle setose.

* Leaves linear to lanceolate, entire or serrulate; Australian.

1. F. AUSTRALASICA, Hook. Herbaceous, 30 to 60 cm. high; stem striate, glabrate: leaves with base dilate, linear-lanceolate, entire or serrulate, glabrous, 3-nerved, .2 to 1.4 cm. wide, 2 to 7.5 cm. long: heads densely glomerate, glomerules 1 to 2.5 cm. wide, subinvolucrate: involucral bracts 2 to 4: corolla ♀ ligulate, lamina equalling tube or nearly so, upright or oblique, entire, emarginate or tridentate, corolla tube slightly villous: achenes nearly equal, 2.25 mm. long. — In Mitchell's Journ. Trop. Austral. 118; Mueller, Frag. i. 183. — N. AUSTRALIA,

Nichol Bay, N. W. coast, *F. Gregory's* Expedition; Victoria River and Hooker's Creek, *F. Mueller* (hb. Gr., hb. U. S.); islands of the Gulf of Carpentaria, *R. Brown;* in the interior, *M'Douall Stuart's* Expedition; Albert River, *Henne;* Balonne River, Queensland, *Mitchell* (hb. Gr.): these acc. Benth. Fl. Aust. iii. 546 (1866).

* * Leaves lanceolate to elliptical; American.

2. F. REPANDA, Lag. Herbaceous, stem 30 to 120 cm. high, green or reddish, dichotomous, striate, glabrate: leaves opposite, lanceolate, prominently 3-nerved, apex rounded or acute, 2 to 9 cm. long, .6 to 2.5 cm. wide, the base of the lower leaves tapering into a sort of petiole, the two opposite ones being connate at the stalk; the upper leaves are decidedly connate; leaves in three pairs about inflorescence; lanceolate leaves serrate, elliptical leaves repand-dentate: heads usually 1-flowered, gathered in dense axillary or terminal clusters: ♀ bracts 1 to 2, usually 2, both concave, acute or rounded at apex, even emarginate, one larger than the other: corolla subligulate, emarginate, entire or tridentate, a little over .1 cm. long, lamina the length of the tube, oblique, entire corolla a little over ½ length of achene, lower part of tube villous: achene scarcely .1 cm. long: ☿ bracts 1 to 2 as in ♀; corolla tube narrow becoming full at throat, lobes acute, reflexed, tube villous as ♀; anthers with obtuse appendages at apex; filaments thickened immediately below the anther: setae filamentous, slightly flattened and spreading at apex, or flattened throughout as half-aborted bracts; achene of ☿ smaller by ⅓ than that of ♀.— Lag. Gen. et Sp. Nov. 33, no. 406 (1816). *Oedera trinervia,* Spr. in Bot. Gart. Halle, 63 (1800). *Brotera Contrayerva,* Spr. in Schrad. Journ. Bot. iv. t. 5 (1800), not *Millera Contrayerba,* Cav. *Nauemburgia trinervata,* Willd. Sp. Pl. iii. 2393 (1804). *Broteroa trinervata,* DC. Prod. v. 636 (1836). *Brotera Sprengelii,* H. Cass. Dict. xxxiv. 304. *Flaveria trinervata,* Baillon, Hist. Pl. viii. 55 (1886). *Flaveria trinervia* (Spreng.) Mohr, Cont. Nat. Herb. vi. 810 (1901).— ALABAMA: adventive with ballast, Mobile County, *Mohr,* Cont. Nat. Herb. vi. 810 (1901) (hb. Geol. Surv., hb. Mohr). TEXAS: Barstow, *S. M. Tracy,* 8161, Oct. 1902 (hb. Gr.); Rio Grande, *Wright,* 356, Oct. 1849 (hb. Gr., hb. U.S.); near Doñana, valley of Rio Grande, *Parry, Bigelow, Wright,* and *Schott,* 593 (hb. U.S.). NEW MEXICO: Mesilla, *E. O. Wooton,* 51, June, 1898, alt. 1300 m. (hb. M. B. G., hb. U. S.); Roswell, *F. S. Earle* & *E. S. Earle,* 304, Aug. 1900, alt. 1,200 m. (hb. M. B. G., hb. U. S.); Las Cruces, *Vasey,* 184, 1881 (hb. J. D. S., hb. U. S.). COAHUILA: San Carlos, *Berlandier* 2372, Nov. 1831 (hb. Gr.); Parras, *Palmer,* 686, Oct. 1880 (hb. Gr., hb. U. S., hb. J. D. S., hb. M. B. G.),

Saltillo, *Palmer*, 284, Sept. 1898 (hb. Gr., hb. U. S.); Jimulco, *Pringle*, 83, May, 1885 (hb. Gr., hb. J. D. S., hb. U. S.). NUEVA LEON: Monterey, *Enero*, 1422 & 162, 1828 (hb. Gr.). DURANGO: rich moist bottom-lands, *Palmer*, 481, Aug. 1896 (hb. Gr., hb. M. B. G., hb. U. S.). SAN LUIS POTOSI: en route from San Fernando to Santander, *Berlandier*, 839, Oct. 1839 (hb. Gr.); *Parry* & *Palmer*, 499, 1878 (hb. Gr., hb. M. B. G., hb. U. S.). CHIHUAHUA: *Potts* acc. Hemsl. Biol. Cent.-Am. Bot. ii. 216. GUANAJUATO: Jaral, *Walther Schumann*, 64, 1885 (hb. Gr., hb. J. D. S.); *Alfredo Dugès*, Nov. 1897 (hb. Gr.); Irapuato, *Pringle*, 2688, May, 1889 (hb. M. B. G.). OAXACA: *L. C. Smith*, 307, Nov. 1894 (hb. Gr.); *Conzatti* & *González*, 1016, Aug. 1900 (hb. Gr.), *E. W. Nelson*, 130, Sept. 1894, alt. 1,600 to 1,700 m. (hb. Gr.). HIDALGO: Cadena in rich valley, *Gregg*, 22 (hb. Gr.) & 594 (hb. Engel.). TEOTITLAN: Tecomavaca, *Pringle*, 5724, Sept. 1894 (hb. Gr., hb. U. S.); *C. L. Smith*, 264, Sept. 1894, alt. 650 m. (hb. M. B. G., hb. U. S.). TEHUACAN: *Galeotti*, 2639 acc. Hemsl. l. c., alt. 2,000 m. MORELOS: Jojutla, *Pringle*, 9508, June, 1901, alt. 1,000 m. (hb. Gr., hb. U. S.). YUCATAN: ditches near Progreso, *Millspaugh*, 1653, and along railroad south of lagoon, Progreso, *Millspaugh*, 1699 & 1731, Pub. Field Columb. Mus. Bot. ii. 109; downs of Progreso, *Schott*, 973, Dec. 1865, acc. Millspaugh, Pub. Field Columb. Mus. Bot. i. 395. CUBA: Liebmann, 452 (hb. Gr.); salt marshes near Guanimas, *Wright*, 2860 (hb. Gr., hb. Engel., hb. U. S.); waste places, field and roadsides, Cieneguita, *R. Combs*, 473, Aug. 1895 (hb. Gr., hb. M. B. G.); Havana, *Palmer* & *Riley*, 1156, July, 1900 (hb. U. S.). VENEZUELA: Tovar, *A. Fendler*, 692, 1854–5, alt. 1,000 m. (hb. Gr.); Caracas, *A. H. Moore*, 25, Mar. 1899 (hb. Gr.). PERU: Huanuco, acc. Spr. in Schrad. Journ. Bot. iv. 186 (1800). BRITISH GUIANA: *Schomburgk*, 247, acc. Baker in Mart. Fl. Bras. vi. pt. 3, 270. BRAZIL: *Riedel*, 813 (hb. Gr.); Bahia, *Riedel*, 938, and *Luschnath*, acc. Baker, l. c. Cult. specimen: Hort. Cantabr. 1849 (hb. Gr.), Hort. Genev. (hb. Gr.), Hort. Bot. Berol. (hb. Gr.).

§ 2. Receptacle not setose.

* Heads 3- to 8- flowered.

+ Annual.

++ Bracts plain.

= Ligule slender, nearly upright.

3. F. CHILENSIS, Gmel. Herbaceous, erect, .6 to .9 m. high, with a copious dichotomous branching: stem green or rubescent, branches gla-

brous or villous at nodes: leaves opposite, slightly connate, lanceolate-elliptical, narrowing at base, sometimes appearing petiolate, 3-nerved, serrate; lower leaves 6 cm. long, .4 to 5 cm. wide: cymes 2 to 2.5 cm. in diameter; branches densely scorpioid: involucral bracts .4 cm. long, oblanceolate, obtuse, subequal, exterior bracts 1 to 2, lanceolate; heads consist of one ♀ and one ☿, or usually one ♀ and 2 to 5 (sometimes 8) ☿, undeveloped ☿ often present: corolla tube villous or not; ♀ subligulate, ligule narrow, upright, acute, exceeded by lobes of style before they are reflexed, slightly more than $\frac{1}{2}$ length of ☿ corolla; ☿ corolla exceeding achene; corolla tube .3 cm. long: achene of ♀ slightly exceeding ☿ achene. — Gmel. Syst. 1269 (1796). *Flaveria peruviana*, Gmel. Syst. 1549 (1796). *Milleria chiloensis*, Hort. Reg. Paris, acc. to Juss. Gen. 187 (1789), nomen seminudum. *M. Contrayerba*, Cav. Ic. Pl. i. 2, t. 4. *Flaveria Contrayerba*, Pers. Syn. ii. 489. *Vermifuga corymbosa*, Ruiz & Pavon, Fl. Per. 114, t. 24. *Flaveria bonariensis*, DC. Prod. v. 635. *Flaveria capitata*, Juss. ex Sm. in Rees, Cycl. xv. n. 1. *Flaveria peruviana*, Juss. Gen. Pl. 187. *Milleria contrahierba*, Lam. Dict. iv. 183. *Eupatorium chilense*, Mol. Chil. 335, acc. Gay, Flora Chilena, 278. — Georgia: waste places among rosin wharves, Brunswick, *Harper*, 1521 (hb. Gr.). Florida: Pensacola, *Curtiss*, 1504, Sept. 1886 (hb. Gr., hb. J. D. S., hb. M. B. G., hb. U. S.); *Curtiss*, 5, Sept. 1886 (hb. Gr.); *Curtiss*, 6495, July, 1899 (hb. Gr., hb. M. B. G., hb. U. S.), on waste ground. Alabama: Mobile, *Mohr*, 17, 1891 (hb. U. S.). Mexico: Real del Monte, *Berlandier;* Guanajuato, *Mendez*, acc. Hemsl. l. c. Peru: Lima, *Gaudichaud*, 113 (hb. Gr.); in yards and fields of Cercado, Chaucay, Cautae, Huarocheri, Huanuci, and Cuzco provs. Commonly called "contrayerba" and "matagusanos" in Lima, and in Cuzco, "chinapaya," acc. Ruiz et Pavon, Syst. Veg. Fl. Per. et Chil. 216; in Peruvian Andes near Guancabamba, alt. 2,000 m., acc. HBK. Nov. Gen. et. Sp. iv. 285; Calloa and Lima, *U. S. Exploring expedition* under *Capt. Wilkes*. Ecuador: Manobi, *Eggers*, 15704. Bolivia: *Bang*, 2026 (hb. Gr.); Socata, *G. Mandon*, 58, 1859 (hb. Gr.); Cochabamba, *Bang*, 968, 1891 (hb. Gr., hb. J. D. S., hb. M. B. G., hb. U. S.). Chili: Valparaiso, *Mertens* (hb. Gr.); Tarajuaca, *R. A. Philippi*, Chili Museo National Santiago, 1888 (hb. J. D. S.); Conception, about 1709, acc. Feuille in Journ. Obs. Phys. iii. 18, t. 14. Uraguay: *Tweedie* (hb. Gr.); Mendoza, *Gillies*, 146 (hb. Gr.). Argentine Republic: Cordoba, *Lorentz*, 1874 (hb. Gr.); Naporta Grande, *Lorentz*, 1874 (hb. Gr.,); Buenos Ayres, Hb. Parker (hb. Gr.); near S. Juan, Jachal, Cordoba, *Tweedie*, *Jameson*, *Hieronymus*, acc. Baker l. c.; near Buenos Ayres, *Bacle*, acc. Baker, l. c.

= = Ligule oval, reflexed.

a. Leaves lanceolate, bracts 3 (sometimes 4 or 6).

b. Leaves serrate, inflorescence leafy, not compact; stem glabrous or villous at nodes, stout.

4. **F. campestris,** nov. sp. Herbaceous: stem erect, branching dichotomously, green or rubescent, glabrous or villous at nodes: leaves linear to lanceolate, serrulate, 3-nerved, narrowing at base, slightly connate, 2.5 to 6.5 cm. long, 1 to 2.5 cm. wide: inflorescence densely cymose, cymes corymbose-paniculate, involucral bracts 3 nearly equal, .5 cm. long; outer bracts usually 2 unequal, .1 to .6 cm. long, linear-lanceolate: flowers usually one ♀ and 3 to 4 ☿; corolla tube slightly villous, lamina of ♀ large, oval, reflexed, nearly equalling ♀ achene, which is .3 cm. long; achene of ☿ .25 cm. long. — MISSOURI: Courtney, *B. F. Bush,* 51, Sept. 1890 (hb. U. S.). KANSAS: Pawnee Rock, *A. Gordon,* Sept. 1895 (hb. Engel.); Argentine, *K. Mackenzie,* Sept. 1895 (hb. M. B. G.); *M. A. Carleton,* 740, 1896 (hb. Gr., hb. U. S.); Belvidere, *S. F. Ward,* 1897 (hb. Gr., hb. U. S.); Wichita, *B. B. Smyth,* 240, 1890, low sandy dunes and flats near river (hb. U. S.); Medicine Lodge, *Smyth,* 292, 1890, low prairies (hb. U. S.). INDIAN TERRITORY: Cherokee Outlet, *Carleton,* 505, Sept. 1891 (hb. U. S.). TEXAS: Mustang Spring, *V. Havard,* 13 (hb. U. S.); Baylor County, *Reverchon,* 22, 1879 (hb. Gr.); banks of the Brazos, Seymour, *Reverchon,* 506, 1879 (hb. J. D. S., hb. U. S.); alkali flat, Big Spring, Howard County, *V. Havard,* 13, Sept. 1881 (hb. Gr.); Antelope Hills of the Canadian, *Bigelow,* 1853–4 (hb. U. S.); Cimarroon Creek, low prairies, *A. Fendler,* 536 (hb. Gr., hb. Engel.). COLORADO: Pueblo, *R. W. Woodward,* 1882 (hb. Gr.); Huerfano, *G. Engelmann,* Sept. 1881 (hb. Engel.).

b. b. Leaves remotely denticulate, tapering at base; inflorescence naked, compact; upper stem usually pubescent, stout; outer bracts large.

5. F. ANGUSTIFOLIA, Pers. Herb erect, 30 to 90 cm. high, branching dichotomous, upper part puberulent, branching little up to the inflorescence which is corymbiform or almost umbellate: leaves linear-lanceolate, 2.5 to 11.5 cm. long, 2.2 to 4 cm. wide, obsoletely denticulate or entire, glabrous: inflorescence naked, densely glomerate: bracts usually 3, rarely 4 or 6; two outside bracts conspicuous, unequal: 1 ♀ and 3 to 5 ☿, sometimes 6 to 8 ☿ and no ♀; corolla tube villous: achenes about equal. — Pers. Syn. ii. 489 (1807). *Milleria angustifolia,* Cav. Ic. Pl. iii. 12, t. 223 (1795). *F. integrifolia,* Moc. & Sessé, acc. to DC. Prod. v. 635. *F. elata,* Klatt, Leopold. xxiii. 146 (1887). *F. contrayerba,*

Sch. Bip. acc. to Klatt, l. c. — OAXACA: *Galeotti*, 2122, *Andrieux*, 345; Tehuacan, *Liebmann*, 267 (drawing in hb. Gr.), *Schmitz*, 1027 (hb. Gr.); Coixlahuaca, *Nelson*, 1935, alt. 2,000 to 2,500 m., 1894 (hb. Gr.). PUEBLA: Puebla, *Smith*, 912, alt. 2,000 m., 1895 (hb. Gr.), *Pringle*, 4749, fields, 1894 (hb. Gr., hb. J. D. S., hb. M. B. G., hb. U. S.); Chapultepec, *Schaffner*, 16 (hb. Gr.); Hort. Mex., *Berlandier*, 640 (hb. Gr.).

b. b. b. Leaves remotely denticulate, narrow at base; inflorescence naked, not compact; upper stem pubescent, slender; outer bracts usually conspicuous.

6. **F. intermedia**, nov. sp. Stem about 30 cm. high, slender, erect, angled, purplish, pubescent, dichotomously branching, primary branches in the two specimens all simple, terminated by a loose corymbiform glomerule resembling that of *F. campestris:* leaves minutely serrate, 1.2 to 5 cm. long, .2 to .4 cm. wide, usually opposite, sometimes whorled in axils: heads few-flowered, bracts minute to 3 mm. long: ♀ achene $\frac{1}{3}$ larger than ☿ achene. — DURANGO: Plains near Yermo, *Pringle*, 7359, Oct. 1896 (hb. U. S., hb. Gr.).

This species differs from *F. linearis* in having only three bracts, in possessing a ♀ achene much larger than the achene of ☿ flower, and in form of leaves. It differs from *F. campestris* in its very slender stem, in shape of leaves, and in size of flowers; and from *F. angustifolia* in its slender stem, comparatively loose inflorescence, and size of floral organs.

b. b. b. b. Leaves remotely denticulate, narrowing to a petiole-like base; stem pubescent; outer bracts minute or absent.

7. F. ROBUSTA, Rose. About 1.2 m. high, pubescent or glabrate below: leaves lanceolate or linear above, 4.5 to 13 cm. long, acute to acuminate, tapering into a slender petiole, 3-nerved, entire or slightly serrate: inflorescence open, corymbose; heads small, with 3 involucral bracts: flowers 3; ray 1, orbicular, about .2 cm. long; disk flowers 2: achenes .15 cm. long. — Rose, Cont. Nat. Herb. i. 337 (1895). — MEXICO: Colima, *Palmer*, 1299, Feb. 27–28, 1891 (hb. Gr., hb. J. D. S., hb. M. B. G., hb. U. S.); Armeria, *Marcus E. Jones*, 276, June 28, 1892 (hb. Gr., hb. U. S.); Chihuahua, Batopilas, *E. A. Goldman*, 240, Oct. 1898 (hb. Gr., hb. U. S.).

a. a. Leaves linear, bracts 5.

b. Stem strict.

8. F. LINEARIS, Lag. Stem shrubby, erect or more commonly sprawling, branching indefinite, branches of unequal length; lower internodes short, made conspicuous by the remnants of the leaves, striate, glab-

rate: leaves linear, connate to connate-vaginate, the lower ones breaking away, typically opposite, but this character becomes inconspicuous by the presence of numerous leaves whorled in the axils of the opposite ones, entire, from 2 cm. to 10 cm. long, and from .1 to .4 cm. wide: inflorescence irregular, consisting of small loose glomerules or larger compact aggregations; involucral bracts equal, enclosing 3 to 8 flowers, usually one ligulate; outside bracts small; ligule of ♀ oval, almost equalling the long slender tube: achenes about equal, ♀ somewhat stouter than ☿. — Lag. Gen. et Sp. Nov. 33, n. 40 (1816). *F. maritima*, HBK. Nov. Gen. et Sp. iv. 285. *F. tenuifolia*, Nutt. in Journ. Acad. Phil. N. S. vii. 81. *Selloa nudata*, Nutt. in Am. Journ. Sc. v. 300 (1822). *Gymnosperma nudata*, DC. Prod. v. 312. — FLORIDA: Biscayne Bay, *Palmer*, 292, 1874 (hb. Gr., hb. U. S.); Miami, *Garber*, May, 1877 (hb. Gr., hb. U. S.); New Found Harbor Key, *Pollard*, *Collins*, and *Morris*, 79, Mar. 1898 (hb. U. S.); No Name Key, *J. H. Simpson*, 185, May, 1891 (hb. U. S.); Key West, *Blodgett* (hb. Gr.). CUBA: Toscano seashore, *Wright*, 2859, 1860–64 (hb. Engel., hb. U. S., hb. Gr.); Mariel, Province of Pinar del Rio among coral rocks near sea, *Wm. Palmer* and *J. H. Riley*, 713, May, 1900, and 811, June, 1900 (hb. U. S.). BAHAMAS: Red Bays, Andros, *John I.* and *Alice R. Northrop*, 462, Apr. 1900 (hb. Gr.). MEXICO: *Galeotti*, 23 (hb. Gr.). YUCATAN: *G. F. Gaumer* 1141 (hb. Gr.); Holbox Is., Bay of Honduras, *Gaumer*, 1886 (hb. Gr.)

Var. **latifolia**, nov. var. Stem herbaceous, erect, greenish or rubescent, slender, striate, glabrate; lower internodes not conspicuously shorter than the upper ones: leaves opposite, linear to lanceolate, conspicuously narrowed above the expanded connate base; whorls of leaves inconspicuous; leaves entire or denticulate, 2.5 cm. to 10.5 cm. long, and .4 cm. to 3 cm. wide: heads gathered in rather small glomerules which are arranged in an open quite regular corymb; heads similar to type of the species. Although this variety is very distinct in habit from the type of the species, there are intermediate and dubious forms. — FLORIDA: shore of Lake Worth near Palm Beach, *Curtiss*, 5524, Aug. 1895 (hb. M. B. G., hb. U. S); Key West, *Palmer*, 292; Sneed's Is., *Tracy*, 6356, Sept. 1899 (hb. Gr., hb. U. S., hb. M. B. G.); shore of Indian River, *Curtiss*, 1504 (hb. Gr., hb. M. B. G.); Titusville, Breward Co., *G. V. Nash*, 2301, Jul.–Aug. 1895 (hb. M. B. G., hb. U. S., hb. Gr.). YUCATAN: Cozumel Is., *Gaumer*, Aug. 1885 (hb. Gr.).

b. b. Stem branching copiously.

9. F. RAMOSISSIMA, Klatt. Stem purplish, terete, striate, glabrate,

branching; branches diffuse, fastigiate-corymbose, leafy; upper stem somewhat villous, leaves linear-lanceolate, acute, remotely dentate, 1-nerved, 2.5 cm. long, .2 cm. wide, base connate-vaginate: peduncles and pedicels winged: heads .3 cm. long, with a bracteate base, 1-ligulate, 5-flowered: ♀ achene 1.5 mm. long, ☿ 1 mm. long. — Klatt, Leopold. xxiii. 146 (1887); *F. angustifolia*, Sch. Bip. non Pers. var. *ramosissima*, Klatt, l. c. — Mexico: Tehuacan, *Pringle*, 6756, Aug. 1897, alt. 1500 m. (hb. Gr., hb. U. S.); Puebla near Tehuacan, *Pringle*, 7031, 1895 (hb. U. S., hb. Gr.); *Liebmann*, 456 and 457 (hb. Gr.).

++ ++ Bracts swollen.

10. F. anomala, Rob. A glabrous annual, a span high: stem striate, angulate, much branched: leaves linear or lanceolate-linear, gradually narrowed to a slightly connate base, acute, 4 cm. to 5 cm. long, .3 cm. to .5 cm. wide: heads numerous, aggregated at the ends of the branches in dense corymbs, very small, subtended by minute dark-tipped bracts, 1-(rarely 2-) flowered; the single flower either tubular or ligulate: involucral scales of unequal breadth, lanceolate-linear or oblong, more or less narrowed but obtuse at the apex, persisting in fruit, and becoming swollen and bulbous on the back near the base; corollas bright yellow; in the tubular flowers the campanulate throat and spreading limb equalling the tube, the latter hairy on the outer surface; the ligules .2 cm. or less in length: achenes about equal. — Rob. Proc. Am. Acad. xxvii. 178 (1892). — Mexico: San Luis Potosi, Sept. 1890, *Pringle*, 3669; Plains of Venegas, *Pringle*, 5367, Nov. 1892 (hb. U. S.); *Parry*, in 1878, North Mexico, 31, and in 1878 en route from San Luis Potosi to San Antonio, Texas, 500 (hb. Gr., hb. U. S.).

+ + Perennial.

++ Inflorescence much branched; glomerules of few heads.

11. **F. Palmeri**, nov. sp. Thickened perennial root, giving rise to 1 to several more or less reclining stems; stem pale, branching copiously, glabrous: leaves linear-lanceolate, sparingly denticulate, slightly connate, 1 to 3.5 cm. long: heads in rather small cymes; cymes in compound corymbiform panicles: involucral bracts 3, equal, slightly keeled, .5 cm. long, the 1–2 outside bracts minute: lamina ♀ oval, .3 cm. long, exceeding slightly the achene; corolla tube villous. — Mexico: San Lorenzo de Laguna, 93–114 kms. southwest of Parras, Coahuila, *Palmer*, 684, May, 1880 (hb. Gr., hb. U. S.).

++ ++ Inflorescence densely corymbose, terminal on long stalks.

12. F. VAGINATA, Rob. & Greenm. Perennial with stout lignescent root: stems several, ascending from a decumbent or even prostrate somewhat branched base, terete, striate, purplish, with biliniate short grayish woolly pubescence, leafy above, naked below except for the persistent and sheathing bases of the fallen leaves: internodes very short: leaves linear-lanceolate, clasping at the base, very gradually attenuate, often fascicled in the axils, 1(–3)-nerved, rather pale green, finely ciliated toward the base: heads small, closely aggregated into terminal solitary or corymbose-paniculate glomerules; these simulating the normal involucrate heads of the order: glomerules .12 to .16 cm. in breadth, subtended by a few short recurved foliaceous bracts, and containing 30 or more heads: involucral scales 3 to 4 in each head, hyaline: ray-flower solitary, conspicuous, .5 cm. long, with oblong slightly 2 to 3 toothed yellow ligule: disc flowers 5 to 7, yellow: style of ♀ 3-cleft: achenes black, lucid, about 10-nerved, ♀ 2.25 mm. long, ☿ achene 1.75 mm. long. — Rob. & Greenm. Proc. Am. Acad. xxxii. 48 (1896). *E. W. Nelson,* 1933, between Coixlahuaca and Tamazulapam, Oaxaca, alt. 2,000 to 2,500 m. Nov. 1894.

* * Heads 10- to 15-flowered.

+ Leaves linear to lanceolate, slightly connate.

++ Ligulate flowers not present, inflorescence usually naked.

13. F. LONGIFOLIA, Gray. Rather stout, 3 to 9 dm. high, pale: leaves glabrous, broadest or not narrowed at the closely sessile base, 5 to 12.5 cm. long, entire or with rare spinulose denticulations: heads in regular very ample cymes, which are often destitute of leaves. — Gray, Pl. Fendl. 88. *Gymnosperma oppositifolia,* DC. Prod. v. 312. — MEXICO: San Luis Potosi, *Parry* and *Palmer,* 498, 1878 (hb. Gr., hb. Engel., hb. U. S.), also *Pringle,* 3767, July, 1891 (hb. Gr.); Parras, Coahuila, *Palmer,* 685, 1880 (hb. Gr., hb. J. D. S., hb. U. S.); Saltillo, Coahuila, *Palmer,* 304, Sept. 1898 (hb. Gr.); also *Palmer,* 681, 1880 (hb. Gr., hb. J. D. S., hb. U. S.); Tehuacan, *Liebmann* acc. Hemsl. l. c.; *Berlandier,* without locality (hb. Gr.); Cienega Grande, *Gregg,* 705, May, 1847 (hb. Engel.); Tamaulipas, Jaumam Valley, *E. W. Nelson,* 4450, June, 1898 (hb. U. S., hb. Gr.).

++ ++ Ligulate flowers present, inflorescence usually subtending a small whorl of leaves.

14. F. **floridana,** nov. sp. Herbaceous, erect or ascending, branching dichotomous, branches often unequal in length, striate, glabrate:

leaves opposite, linear to lanceolate, base narrowing somewhat above the expanded often connate portion, rarely denticulate, 2.5 cm. to 7.5 cm. long, .3 cm. to 1.25 cm. wide: inflorescence irregular in form; glomerules fairly compact, subtending a whorl of leaves which are usually broad and conspicuous: involucral bracts subequal; head 10- to 13-flowered, sessile, the ray flower has a large oval ligule equalling in length the slender corolla tube: achenes of ♀ and of ☿ about equal, that of the ♀ usually stouter: most nearly allied to *F. linearis*, Lag., from which it differs mainly in the number of flowers in a head, in possessing a whorl of leaves about the glomerules, and in habit; and to *F. longifolia*, Gray, from which it differs in the possession of ligulate flowers, and the whorl of leaves about the glomerule, and in habit. — FLORIDA: Sonibel Is., *H. J. Webber*, 175, Jan. 1896 (hb. U. S.); Manatee, *J. H. Simpson*, 1889 (hb. U. S.); Hog Is., *S. M. Tracy*, 7341, Nov. 1901 (hb. Gr., hb. M. B. G.).

+ + Leaves ovate, conspicuously perfoliate.

15. F. CHLORAEFOLIA, Gray. Stem glaucous, striate, 3 to 9 dm. high: leaves ovate-oblong to narrowly lanceolate, broadest and connate or connate-perfoliate at base, glabrous, 2.5 to 9 cm. long, 1.2 to 3.5 cm. wide: heads more or less clustered in a broad and open naked pedunculate compound terminal corymbiform cyme: heads 11- to 13-flowered, involucral bracts 5, the 2 smaller and outer bracts quite distinct; ray flowers 0, disk flowers .6 cm. long; 2 to 3 small pales on most achenes (4 acc. Gray). — Gray, Pl. Fendl. 88 (1849), & Syn. Fl. ii. pt. 2, 353. — TEXAS: Screw Bean, *G. C. Nealley*, 688, 1889 (hb. U. S.); on the banks of the Rio Grande, *Wright*, 357 (hb. Gr.); below Doñana, valley of Rio Grande, *Parry*, *Bigelow*, *Wright*, and *Schott*, 592 (hb. U. S.). MEXICO: Coahuila, *Palmer*, 682, 1880 (hb. Gr., hb. J. D. S., hb. M. B. G., hb. U. S.), and 2083, 1897 (hb. Gr., hb. J. D. S., hb. M. B. G.).

DOUBTFUL AND EXCLUDED SPECIES.

F. HUMILLIMA, Sch. Bip. in Linnaea, xxxiv. 529 (1865–66), without description.

F. SPICATA, Juss. mss. acc. Smith in Rees, Cyclopedia, xv. Collected by Dombey in Peru. Acc. to Hook. f. & Jacks., in Index Kewensis, it is *Piqueria artemisioides*.

F. PERUVIANA, Juss. acc. DC. Prod. v. 635, is *Piqueria artemisioides*.

F. PERFOLIATA, Klatt, Leopold. xxiii. 146 (1887), is *Desmanthodium perfoliatum*.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY,
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ON THE LINES OF CERTAIN CLASSES OF SOLENOIDAL OR LAMELLAR VECTORS, SYMMETRICAL WITH RESPECT TO AN AXIS.

By B. O. Peirce.

ON THE LINES OF CERTAIN CLASSES OF SOLENOIDAL OR LAMELLAR VECTORS, SYMMETRICAL WITH RESPECT TO AN AXIS.

BY B. O. PEIRCE.

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THE lines of force due to a homogeneous body bounded by a surface of revolution are curves, each of which lies in a plane passing through the fixed axis of symmetry of the body. The force in such a case is an example of a vector "symmetrical with respect to a straight line," and it is evident that the whole field of a vector of this kind may be studied by examining the lines of the vector in any plane which passes through the axis. If we represent by R, Φ, X the components of a vector taken in the directions in which the columnar coördinates r, ϕ, x increase most rapidly, Φ is everywhere zero if the vector is symmetrical about the x axis.

If $u \equiv f_1(x, y) = k$ is the equation of a family of curves of one parameter in the $x\,y$ plane, which are lines of a vector symmetrical with respect to the x axis, we may regard $f_1(x, r) = k$, $\phi = m$ as the equations of all the lines of the vector, and the components of the vector satisfy the equations

$$\Phi = 0, \qquad R \cdot \frac{\partial f_1}{\partial r} + X \cdot \frac{\partial f_1}{\partial x} = 0. \tag{1}$$

Given any family of curves, $u = k$, in the $x\,y$ plane, without multiple points or points of intersection with each other or with the x axis, it is possible to form an infinite number of vectors symmetrical about the x axis which shall have the u curves which lie on one side of it, as lines; for we may choose either R or X wholly at pleasure and determine the other by means of equation (1). It is evident that if two such vectors, $[R_1, 0, X_1]$, $[R_2, 0, X_2]$, have the same lines, $R_1 / R_2 = X_1 / X_2$.

If $v \equiv f_2(x, y) = l$ represents a family of curves in the $x\,y$ plane orthogonal to the family $u = k$, so that

$$\frac{\partial f_1}{\partial x} \cdot \frac{\partial f_2}{\partial x} + \frac{\partial f_1}{\partial y} \cdot \frac{\partial f_2}{\partial y} = 0, \tag{2}$$

and if $F(v)$ is any single-valued, differentiable function of v, a vector the components of which are

$$R \equiv F'(v) \cdot \frac{\partial f_2(x, r)}{\partial r}, \; \Phi \equiv 0, \; X \equiv F'(v) \cdot \frac{\partial f_2(x, r)}{\partial x} \tag{3}$$

has the u curves as lines, and it is lamellar, for

$$\frac{\partial R}{\partial x} = \frac{\partial X}{\partial r}.$$

Of all the vectors symmetrical about the x axis which have the given u curves as lines, an infinite number are lamellar.

Since the divergence of a vector symmetrical about the x axis is equal to

$$\frac{\partial (r \cdot R)}{r \cdot \partial r} + \frac{\partial X}{\partial x}, \tag{4}$$

it is evident that if $[R, 0, X]$ is a solenoidal vector which has the u curves as lines, $[R \cdot F(u), 0, X \cdot F(u)]$, where F is any single-valued, differentiable function, is another solenoidal vector which has the same lines. If two solenoidal vectors, $[R_1, 0, X_1]$, $[R_2, 0, X_2]$, symmetrical about the x axis have the u curves as lines, the ratio R_1 / R_2, or X_1 / X_2 is a function of u only.

Whatever u is, the vector which has the components

$$-\frac{W}{h_u} \cdot \frac{\partial u}{\partial x}, \; 0, \; +\frac{W}{h_u} \cdot \frac{\partial u}{\partial r}, \tag{5}$$

or the components

$$\frac{W}{h_v} \cdot \frac{\partial v}{\partial r}, \; 0, \; \frac{W}{h_v} \cdot \frac{\partial v}{\partial x}, \tag{6}$$

— where W is a single-valued function of the space coördinates and h_u, h_v are the gradients of the functions u and v, — has the u curves as lines. The tensor of this vector involves u alone or v alone, according as W is expressible in terms of u alone or in terms of v alone.

It is to be remembered that the field of a physical vector may be a restricted region in space, so that a family of u curves which have double points, or points of intersection with each other, or with the x axis, may still be lines of a vector symmetrical with respect to the x axis if the field of the vector is free from such singular points.

If a set of orthogonal curvilinear coördinates in the plane $\phi = 0$ be defined by the functions, $u \equiv f_1(x, r)$, $v \equiv f_2(x, r)$, and if

$$U \equiv \xi(x, r),\ \ V \equiv \eta(x, r),\ \ \Phi = 0, \tag{7}$$

represent the magnitudes, at the point (x, r) of the components, taken in the directions in which u, v, ϕ increase most rapidly, of a vector Q, symmetrical with respect to the x axis, it is easy to prove that the divergence of Q is given by the expressions

$$\frac{h_u \cdot h_v}{r}\left\{\frac{\partial}{\partial u}\left(\frac{r \cdot U}{h_v}\right) + \frac{\partial}{\partial v}\left(\frac{r \cdot V}{h_u}\right)\right\}, \tag{8}$$

$$h_u^2 \cdot \frac{\partial}{\partial u}\left(\frac{U}{h_u}\right) + L(u) \cdot \left(\frac{U}{h_u}\right) + h_v^2 \cdot \frac{\partial}{\partial v}\left(\frac{V}{h_v}\right) + L(v) \cdot \left(\frac{V}{h_v}\right), \tag{9}$$

$$\text{where } L(u) \equiv \frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{r \cdot \partial r} + \frac{\partial^2 u}{\partial x^2} \text{ and } h_u^2 \equiv \left(\frac{\partial u}{\partial r}\right)^2 + \left(\frac{\partial u}{\partial x}\right)^2. \tag{10}$$

The components, (K_u, K_v, K_ϕ) of the curl of Q are

$$0,\ 0, \text{ and } h_u \cdot h_v \left\{\frac{\partial}{\partial u}\left(\frac{V}{h_v}\right) - \frac{\partial}{\partial v}\left(\frac{U}{h_u}\right)\right\}. \tag{11}$$

It is to be noticed that

$$\frac{\partial r}{\partial u} = \frac{1}{h_u^2} \cdot \frac{\partial u}{\partial r}, \qquad h_u \cdot \frac{\partial h_u}{\partial r} = \frac{\partial u}{\partial r} \cdot \frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{\partial x} \cdot \frac{\partial^2 u}{\partial r \cdot \partial x},$$

$$h_u^2 \cdot \frac{\partial h_u}{\partial u} = \frac{\partial h_u}{\partial r} \cdot \frac{\partial u}{\partial r} + \frac{\partial h_u}{\partial x} \cdot \frac{\partial u}{\partial x}, \qquad \frac{L(u)}{h_u^2} = \frac{\partial}{\partial u} \log\left(\frac{r \,.\, h_u}{h_v}\right),$$

$$\text{and } \frac{L(v)}{h_v^2} = \frac{\partial}{\partial v} \log\left(\frac{r \,.\, h_v}{h_u}\right). \tag{12}$$

If the lines of Q in the plane $\phi = 0$ coincide with the u curves, the vector has no component perpendicular to these curves, and U is everywhere zero, so that

$$\text{Divergence } Q = h_v^2 \,.\, \frac{\partial}{\partial v}\left(\frac{V}{h_v}\right) + L(v)\left(\frac{V}{h_v}\right), \tag{13}$$

$$= \frac{h_u \cdot h_v}{r} \cdot \frac{\partial}{\partial v}\left(\frac{r V}{h_u}\right), \tag{14}$$

$$\text{Tensor curl } Q = h_u \cdot h_v \cdot \frac{\partial}{\partial u}\left(\frac{V}{h_v}\right). \tag{15}$$

If P_u, P_v, P_ϕ are the components of a vector P taken in the directions in which u, v, ϕ increase most rapidly, the components of the curl of P are

$$\Pi_u = \frac{h_v}{r}\left\{\frac{\partial}{\partial v}(r \cdot P_\phi) - \frac{\partial}{\partial \phi}\left(\frac{P_v}{h_v}\right)\right\}, \tag{16}$$

$$\Pi_v = \frac{h_u}{r}\left\{\frac{\partial}{\partial \phi}\left(\frac{P_u}{h_u}\right) - \frac{\partial}{\partial u}(r \cdot P_\phi)\right\}, \tag{17}$$

$$\Pi_w = h_u \cdot h_v \left\{\frac{\partial}{\partial u}\left(\frac{P_v}{h_v}\right) - \frac{\partial}{\partial v}\left(\frac{P_u}{h_u}\right)\right\}; \tag{18}$$

and, if P is to be a vector potential function of a given solenoidal vector, $Q = [0,\ V,\ 0]$, which has the u curves as lines and is symmetrical about the x axis, we may assume that P_u and P_v involve u and v only, and write

$$P_\phi = \frac{f(u)}{r}, \tag{19}$$

where

$$V = -\frac{h_u}{r} \cdot \frac{\partial}{\partial u} f(u). \tag{20}$$

Any vector of the form $[P_u, P_v, f(u)/r]$, where P_u and P_v are any functions of u and v subject only to the condition

$$\frac{\partial}{\partial u}\left(\frac{P_v}{h_v}\right) = \frac{\partial}{\partial v}\left(\frac{P_u}{h_u}\right), \tag{21}$$

is a vector potential function of a solenoidal vector, symmetrical about the x axis, which has the u curves for lines; and there is no vector of the kind last mentioned which does not have as a potential function a vector P of the form given. It is usually convenient to make $P_u = P_v = 0$.

It is easy to see from the foregoing equations that the statements which follow are true.

(*a*) If Q is to be solenoidal, $r \cdot V/h_u$ must be either constant or a function of u only; that is,

$$V = \frac{h_u \cdot F(u)}{r}, \tag{22}$$

where $F(u)$ is some single-valued function: if V is of this form it is solenoidal. It follows from this that of all the vectors symmetrical about the x axis which have the u curves as lines an infinite number are solenoidal. If the u curves are straight lines parallel to the x axis, the tensor of Q is some function of r or else constant.

(b) If Q is to be solenoidal and if its tensor is to be either constant or expressible in terms of u, the gradient of the function u must satisfy an equation of the form

$$h_u = r \cdot F(u). \tag{23}$$

If for the function u in this equation we substitute a new function w defined by the equation

$$w = \int \frac{du}{F(u)},$$

we shall get the simpler equation

$$h_w = r. \tag{24}$$

It is to be noticed that w is constant on any line of constant u, and that (23) and (24) may be said to define the same curves.

(c) If Q is to be solenoidal and if its tensor is to be expressible in terms of v only, h_u must be of the form

$$h_u = r \cdot F(u) \cdot \psi(v), \tag{25}$$

and if for u we substitute w in the manner indicated in (b) we shall obtain the equation

$$h_w = r \cdot \psi(v), \tag{26}$$

or

$$\frac{\partial}{\partial w}\left(\frac{h_w}{r}\right) = 0. \tag{27}$$

(d) If a solenoidal vector symmetrical about the x axis has the u curves for lines, its curl is of the form

$$\left[0,\ 0,\ h_u \cdot h_v \cdot \frac{\partial}{\partial u}\left(\frac{h_u \cdot F(u)}{r \cdot h_v}\right)\right] \quad \text{or}$$

$$\left[0,\ 0,\ r \cdot h_u^2 \cdot \frac{\partial}{\partial u}\left(\frac{F(u)}{r^2}\right) + \frac{L(u)}{r} \cdot F(u)\right]. \tag{28}$$

If, for instance, the lines of such a vector are straight lines parallel to the x axis, its curl is either constant or a function of r alone.

(e) If Q is to be lamellar, it must be of the form

$$V = h_v \cdot F(v). \tag{29}$$

Any vector of the form $[0, h_v \cdot F(v), 0]$ is lamellar.

(*f*) If Q is to be lamellar and if its tensor is to involve v only, we must have

$$h_v = \psi(v), \tag{30}$$

and the v curves must form a system of parallel, curved or straight, lines (for instance, a set of concentric circumferences); the u curves are, therefore, straight lines.

If for v in (30) we substitute a new function z such that

$$z \equiv \int \frac{dv}{\psi(v)},$$

we shall get the new equation

$$h_z = 1. \tag{31}$$

(*g*) If Q is to be lamellar while its tensor involves u only, we must have

$$h_v = F(u) \cdot \psi(v), \tag{32}$$

and the substitution used in (*f*) leads to the condition

$$h_z = F(u). \tag{33}$$

We may consider that (32) and (33) define the same systems of curves. By making use of the so-called "Principle of Duality," and putting

$$m = -2 \log (x^2 + r^2), \qquad n = \tan^{-1} (r/x), \tag{34}$$

it is possible* to reduce (33) to the equivalent of Fourier's equation for the linear flow of heat.

If the v curves were a family of straight lines emanating from some fixed point (x_0, r_0) the equations of these lines might be written in the form

$$v = \frac{r - r_0}{x - x_0},$$

and the equation of the orthogonal curves in the form

$$u^2 = (x - x_0)^2 + (r - r_0)^2:$$

in this case we should have

$$h_v = (1 + v^2)/u,$$

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* Peirce, These Proceedings, **38**, p. 663.

so that (32) would be satisfied. If the equation of the same family of straight lines had been written in the form

$$v = \tan^{-1}\left(\frac{r - r_0}{x - x_0}\right)$$

we should have had

$$h_v = 1 / u,$$

so that v would have satisfied an equation of the form (33).

(h) If Q is lamellar, and if Ω is a scalar potential function of Q, Ω must be expressible in terms of v, and the divergence of Q is equal to

$$h_v^2 \cdot \frac{d^2\Omega}{dv^2} + L(v) \cdot \frac{d\Omega}{dv}, \tag{35}$$

(i) If the tensor of Q has the same value at all points of the $x\,r$ plane Q is lamellar if, and only if, h_v is constant or a function of v alone.

(j) If the tensor of a vector, Q, which has the u curves as lines, is a function of u only, its divergence is

$$\frac{h_u \cdot h_v \cdot V}{r} \frac{\partial}{\partial v}\left(\frac{r}{h_u}\right). \tag{36}$$

(k) If the tensor of Q is expressible in terms of v, the tensor of its curl is

$$h_u \cdot h_v \cdot V \cdot \frac{\partial}{\partial u}\left(\frac{1}{h_v}\right). \tag{37}$$

If, for instance, the u curves are a family of straight lines, the tensor of the curl of such a vector must be zero.

(l) If Q is to be solenoidal as well as lamellar, equations (13) and (15) yield Lamé's well-known condition that $L(v)/h_v^2$ must be expressible in terms of v alone, so that

$$\frac{\partial}{\partial u}\left(\frac{L(v)}{h_v^2}\right) = 0. \tag{38}$$

(m) A vector symmetrical about the x axis and directed everywhere parallel to it, is solenoidal only when its tensor has the same value throughout every one of its lines.

Systems of Straight Lines and Circles in the $x\,r$ Plane which satisfy Lamé's Condition.

If a set of curves $(u = k)$ in the $r\,x$ plane yield when revolved about the x axis a set of isothermal * surfaces, the function

$$\frac{L(u)}{h_u^2} \tag{39}$$

must be expressible as a function of u alone. The families of curves which satisfy this condition are generally, of course, quite different from those which satisfy the condition

$$\frac{\nabla^2(u)}{h_u^2}, \text{ a function of } u, \tag{40}$$

for isothermal lines in the plane. A set of confocal conics with foci on the x axis would, however, satisfy † both conditions.

To determine what systems of straight lines in the $x\,y$ plane satisfy (39) we may represent any such system by the equation $\alpha\,x + \beta\,y = 1$, where α and β are functions of a single parameter u, and write

$$\alpha' \equiv \frac{d\alpha}{du}, \qquad \beta' \equiv \frac{d\beta}{du}.$$

$$\frac{\partial u}{\partial x} = \frac{-\,\alpha}{\alpha'\,x + \beta'\,y}, \qquad \frac{\partial u}{\partial y} = \frac{-\,\beta}{\alpha'\,x + \beta'\,y}, \qquad h_u^2 = \frac{\alpha^2 + \beta^2}{(\alpha'\,x + \beta'\,y)^2},$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{2\,\alpha\,\alpha'}{(\alpha'\,x + \beta'\,y)^2} - \frac{\alpha^2\,(\alpha''\,x + \beta''\,y)}{(\alpha'\,x + \beta'\,y)^3},$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{2\,\beta\,\beta'}{(\alpha'\,x + \beta'\,y)^2} - \frac{\beta^2\,(\alpha''\,x + \beta''\,y)}{(\alpha'\,x + \beta'\,y)^3},$$

$$\frac{L(u)}{h_u^2} = \frac{2\,(\alpha\,\alpha' + \beta\,\beta')}{\alpha^2 + \beta^2} - \frac{\alpha''\,x + \beta''\,y}{\alpha'\,x + \beta'\,y} - \frac{\alpha\,(\alpha'\,x + \beta'\,y)}{y\,(\alpha^2 + \beta^2)}. \tag{41}$$

In order that the sum of the last two terms of (41) may be expressible in terms of u only, it is necessary that α' shall be zero, so that α is only a constant, and the equation of any system of straight lines which satisfy (39) is of the form

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* Lamé, Leçons sur les coordonnées curvilignes, p. 32; Leçons sur les fonctions inverses, p. 5; Somoff-Ziwet, Theoretische Mechanik, I. 113 and 138.

† Peirce, American Journal of Mathematics, 1896.

$$\frac{1 - c\,x}{y} = \text{parameter}.$$

All the lines of such a system pass through a fixed point $(1/c,\ 0)$ on the x axis.

Again: let

$$x^2 + y^2 - 2\,\alpha\,x - 2\,\beta\,y - \gamma = 0 \tag{42}$$

where α, β, γ are functions of a single parameter u, represent a family of circles in the $x\,y$ plane so that

$$\frac{\partial u}{\partial x} = \frac{2\,(x - \alpha)}{2\,\alpha'\,x + 2\,\beta'\,y + \gamma'}, \qquad \frac{\partial u}{\partial y} = \frac{2\,(y - \beta)}{2\,\alpha'\,x + 2\,\beta'\,y + \gamma'},$$

$$h_u^{\,2} = \frac{4\,(\alpha^2 + \beta^2 + \gamma)}{(2\,\alpha'\,x + 2\,\beta'\,y + \gamma')^2}.$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{2}{2\,\alpha' x + 2\,\beta'\,y + \gamma'} - \frac{8\,\alpha'\,(x - \alpha)}{(2\,\alpha'\,x + 2\,\beta'\,y + \gamma')^2} - \frac{4\,(x - \alpha)^2\,(2\,\alpha''\,x + 2\,\beta''\,y + \gamma'')}{(2\,\alpha'\,x + 2\,\beta'\,y + \gamma')^3}.$$

$$\frac{L\,(u)}{h_u^{\,2}} = \frac{2\,\alpha\,\alpha' + 2\,\beta\,\beta' + \gamma'}{\alpha^2 + \beta^2 + \gamma} - \frac{2\,\alpha''\,x + 2\,\beta''\,y + \gamma''}{2\,\alpha'\,x + 2\,\beta'\,y + \gamma'} + \frac{(y - \beta)\,(2\,\alpha'\,x + 2\,\beta'\,y + \gamma')}{4\,y\,(\alpha^2 + \beta^2 + \gamma)}. \tag{43}$$

The first term in the second number of (43) is already expressed as a function of u: the sum of the last two terms is a function of u if, and only if, $\alpha' = 0$, $\beta = 0$, so that α is a constant (c) and (42) takes the form

$$(x - a)^2 + y^2 = u. \tag{44}$$

This represents a set of concentric circles with centre at same point on the x axis.

THE EQUATION $h_u = y$.

If the tensor of a solenoidal vector, symmetrical about the x axis, is to have at every point of each of its lines a value constant for that line, the tensor is a function of the parameter of the lines and the equation of the family must be found among the solutions of equation (24).

If u is a differentiable function of x and y, the partial derivations of which with respect to these variables are p and q respectively, we may use the usual notation and rewrite (24) in the form

$$p^2 = y^2 - q^2. \tag{45}$$

Equating each side* to a constant, (a^2), we get

$$u = \pm\, a\, x + \tau\,(y) = \tfrac{1}{2}\, y\, \sqrt{y^2 - a^2} - \tfrac{1}{2}\, a^2 \,.\, \log\,(y + \sqrt{y^2 - a^2}) + \chi\,(x),$$

so that the complete integral of (45) is

$$u = \pm\, a\, x + \tfrac{1}{2}\, y\, \sqrt{y^2 - a^2} - \tfrac{1}{2}\, a^2 \,.\, \log\,(y + \sqrt{y^2 - a^2}) + b, \tag{46}$$

and the general integral can be formed from this directly.

A special solution gives straight lines parallel to the x axis as possible lines of a solenoidal symmetrical vector the tensor of which is a function only of the distance from the axis of symmetry. No solenoidal symmetrical vector the tensor of which involved x only could have these lines.

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* Forsyth, Differential Equations, p. 310.

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ON THE REAL AUTOMORPHIC LINEAR TRANSFORMATION OF A REAL BILINEAR FORM.

By Henry Taber.

ON THE REAL AUTOMORPHIC LINEAR TRANSFORMATION OF A REAL BILINEAR FORM.

By Henry Taber.

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§ 1.

Let

$$\mathfrak{F} \equiv (A \between x_1, x_2, \ldots x_n \between y_1, y_2, \ldots y_n)$$

denote a bilinear form $\underset{1}{\Sigma_r^n}\,\underset{1}{\Sigma_s^n}\, a_{rs}\, x_r\, y_s$ of non-zero determinant. Let the x's and y's be transformed respectively by the linear homogeneous transformations T and T_1, so that

$$(x_1', x_2', \ldots x_n') = (T \between x_1, x_2, \ldots x_n)\,,$$
$$(y_1', y_2', \ldots y_n') = (T_1 \between x_1, x_2, \ldots x_n)\,.$$

It will be assumed as the result of these substitutions that $\mathfrak{F}$ is transformed automorphically, so that

$$(A \between x_1', x_2', \ldots x_n') = (A \between x_1, x_2, \ldots x_n)\,,$$

for which the necessary and sufficient condition is

$$\check{T}_1\ A\ T = A.^*$$

I shall denote the family of transformations T of the x's by

1st, Γ', when the x's and y's are contragredient, in which case we have $T_1 = \check{T}^{-1}$;

2d, by Γ'', when the respective transformations of the x's and of the y's are conjugate, so that $T_1 = \check{T}$;

3d, by Γ''', when the product of the respective transformations of the x's and of the y's is the identical transformation, so that $T_1 = T^{-1}$. The conditions necessary and sufficient that T shall be a transformation of Γ', Γ'', or Γ''', respectively, are then

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* Cayley: Phil. Trans., 1858. Throughout this paper $\check{T}$ will denote the transverse or conjugate of the linear transformation T.

$$(1)\quad T^{-1} A\, T = A,$$

$$(2)\quad T A\, T = A,$$

$$(3)\quad \breve{T}^{-1} A\, T = A.$$

The transformations of Γ' constitute a group, as do also the transformations of Γ'''; the transformations of Γ'' that are commutative form a group.

In a paper entitled "Note on the Automorphic Linear Transformation of a Bilinear Form,"* I have shown that each transformation T of the family Γ' belongs to a group with a single parameter of transformations of Γ', and can thus be generated by an infinitesimal transformation of Γ'; and that a similar theorem holds for the family Γ'''. But on the other hand, for certain forms $\mathfrak{F}$, that Γ'' contains transformations (of determinant $+1$ as well as of determinant -1) that cannot be generated by an infinitesimal transformation of Γ''.† In this paper I consider only real forms $\mathfrak{F}$, and only real transformations of the variables x and y. Thus in what follows the matrix A and the families Γ', Γ'', Γ''' of transformations T will be assumed to be real. And with this restriction to real forms and families of real transformations I show severally that each of the families Γ', Γ'', Γ''', for certain forms $\mathfrak{F}$, contains transformations (of positive as well as of negative determinant) that cannot be generated by infinitesimal transformations of that family.‡ A transformation of either of the families Γ which is the second power of a transformation of that family I term a transformation of the *first kind*, otherwise a transformation of the *second kind;* and, for each of the families Γ, I show that every transformation of the first kind, but no transformation of the second kind, can be generated by an infinitesimal transformation of that family. I also show that a transformation of either of the families Γ is a transformation of the first kind when no negative number is a root of the characteristic equation of the transformation.* Finally, I show that each transformation of the second kind of either of the families Γ is the $(2m+1)^{th}$ power of a transformation of that family for any odd exponent $2m+1$.

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* These Proceedings, **31**, 181.

† If T is a transformation of Γ'', then $|T|^2 = 1$. That Γ'' shall contain transformations of the type mentioned and of determinant $+1$, it is sufficient that two of the real roots of the characteristic equation of $\mathfrak{F}$ shall be of opposite sign and equal in absolute value.

‡ In order that Γ', Γ'', Γ''' shall severally contain transformations of the type mentioned, it is sufficient for Γ' that the real roots of the characteristic equation of A shall not all be distinct; for Γ'', that the ratio of two of the real roots of this equation shall be equal to -1; and for Γ''', that the ratio of two of the real roots of the equation shall be negative.

The theorems given above depend upon considerations relating to the exponential function

$$e^U \equiv 1 + U + \frac{U^2}{2\,!} + \frac{U^3}{3\,!} + \dots$$

of the matrix or linear substitution U. This series is convergent for any finite matrix. We have

$$(e^U)^{-1} = e^{-U},$$

$$\breve{(e^U)} = e^{\breve{U}};$$

and, if U_1 and U_2 are commutative,

$$e^{U_1}\, e^{U_2} = e^{U_1+U_2},$$

in particular, for any integer m,

$$(e^U)^m = e^{mU}.$$

For any linear substitution T of non-zero determinant a polynomial $U = \Sigma_0^n{}_p\, c_p\, T^p$ can be found such that $T = e^U$. Let $\zeta_1, \zeta_2, \dots \zeta_\nu$, respectively of multiplicity $\mu_1, \mu_2, \dots \mu_\nu$, be the distinct roots of the characteristic equation of T; let, moreover,

$$G_i^{(k)}\,(T) \equiv \left(\frac{(T-\zeta_i)^{\mu_i} - (\zeta_k-\zeta_i)^{\mu_i}}{-(\zeta_k-\zeta_i)^{\mu_i}}\right)^{\mu_k},$$

$$(i,\, k = 1, 2, \dots \nu \qquad k \mp i)$$

$$F_i\,(T) \equiv \mathrm{G}_i^{(1)}\,(T) \dots G_i^{(i-1)}\;(T)\; G_i^{(i+1)}\;(T) \dots G_i^{(\nu)}\,(T);$$

$$(i = 1, 2, \dots \nu)$$

and let

$$f(T) \equiv \Sigma_1^\nu{}_i \Big[\log \zeta_i + 2\,m_i\sqrt{-1} + \frac{1}{\zeta_i}\,(T-\zeta_i) - \frac{1}{\zeta_i^2}\,\frac{(T-\zeta_i)^2}{2}$$

$$+ \frac{1}{\zeta_i^3}\,\frac{(T-\zeta_i)^3}{3} - \dots + (-1)^{\mu_i-1}\,\frac{1}{\zeta_i^{\mu_i-1}}\,\frac{(T-\zeta_i)^{\mu_i-1}}{\mu_i-1}\Big]\,F_i\,(T),$$

where m_1, m_2, etc., are integers. Then

$$T = e^{f(T)}.$$

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* The determinant of a transformation of the first kind is positive.

Moreover, if $\phi\,(T)$ is any polynomial in T, we have

$$\phi\,(T^{-1}) = \Sigma_i{}_1^{\nu} \Bigg[\phi\,(\zeta_i^{-1}) + (T_i - \zeta_i)\,\frac{\partial\phi\,(\zeta_i^{-1})}{\partial\zeta_i} + \frac{(T-\zeta_i)^2}{2\,!}\,\frac{\partial^2\phi\,(\zeta_i^{-1})}{\partial\zeta_i^{\,2}} + \ldots + \frac{(T-\zeta_i)^{\mu_i-1}}{(\mu_i-1)\,!}\,\frac{\partial^{\mu_i-1}\phi\,(\zeta_i^{-1})}{\partial\zeta_i^{\,\mu_i-1}}\Bigg] F_i\,(T)\,.$$

In particular, if $\zeta_k = \pm\ 1$,

$$F_k\ (T^{-1}) = F_k\ (T)\,;$$

and if $\zeta_k \neq \pm\ 1$, and $\zeta_k = \zeta_k^{-1}$ has the same multiplicity as ζ_k, then

$$F_{k'}\ (T^{-1}) = F_k\ (T)\,.$$

We have now the following theorems:

I. If U is real and $T = e^{U\sqrt{-1}}$ is also real, then $T^2 = 1$.*

II. If T is real and no negative number is a root of the characteristic equation of T, there is a real polynomial $f\,(T)$ satisfying the condition $T = e^{f(T)}$ †

III. If T is real and each negative root other than -1 of the characteristic equation of T is paired with its inverse, there is a polynomial $f\,(T)$ satisfying the equation $T = e^{f(T)}$ and such that $f\,(T) - f\,(T^{-1})$ is real.†

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* For if $T = e^{U\sqrt{-1}}$ is real, then

$$2\sqrt{-1}\,(U - \frac{U^3}{3\,!} + \frac{U^5}{5\,!} - \ldots) = e^{U\sqrt{-1}} - e^{-U\sqrt{-1}} = T - T^{-1}$$

is real. Therefore, if U is real, $T - T^{-1} = 0$; that is $T^2 = 1$.

† If T is real, each imaginary root of the characteristic equation of T is paired with its conjugate imaginary. Therefore, if this equation has no negative root, $f\,(T)$ may be taken real by a proper choice of m_1, m_2, etc.

If the distinct negative roots of the characteristic equation are

$$\zeta_i,\ \zeta_{\nu'+i} = \zeta_i^{-1}\ (i = 1, 2, \ldots \nu')$$

and $\zeta_{2\nu'+1} = -1$, and moreover (for $i = 1, 2, \ldots \nu'$) ζ_i and ζ_i^{-1} have the same multiplicity, then, for a proper choice of m_1, m_2, etc., the imaginary part of the polynomial $f\,(T)$ is

$$\pi\sqrt{-1}\ \Sigma_i{}_1^{\nu'}\,(F_i\,(T) + F_{\nu'+i}\,(T)) + \pi\sqrt{-1}\,F_{2\nu'+1}\,(T)\,;$$

and, since now

$$F_i\,(T^{-1}) = F_{\nu'+i}\,(T),\quad F_{\nu'+i}\,(T^{-1}) = F_i\,(T),$$
$$(i = 1, 2, \ldots \nu')$$
$$F_{2\nu'+1}\,(T^{-1}) = F_{2\nu'+1}\,(T),$$

therefore $f\,(T) - f\,(T^{-1})$ will be real.

IV. If T is real and each negative root of the characteristic equation of T is in absolute value less than unity, there is a real polynomial $\mathrm{f}(T)$ satisfying the equation $1 + T = e^{\mathrm{f}(T)}$.

Finally, from the identity,

$$e^{-AUA^{-1}} A\, e^{U} = A\, e^{-U} A^{-1} \cdot A\, e^{U} = A\, e^{-U} e^{U} = A\,,$$

we derive the theorems:

V. If U satisfies the equation

$$U = AUA^{-1} \tag{4}$$

and $T = e^{U}$ is real, in particular if U is real, then T is a transformation of Γ'.*

VI. If U satisfies the equation

$$U = -A\, U A^{-1} \tag{5}$$

and $T = e^{U}$ is real, in particular if U is real, then T is a transformation of Γ''.

VII. If U satisfies the equation

$$\breve{U} = AUA^{-1}, \tag{6}$$

and $T = e^{U}$ is real, in particular if U is real, then T is a transformation of Γ'''.

§ 2. $T^{-1} A\, T = A$.

If U is real and satisfies the equation

$$U = A\, U A^{-1}, \tag{4}$$

then, by Theorem V, every transformation of the group $e^{\zeta U}$, with real parameter ζ, is a transformation of Γ'; in particular the infinitesimal transformation $e^{\delta\zeta U}$ of this group is a transformation of Γ'.

If the real infinitesimal transformation $e^{\delta\zeta U} = 1 + \delta\zeta U$ is a transformation of Γ', then first U is real; moreover,

$$A + \delta\zeta\,(-UA + A\,U) = (1 - \delta\zeta\, U)\, A\, (1 + \delta\zeta U) = e^{-\delta\zeta U} A\, e^{\delta\zeta U} = A\,,$$

and therefore $UA = A\,U$, that is, U satisfies equation (4). Therefore, for any real scalar ζ, the transformation $e^{\zeta U}$ generated by $e^{\delta\zeta U}$ is a transformation of Γ'. Whence it follows that *every transformation generated*

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* In this case the application of the above identity is unnecessary; for $UA = A\,U$, and therefore $T^{-1} A\, T = e^{-U} A\, e^{U} = A\, e^{-U} e^{U} = A$.

by an infinitesimal transformation of Γ' *is the* m^{th} *power for any exponent* m *of a transformation of* Γ'*; in particular, every such transformation is the second power of a transformation of* Γ'*, and is therefore of the first kind.*[*]

Conversely, *every transformation of the first kind can be generated by an infinitesimal transformation of* Γ'. For let T be any transformation of Γ', then

$$(7) \qquad T = A\,T A^{-1}.$$

Let V and $W\sqrt{-1}$, respectively, be the real and imaginary parts of the polynomial $U = f(T)$, satisfying the equation $T = e^U$. Since T is real, both V and W are polynomials in T, and therefore commutative. Let

$$T_1 = e^V, \quad T_0 = e^{W\sqrt{-1}}.$$

Then since V, and therefore $T_1 = e^V$, is real, and since

$$T = e^U = e^{V+W\sqrt{-1}} = e^V e^{W\sqrt{-1}} = T_1\,T_0$$

is also real, it follows that $T_0 = T_1^{-1}\,T$ is real; and therefore, by Theorem I,

$$T_0^2 = (e^{W\sqrt{-1}})^2 = 1.$$

Whence,

$$T^2 = (T_1\,T_0)^2 = T_1^2\,T_0^2 = T_1^2.$$

But, since $V = \phi(T)$ is a polynomial in T, we have by equation (7)

$$V = \phi(T) = \phi(A T A^{-1}) = A\,\phi(T)\,A^{-1} = A\,V A^{-1};$$

and therefore, since V is real, $e^{\zeta V}$ for any real scalar ζ, is generated by the infinitesimal transformation $e^{\delta\zeta V}$ of Γ'. In particular,

$$T^2 = T_1^2 = (e^V)^2 = e^{2V}$$

is generated by the infinitesimal transformation $e^{\delta\zeta V}$ of Γ'.

A transformation T *of* Γ' *is a transformation of the first kind if the characteristic equation of* T *has no negative root.* For then, by Theorem II, there is a real polynomial $U = f(T)$ satisfying the equation $T = e^U$. In this case $V = U$, $W = 0$; and U satisfies equation (4). Therefore, T is the second power of the transformation $e^{\frac{1}{2}U}$ of Γ', and is generated by the infinitesimal transformation $e^{\delta\zeta U}$ of Γ'.

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[*] See p. 308.

The characteristic equation of a transformation of Γ' of the first kind may have negative roots. But in such case the numbers belonging to each of the negative roots of this equation are all even; and therefore, for each negative root of this equation, the elementary divisors (*elementar Theiler*) of the characteristic function corresponding to such root occur in pairs with equal exponent.*

No transformation of Γ' with negative determinant is of the first kind. Let $n = 2$, and

$$\mathfrak{F} \equiv \left(\begin{matrix} 1, & b \\ 0, & 1 \end{matrix} \between x_1, x_2 \between y_1, y_2\right) \equiv (x_1 y_1 - x_2 y_2) + b\, x_2 y_1.$$

The form $\mathfrak{F}$ is transformed automorphically if

$$(x_1', x_2') = T(x_1, x_2) = \left(\begin{matrix} -1, & 1 \\ 0, & -1 \end{matrix} \between x_1\ x_2\right),$$

$$(y_1', y_2') = \breve{T}^{-1}(y_1, y_2) = \left(\begin{matrix} -1, & 0 \\ -1, & -1 \end{matrix} \between y_1, y_2\right).$$

We have $|\,T\,| = +1$, but T is a transformation of the second kind. Whence it follows, for any value of n, that there are forms $\mathfrak{F}$ such that Γ' contains transformations of the second kind with positive determinant.†

By definition no transformation of the second kind is an even power of a transformation of Γ'; but *every transformation of* Γ' *of the second kind is the* $(2m + 1)^{th}$ *power, for any odd exponent* $2m + 1$, *of a transformation of* Γ'. Thus, let T be any transformation of Γ', and, as before, let V and W be real polynomials in T satisfying the equation

$$T = e^{V + W\sqrt{-1}}.$$

As shown above, for any real scalar ζ, $e^{\zeta V}$ is a transformation of Γ'; and, therefore, so also is $e^{W\sqrt{-1}} = Te^{-V}$. Consequently

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* For the roots of the characteristic equation of T^2 are the squares of the roots of the characteristic equation of T; and, if T is real, each imaginary root of the characteristic equation of T is paired with its conjugate imaginary. Compare These Proceedings, **31**, 189.

For the relation between the numbers belonging to the roots of the characteristic equation of a linear transformation T and the exponents of the elementary divisors of the characteristic function of T, see Bull. Am. Math. Soc., 2d series, **3**, 156.

† The condition, given on p. 308, sufficient that Γ' shall contain transformations of the second kind, is readily obtained.

$$e^{\zeta V + W\sqrt{-1}} = e^{\zeta V} e^{W\sqrt{-1}}$$

is a transformation of Γ'.[*] Moreover,

$$e^{2W\sqrt{-1}} = (e^{W\sqrt{-1}})^2 = 1\,;$$

and therefore, for any integer m,

$$(e^{W\sqrt{-1}})^{2m+1} = (e^{2W\sqrt{-1}})^m\, e^{W\sqrt{-1}} = e^{W\sqrt{-1}}.$$

Whence we have

$$\begin{aligned} T = e^V e^{W\sqrt{-1}} &= (e^{\frac{1}{2m+1}V})^{2m+1} (e^{W\sqrt{-1}})^{2m+1} \\ &= (e^{\frac{1}{2m+1}V} e^{W\sqrt{-1}})^{2m+1} = (e^{\frac{1}{2m+1}V + W\sqrt{-1}})^{2m+1}. \end{aligned}$$

§3. $T A T = A$.

If U is real and satisfies the equation

$$U = -A\,U A^{-1}, \tag{5}$$

then, by Theorem VI, every transformation of the group $e^{\zeta U}$, with real parameter ζ, is a transformation of Γ''; in particular, the infinitesimal transformation $e^{\delta\zeta U}$ of the group is a transformation of Γ''.

If the real infinitesimal transformation $e^{\delta\zeta U} = 1 + \delta\,\zeta\,U$ is a transformation of Γ'', then first U is real; moreover

$$A + \delta\,\zeta\,(U A + A\,U) = (1 + \delta\,\zeta\,U)\,A\,(1 + \delta\,\zeta\,U) = e^{\delta\zeta U} A\, e^{\delta\zeta U} = A\,;$$

and therefore $U A + A\,U = U$, that is, U satisfies equation (5). Therefore, for any real scalar ζ, the transformation $e^{\zeta U}$ generated by $e^{\delta\zeta U}$ is a transformation of Γ''. Whence it follows that *every transformation generated by an infinitesimal transformation of* Γ'' *is the mth power for any exponent m, of a transformation of this family ; in particular, every such transformation is the second power of a transformation of* Γ'', *and is therefore of the first kind.*

Conversely, *every transformation of the first kind can be generated by an infinitesimal transformation of* Γ''. For let T be any transformation of Γ'', then

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[*] Since the transformations of Γ' form a group, if T and T_1 are transformations of Γ', so also is $T\,T_1$.

$$
\begin{aligned}
T &= A\,T^{-1}A^{-1}, \\
T^{-1} &= A\,TA^{-1};
\end{aligned} \tag{8}
$$

and therefore each root, other than ± 1, of the characteristic equation of T is paired with its inverse, so that if $\zeta_i \neq \pm 1$ is a root of this equation, $\zeta_{i'} = \zeta_i^{-1}$ is also a root, and the numbers belonging to ζ_i and $\zeta_{i'}$ (in particular the multiplicity of these roots) are the same. Therefore, by Theorem III, there is a polynomial $f(T)$ satisfying the equation $T = e^{f(T)}$ and such that $f(T) - f(T^{-1})$ is real. Let now

$$2\,U_0 = f(T) + f(T^{-1}), \qquad 2\,U_1 = f(T) - f(T^{-1});$$

and let

$$T_0 = e^{U_0}, \qquad T_1 = e^{U_1}.$$

We have by (8)

$$
\begin{aligned}
T_0^{\,2} &= e^{f(T)+f(T^{-1})} \\
&= e^{f(T)}\,e^{f(T^{-1})} \\
&= e^{f(T)}\,e^{f(ATA^{-1})} \\
&= e^{f(T)}\,e^{Af(T)A^{-1}} \\
&= e^{f(T)} \cdot A\,e^{f(T)}\,A^{-1} \\
&= T\,A\,T \cdot A^{-1} = A\,A^{-1} = 1.
\end{aligned}
$$

Therefore,

$$T^2 = (T_0\,T_1)^2 = T_0^{\,2}\,T_1^{\,2} = T_1^{\,2}.$$

But, by equation (8),

$$
\begin{aligned}
U_1 &= \tfrac{1}{2}\,(f(T) - f(T^{-1})) \\
&= \tfrac{1}{2}\,(f(A\,T^{-1}A^{-1}) - f(A\,TA^{-1})) \\
&= \tfrac{1}{2}\,A(f(T^{-1}) - f(T))\,A^{-1} = -\,A\,U_1\,A^{-1};
\end{aligned}
$$

and therefore, since U_1 is real, it follows that $e^{\zeta U_1}$, for any real scalar ζ, is generated by the infinitesimal transformation $e^{\delta\zeta U_1}$ of Γ''. In particular

$$T^2 = T_1^{\,2} = (e^{U_1})^2 = e^{2U_1}$$

is generated by $e^{\delta\zeta U_1}$.

A transformation T *of* Γ'' *is of the first kind if the characteristic* *equation of T has no negative root.* For in this case, since $|1 + T| \neq 0$, we may put

$$S = \frac{1 - T}{1 + T};$$

whence we derive

$$T = \frac{1 - S}{1 + S},$$

and by (8)

$$(9) \qquad A S A^{-1} = \frac{1 - A T A^{-1}}{1 + A T A^{-1}} = \frac{1 - T^{-1}}{1 + T^{-1}} = \frac{T - 1}{T + 1} = - S.$$

If ζ_1, ζ_2, etc., are the roots of the characteristic equation of T, the roots of the characteristic equation of S are $\frac{1 - \zeta_i}{1 + \zeta_i}$ *for* $i = 1, 2$, *etc.*; and since no negative number is a root of the characteristic equation of T, the real roots of the characteristic equation of S are in absolute value less than unity. Therefore, by Theorem IV, there is a real polynomial $\mathbf{f}(S)$ satisfying the equation

$$1 + S = e^{\mathbf{f}(S)}.$$

From equation (9) we derive

$$1 - S = 1 + A S A^{-1} = A (1 + S) A^{-1} = e^{A \mathbf{f}(S) A^{-1}} = e^{\mathbf{f}(A S A^{-1})} = e^{\mathbf{f}(-S)};$$

and therefore, if $U = \mathbf{f}(- S) - \mathbf{f}(S)$,

$$T = \frac{1 - S}{1 + S} = e^{\mathbf{f}(-S)} e^{-\mathbf{f}(S)} = e^{\mathbf{f}(-S) - \mathbf{f}(S)} = e^{U}.$$

But, by equation (9),

$$\begin{aligned} U = \mathbf{f}(- S) - \mathbf{f}(S) &= \mathbf{f}(A S A^{-1}) - \mathbf{f}(- A S A^{-1}) \\ &= A (\mathbf{f}(S) - \mathbf{f}(- S)) A^{-1} = - A U A^{-1}; \end{aligned}$$

and therefore, since U is real, $e^{\zeta U}$, for any real scalar ζ, is a transformation of Γ''. Consequently, T is the second power of a transformation $e^{\frac{1}{2} U}$ of Γ'', and is generated by the infinitesimal transformation $e^{\delta \zeta U}$ of Γ''.

The characteristic equation of a transformation of the first kind may have negative roots. But in such case, the numbers belonging to each of these roots are all even.*

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* Cf. note, p. 313.

Every transformation of Γ'' with negative determinant is of the second kind. Let $n = 2$, and let

$$\mathfrak{F} \equiv \begin{pmatrix} 1, & 0 \\ 0, & -1 \end{pmatrix}(x_1, x_2)(y_1, y_2) \equiv x_1 y_1 - x_2 y_2.$$

The form $\mathfrak{F}$ is transformed automorphically if

$$(x_1', x_2') = T(x_1, x_2) = \begin{pmatrix} -1, & 1 \\ 0, & -1 \end{pmatrix}(x_1, x_2)$$

$$(y_1', y_2') = \breve{T}(y_1, y_2) = \begin{pmatrix} -1, & 0 \\ 1, & -1 \end{pmatrix}(y_1, y_2).$$

We have $| T | = + 1$, but T is a transformation of the second kind. Whence, for any value of n, it follows that there are forms $\mathfrak{F}$ such that Γ'' contains transformations of the second kind with determinant $+1$.* Again, the form

$$\mathfrak{F} \equiv \begin{pmatrix} 0, & 0, & b, & d \\ 0, & 0, & 0, & -b \\ a, & c, & 0, & 0 \\ 0 & -a, & 0, & 0 \end{pmatrix}(x_1, x\ , x_3, x_4)(y_1, y_2, y_3, y_4) \equiv a(x_1 y_3 - x_2 y_4) + b(x_3 y_1 - x_4 y_2) + c\, x_2 y_3 + d\, x_4 y_1$$

is transformed automorphically by the transformation

$$T = \begin{pmatrix} -\lambda^2, & 1, & 0, & 0 \\ 0, & -\lambda^2, & 0, & 0 \\ 0, & 0, & -\lambda^{-2}, & \lambda^{-4} \\ 0, & 0, & 0, & -\lambda^{-2} \end{pmatrix}$$

of Γ''; and T is a transformation of the second kind if $\lambda \neq \pm 1$.

By definition no transformation of the second kind is an even power of any transformation of Γ''; but *every transformation of the second kind is the* $(2m + 1)^{th}$ *power, for any odd exponent* $2m + 1$, *of a transformation of* Γ''. Thus let T be any transformation of Γ''; and, as before, let $f(T)$ be a polynomial satisfying the equation $T' = e^{f(T)}$, and such that $f(T) - f(T^{-1})$ is real. Then, as shown above, if

$$2\, U_0 = f(T) + f(T^{-1}), \qquad 2\, U_1 = f(T) - f(T^{-1}),$$

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* The condition, given in note on p. 308, as sufficient that Γ'' shall contain transformations, is readily proved.

$e^{\zeta U_1}$, for any real scalar ζ, is a transformation of Γ''; and therefore

$$e^{U_0} = T e^{-U_1}$$

is a transformation of Γ''. Whence it follows that

$$e^{U_0 + \zeta U_1} = e^{U_0}\, e^{\zeta U_1},$$

for any real scalar ζ, is a transformation of Γ''.* Moreover, we have

$$e^{2U_0} = (e^{U_0})^2 = 1\,;$$

and therefore for any integer m

$$(e^{U_0})^{2m+1} = (e^{2U_0})^m\, e^{U_0} = e^{U_0}.$$

Wherefore,

$$T = e^{U_0}\, e^{U_1} = (e^{U_0})^{2m+1}\, (e^{\frac{1}{2m+1} U_1})^{2m+1} = (e^{U_0}\, e^{\frac{1}{2m+1} U_1})^{2m+1} = (e^{U_0 + \frac{1}{2m+1} U_1})^{2m+1}.$$

§ 4. $\breve{T}^{-1} A\, T = A.$

If U is real and satisfies the equation

$$\breve{U} = A\, U A^{-1}, \tag{5}$$

then, by Theorem VI, every transformation of the group $e^{\zeta U}$, with real parameter ζ, is a transformation of Γ'''. In particular, the infinitesimal transformation $e^{\delta\zeta U}$ of the group is a transformation of Γ'''.

If the real infinitesimal transformation $e^{\delta\zeta U} = 1 + \delta\,\zeta\, U$ is a transformation of Γ''', then first U is real; moreover,

$$A + \delta\,\zeta\,(-\,\breve{U} A + A U) = (1 - \delta\,\zeta\,\breve{U})\, A\, (1 + \delta\,\zeta\, U) = e^{-\delta\zeta\breve{U}}\, A\, e^{\delta\zeta U} = A,$$

and therefore $-\,\breve{U} A + A\, U = 0$, that is, U satisfies equation (5). Therefore, for any real scalar ζ, the transformation $e^{\zeta U}$ generated by $e^{\delta\zeta U}$ is a transformation of Γ'''. Whence it follows that *every transformation generated by an infinitesimal transformation of* Γ''' *is the* m^{th} *power for any exponent* m *of a transformation of* Γ'''. *In particular, every such transformation is the second power of a transformation of* Γ''', *and is therefore of the first kind.*

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* The transformations of Γ'' that are commutative form a group. Thus, if T and $T^{(1)}$ are transformations of Γ'' and $T\, T^{(1)} = T^{(1)}\, T$, then $T\, T^{(1)}$ is also a transformation of Γ''.

Conversely, *every transformation of the first kind can be generated by an infinitesimal transformation of* Γ'''. For let T be any transformation of Γ''', then

$$\breve{T} = A\,T A^{-1}. \tag{10}$$

Let V and $W\sqrt{-1}$, respectively, be the real and imaginary parts of the polynomial $U = f(T)$ satisfying the equation $T = e^U$. Then, since T is real, both V and W are polynomials in T, and therefore commutative. Let

$$T_1 = e^V, \quad T_0 = e^{W\sqrt{-1}}.$$

Then since V, and therefore $T_1 = e^V$, is real, and since

$$T = e^{V + W\sqrt{-1}} = e^V e^{W\sqrt{-1}} = T_1\,T_0$$

is also real, it follows that $T_0 = T_1^{-1}\,T_0$ is real; and therefore, by Theorem I,

$$T_0^{\,2} = (e^{W\sqrt{-1}})^2 = 1.$$

Wherefore,

$$T^2 = (T_1\,T_0)^2 = T_1^{\,2}\,T_0^{\,2} = T_1^{\,2}.$$

But, since $V = \phi\,(T)$ is a polynomial in T, we have by (10)

$$\breve{V} = \phi\,(\breve{T}) = \phi\,(A\;T A^{-1}) = A\,\phi\,(T)\,A^{-1} = A\,V A^{-1};$$

and therefore, since V is real, $e^{\zeta V}$ for any real scalar ζ is generated by the infinitesimal transformation $e^{\delta\zeta V}$ of Γ'''. In particular,

$$T^2 = T_1^{\,2} = (e^V)^{\,2} = e^{2V}$$

is generated by $e^{\delta\zeta V}$.

A transformation T of Γ''' is of the first kind if the characteristic equation of T has no negative root. For then, by Theorem II, there is a real polynomial $U = f\,(T)$ satisfying the equation $T = e^U$. In this case $V = U$, $W = 0$; and U satisfies equation (5). Therefore T is the second power of the transformation $e^{\frac{1}{2}U}$ of Γ''', and is generated by the infinitesimal transformation $e^{\delta\zeta U}$ of Γ'''.

The characteristic equation of a transformation of the second kind may have negative roots. But in such case the numbers belonging to the negative root of this equation are all even.*

Every transformation of Γ''' with negative determinant is a transformation of the second kind. Let $n = 2$, and let

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* Cf. p. 312.

$$\mathfrak{F} \equiv \left(\begin{matrix}0, & 1\\ 1, & b\end{matrix}\right| \between x_1, x_2 \between y_1, y_2) \equiv (x_1 y_2 + x_2 y_1) + b\, x_2 y_2.$$

The form $\mathfrak{F}$ is transformed automorphically if

$$(x_1', x_2') = (T \between x_1, x_2) = \left(\begin{matrix}-1, & 1\\ 0, & -1\end{matrix}\right| \between x_1, x_2)$$

$$(y_1', y_2') = (T^{-1} \between y_1, y_2) = \left(\begin{matrix}-1, & -1\\ 0, & -1\end{matrix}\right| \between y_1, y_2).$$

We have $|T| = +1$, but T is a transformation of the second kind. Whence it follows, for any value of n, that there are bilinear forms $\mathfrak{F}$ such that Γ''' contains transformations of the second kind with positive determinant.[*] The form

$$\mathfrak{F} \equiv \left(\begin{matrix}-a^2, & 0\\ 0, & 1\end{matrix}\right| \between x_1, x_2 \between y_1, y_2) \equiv -a^2 x_1 y_1 + x_2 y_2$$

is transformed automorphically if

$$(x_1', x_2') = (T \between x_1, x_2) = \left(\begin{matrix}-1 + a\lambda, & \lambda\\ -a^2\lambda, & -1 - a\lambda\end{matrix}\right| \between x_1, x_2),$$

$$(y_1', y_2') = (T^{-1} \between y_1, y_2) = \left(\begin{matrix}-1 - a\lambda, & -\lambda\\ a^2\lambda, & -1 + a\lambda\end{matrix}\right| \between y_1, y_2);$$

and T, of determinant $+1$, is a transformation of the second kind if λ ‡ 0.

Every transformation Γ''' *of the second kind is the* $(2m + 1)^{th}$ *power of a transformation of* Γ''' *for any odd exponent* $2m + 1$. Thus let T be any transformation of Γ'''. If V and W are real polynomials in T satisfying the equation $T = e^{V + W\sqrt{-1}}$, we may then show, precisely as for the similar theorem in the case of the family Γ' that $e^{\frac{1}{2m+1}V + W\sqrt{-1}}$ is a transformation of Γ''', and that

$$T = e^{V + W\sqrt{-1}} = \left(e^{\frac{1}{2m+1}V + W\sqrt{-1}}\right)^{2m+1}.\dagger$$

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[*] For a sufficient condition that Γ''' shall contain a transformation of the second kind see p. 308.

[†] See p. 313. The family of transformations Γ''' also constitute a group.

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INFLUENCE OF OCCLUDED HYDROGEN ON THE ELECTRICAL RESISTANCE OF PALLADIUM.

By W. E. McElfresh.

INFLUENCE OF OCCLUDED HYDROGEN ON THE ELECTRICAL RESISTANCE OF PALLADIUM.

By W. E. McElfresh.

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It was observed by Graham,* in 1869, and later by Dewar,† Knott,‡ and others, that the electrical resistance of a palladium wire is much greater when the wire is charged with hydrogen than when it is in its normal state.

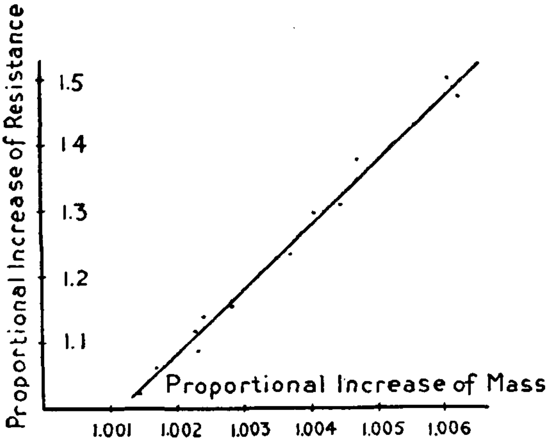

Figure 1.

Knott made a quantitative study of the phenomenon, charging a wire step by step and measuring its resistance after each charging. The curve plotted from these results is reproduced here (Fig. 1) and challenges attention immediately in regard to one or two particulars. For it is practically a straight line for all points between 175 volumes absorbed (0.0014 increase in mass of wire) and saturation (nearly 900 volumes absorbed), but this line does not point toward the origin. The resistance was first measured with no hydrogen in the wire, then with about 175 volumes, then with still greater amounts. No observations were taken to obtain points on the curve between the origin and that for 175 volumes, but, if the results as plotted are correct, it is evident that the curve, as it approaches the origin, must either bend very sharply toward the left or else it must drop below the horizontal axis. In other words, it would appear from the curve that a small quantity of occluded hydrogen, say 50 volumes, would change the resistance of the wire very slightly or, possibly, make it actually smaller than when there is no hydrogen in the wire. Knott comments on the peculiar shape of the curve, but offers no explanation, and there appears to be no record of any later attempt to repeat the experiment or to seek for a more accurate method of studying the phenomenon. The work outlined in this paper was undertaken in the hope of making a more careful determination of the curve given by Knott and, more particularly, with a view to studying its form near the origin.

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* Poggendorff's Annalen, 136 (1869).

† Trans. Roy. Soc. Edin., Vol. **27**.

‡ Proc. Roy. Soc. Edin., Vol. **12** (1882–1883).

The method, employed by Knott and others, of determining the amount of hydrogen occluded — by measuring the increase in weight of the wire — appears to be incapable of giving very exact results. Accordingly, in the work described below, the wire is charged with hydrogen by being made the negative electrode of a gas voltameter and the amount absorbed is determined as follows. The electrolysing current is accurately measured at regular intervals during the charging process and the total quantity of hydrogen liberated from the electrolyte is thus easily calculated; the hydrogen which escapes from the wire is carefully collected, and its volume measured, and this volume being subtracted from the total volume liberated by the current it is assumed that the difference represents the volume of hydrogen occluded by the wire.

In the earlier experiments the voltameter was simply a glass jar of about three liters' capacity filled with a two per cent solution of sulphuric acid; a light hard-rubber frame held in place the two electrodes, the palladium wire being coiled in a slender spiral in the middle of the jar, while a platinum wire, coiled around the inner wall of the jar, formed the positive electrode; a burette with an inverted funnel fused to its lower end served to collect and measure the escaping hydrogen. Two objections to this form of apparatus at once suggested themselves. In the first place it was found that even a large funnel would not catch all of the rising hydrogen bubbles, especially toward the end of the experiment. At this time, when the wire is nearly saturated, the hydrogen escapes as a milky cloud of very small bubbles, many of which stream around beneath the edge of the funnel and so escape to the open air. In the second place, hydrogen is slightly soluble in the electrolyte, and where a large quantity of the latter is used the amount of hydrogen dissolved may be great enough to introduce a considerable error. For, by this method, the computed value of the quantity of hydrogen absorbed by the wire will include both that actually absorbed and also that dissolved by the electrolyte.

In order to overcome these difficulties, the final experiments employed a voltameter whose capacity was only about 475 cubic centimeters, and in which the palladium wire was surrounded by walls of filter paper so arranged that the liberated hydrogen could not escape otherwise than into the burette.

The accompanying diagrams illustrate the construction of the apparatus. The voltameter is made by clamping quarter-inch glass plates, *bb*, in Fig. 2, to a rectangular frame, *aa*, of hard-rubber, a thin strip of pure-gum rubber sheeting serving as packing. As thus constructed, the voltameter is slightly more than one foot in depth, while its cross-section measurements are shown in Fig. 2, which is one-half of actual size. The increased width in the middle portion of the interior allows the electrical lines of flow to be uniformly distributed around the circumference of the palladium wire, which will thus be charged evenly from all sides. The bevelled glass plates, *cccc*, fit loosely in narrow recesses provided for them in the hard-rubber frame, *aa*. Their purpose is solely to diminish the volume of electrolyte used. Fig. 3 — one-fourth of actual size — shows the same piece of apparatus in elevation, and Fig. 4 shows a cross-section at a level just below that of the lower end of the burette. The platinum wires, *dd*, parallel to the palladium wire, form the positive electrode of the voltameter. In the process of charging, the current, for a time, is sent *in* at the tops of the platinum wires and *out*

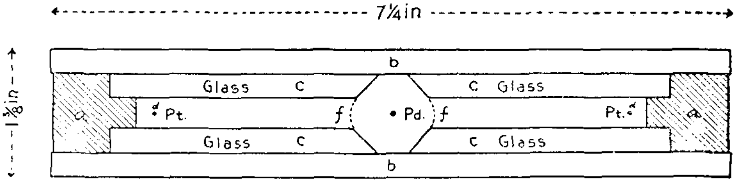

FIGURE 2.

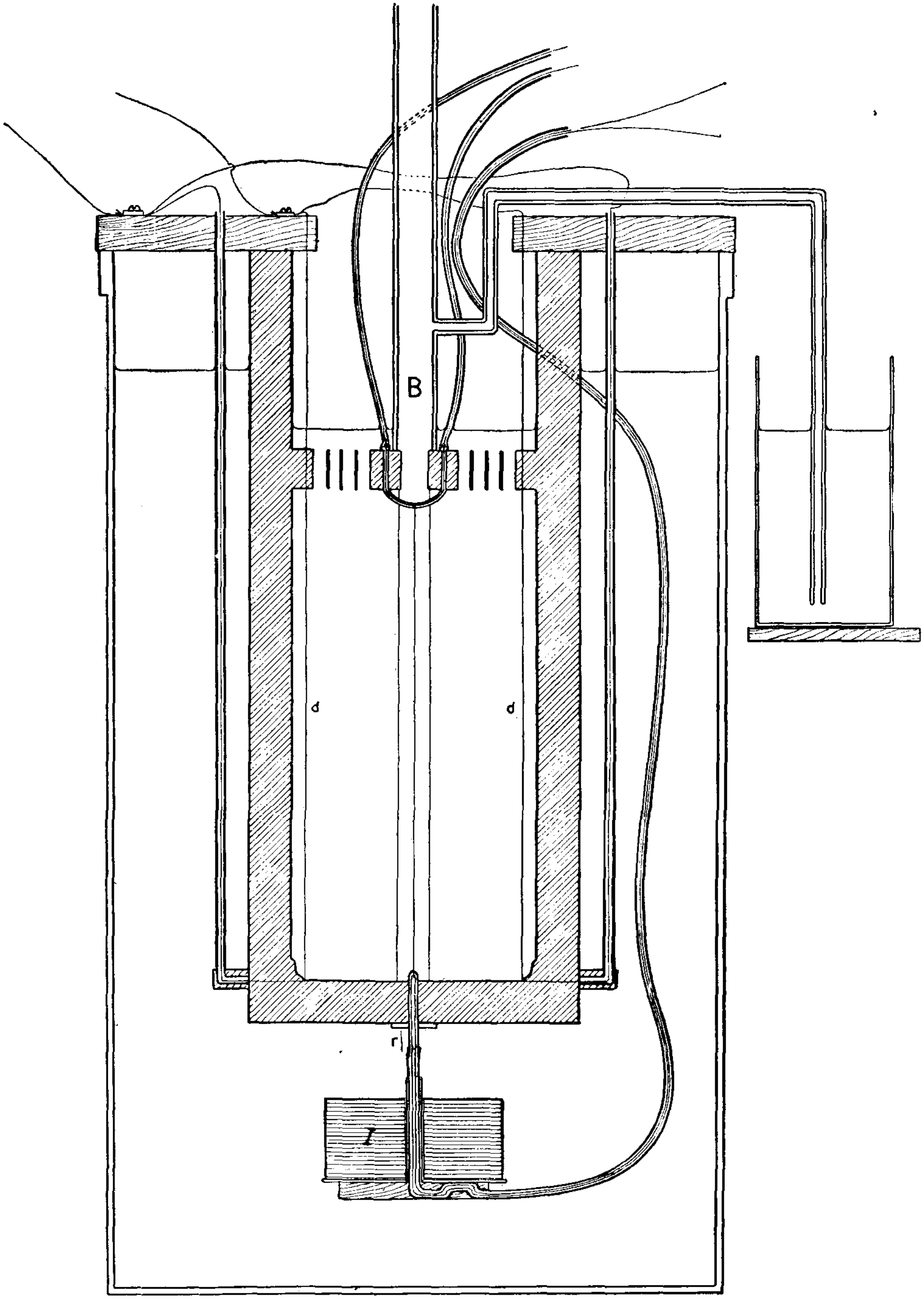

Figure 3.

at the bottom of the palladium; then by a throw of a commutator key it is sent *in* at the lower ends of the platinum wires and *out* at the top of the palladium. This should insure an even distribution of the hydrogen throughout the length of the wire. Two filter-paper partitions, *ff*, in Fig. 2, are introduced, one on either side of the palladium, and are held in position by having their edges caught between the glass plates *b* and *c*. These filter-paper partitions serve to prevent any possible mixing of oxygen with the hydrogen and at the same time force all of the liberated hydrogen that is not absorbed by the palladium to rise into the burette, *B*, where its volume may be measured.

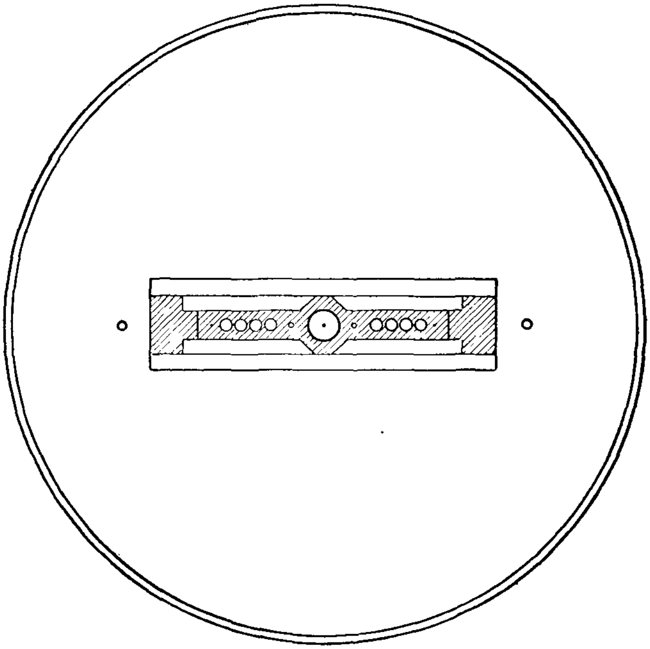

FIGURE 4.

In order to measure the resistance of the palladium wire without removing it from the voltameter it is necessary that two insulated lead-wires be permanently connected with each end. Accordingly (see Fig. 5), the upper end of the palladium wire is soldered to the middle of a copper wire whose ends are carried through small holes in the hard-rubber crosspiece and thence to outside connections. This copper wire is very carefully protected from the electrolyte by means of a special acid-proof cement which covers the submerged portion of the copper wire and the soldered end of the palladium. The two branches of the copper wire are further protected by soft-rubber tubing ("pure-gum"), which forms a water-tight jacket, its lower ends being closed with cement. To the lower end of the palladium wire are soldered two long copper wires which lead to outside connections. Incasing one of these wires in a rubber tube of very small bore serves to insulate them from each other; a rubber tube of larger bore contains the pair of wires, thus insulated from one another, and protects them from contact with the water in the outer jar. The palladium wire increases in length as it absorbs hydrogen, and must be subjected to a considerable tension if it is to be kept straight. To this end a glass tube, about two inches long, is drawn over the lower lead-wires until it encloses the lower end of the palladium; the upper end of this tube is filled with cement; and over its lower end is drawn the larger rubber tube mentioned above. In this way a water-tight jacket is provided for the lead-wires, from the soldered junction to the outside connections, and no metal, excepting the platinum and palladium electrodes, is anywhere exposed to the electrolyte. The glass tube passes through a hole in the bottom of the hard-rubber frame, and a soft-rubber washer cemented to the latter allows freedom of motion to the tube and at the same time prevents escape of electrolyte. This washer is shown as *r* in Fig. 3. A heavy iron weight, *I*, is attached to the rubber-incased lead-wires, and serves to keep the palladium wire straight as its length increases under the charging process.

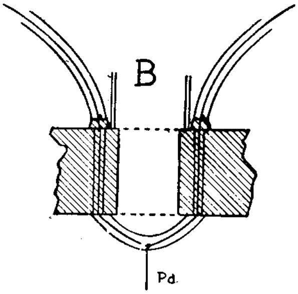

FIGURE 5.

In order that the electrolyte may easily be kept at a constant level in the voltameter, a branch tube in the side of the burette, near its lower end, leads outward, and finally dips into a larger jar of the electrolyte placed near the voltameter. This jar is shown on a small scale in Fig. 3. When the burette is filled with electrolyte this branch tube also fills, and thereafter the level of the electrolyte in the voltameter may be adjusted by raising or lowering this outer vessel.

The voltameter, with the iron weight hanging below it, is suspended in a glass jar of about 40 liters' capacity, filled with water. This water bath serves to prevent sudden changes of temperature, while a mechanical stirrer, operated continuously by an electric motor, insures a uniform temperature throughout the apparatus and so eliminates disturbing thermo-electric currents.

The method of measuring the resistance of the palladium wire is shown in Fig. 6.

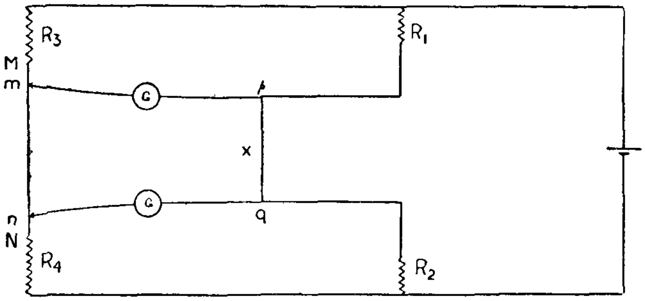

FIGURE 6.

In this diagram, x is the palladium wire, and MN is the wire of a cylindrical rheostat (a Wheatstone-Kirchhoff bridge), while m and n are points on this wire whose potentials are equal respectively to those of the points p and q. If V is the total resistance of the wire MN, and if v is the resistance of the portion included between m and n, then

$$x = \frac{v\,(R_1 + R_2)}{R_3 + R_4 + V - v}.$$

R_3 and R_4 were each equal to about 300 ohms, while R_1 and R_2 were 5-ohm coils. The various resistances were kept at constant temperature (within one degree), and reversals of direction of current flow should have eliminated any errors due to thermo-electric effects. The resistance of the palladium wire was in the neighborhood of 0.1 of an ohm, and the method of measuring gave results which agreed amongst themselves to within one-tenth of one per cent.

The charging current was between 0.02 and 0.03 of an ampere, and was measured by a shunted ammeter which gave readings to within a hundred-thousandth of an ampere. The value of the current strength was needed solely for the purpose of calculating the volume of hydrogen liberated in a given length of time, and therefore it seemed best to calibrate the ammeter by actually measuring the volume of hydrogen liberated per minute under the conditions of the experiment. Accordingly, at the close of one of the regular experiments as described below, the charging current was allowed to continue for a number of hours (twelve in one case, thirty in another) after the wire had apparently reached a state of saturation. At the end of this time, the palladium being unquestionably saturated, the volume of hydrogen liberated per minute was carefully measured and compared with the average ammeter reading. Two independent trials of this sort gave 6.973 and 6.975 cu. cm. as the volume (reduced to 0°C. and 760 mm. pressure) set free per minute by one ampere of current — as measured by the ammeter employed in this work.

In the final experiment the method of procedure was as follows. A length of palladium wire, freshly cut from the original coil, was cleaned by washing in caustic potash and then in distilled water. It was annealed by bringing it to a white heat in the open air and allowing it to cool quickly, the heating being accomplished by passing a current of electricity through it. This process left no visible tarnish, though the surface of the wire appeared less lustrous than before heating.* After the annealing the palladium was mounted in the voltameter, as already explained, and the latter was placed in its water bath, and this was stirred continuously. This bath was, approximately, at the room temperature, and by keeping the latter fairly constant the temperature of the water bath did not change during the progress of the experiment by more than 1°C. The resistance of the wire was then carefully measured. In this work, the commutator, M, in Fig. 7, was set so that the circuit of the charging battery, B_1, was broken, while the current from another battery, B_2, was allowed to flow through the palladium wire and the resistances R_1, R_2, R_3, R_4, etc. The wheel rheostat was included between R_3 and R_4, and on this rheostat were found balancing points (see Fig. 6) for the two ends of the palladium wire, i. e., points so situated that when one of them was connected, through a sensitive galvanometer, with the appropriate end of the palladium wire, no deflection was observed. To guard against any errors from thermo-electric effects, these balancing points were located four times in each measurement of resistance, the commutator N (in Fig. 7) being used to reverse the current through all the resistances, while, for each position of N, reversal of the commutator M, by turning it through 90°, had the effect of turning the palladium wire end for end with respect to the remainder of the circuit.

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* In a preliminary experiment a piece of palladium cut from the same coil had been annealed in an atmosphere of nitrogen, with the result that the wire retained its high polish, but refused, when exposed to the nascent hydrogen, to absorb the latter in any considerable quantity.

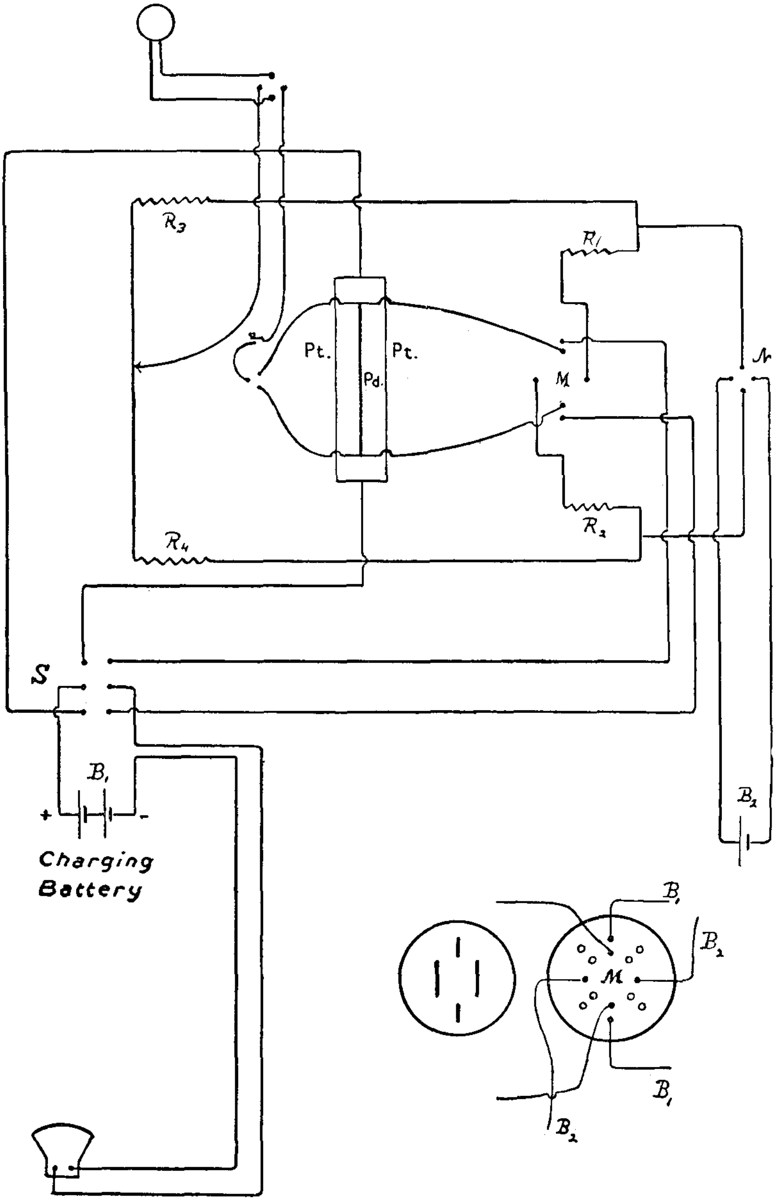
R_3
R_1
Pt.
Pt.
Pd.
M
N
R_2
R_4
S
B_1
+
-
Charging Battery
B_2
B_1
B_2
M
B_1
B_2

FIGURE 7.

By turning the commutator M through 45°, after the measurement of resistance before charging, the resistance measuring circuit was broken and the charging current from B_1 was started. This was allowed to flow for a carefully measured length of time, — fifteen minutes, — and ammeter readings were taken at the end of every minute. A turn of the commutator key, M, stopped the charging current, and at the same time made the proper connections for measuring resistance; but before the latter work was undertaken the burette was examined with a view to recording the volume of hydrogen collected. No hydrogen was found in the burette after this or any other of the earlier chargings; and it was therefore assumed, in such cases, that the total volume liberated during the charging run, as calculated from ammeter readings, had been absorbed by the wire. This was usually, for each fifteen minutes' run, about twenty times the volume of the wire. The resistance of the wire, thus charged, was then carefully measured as before, just fifteen minutes being allowed for this work. Again the wire was charged with an additional twenty volumes (approximately) and its new resistance immediately determined. This process, of alternately charging the wire and measuring its resistance, was continued until the wire appeared to be saturated, the entire experiment requiring about thirty hours of uninterrupted observations. This slow charging rate was employed because preliminary experiments had shown that time is a very important element in the absorption of hydrogen by palladium; if a strong current is used, most of the liberated hydrogen bubbles up to the surface of the liquid, even at the beginning of the run with a fresh wire. But with the weak currents used in these experiments no hydrogen bubbles were seen until after the palladium had absorbed more than six hundred volumes (75 to 80 cu. cm.); oxygen bubbles, of course, rose from the platinum wires from the very start. The first appearance of hydrogen in the burette was carefully watched for, and thereafter, at the close of each charging run, the volume of hydrogen in the burette was carefully recorded and reduced to standard conditions of pressure and temperature. In the subsequent calculations the new volume of hydrogen, collected in the burette during each charging period, was substracted from the total volume liberated by the current during that period, as calculated from ammeter readings, and it was assumed that the difference represented the volume of hydrogen (at 0° C. and 760 mm. pressure) absorbed by the wire during that run. In the earlier part of the work the charging time was, in each case, just fifteen minutes; later, when the hydrogen was escaping very freely, this time was lengthened. In every case just fifteen minutes was allowed, between two consecutive chargings, for measuring resistance. While the charging current was flowing, ammeter readings were taken at intervals of one minute.

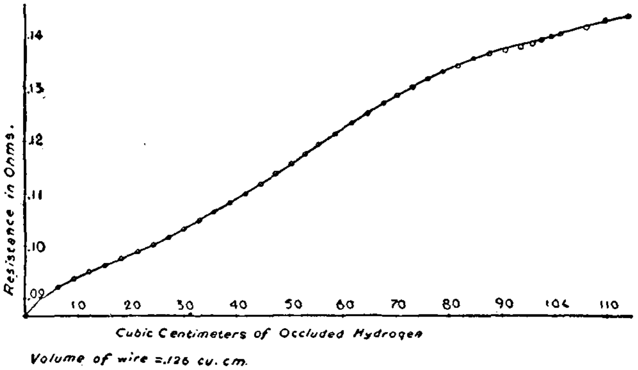

Figure 8.

Figure 8 and Figure 9 show the results of two independent trials. Although three months intervened between these trials, the condi-

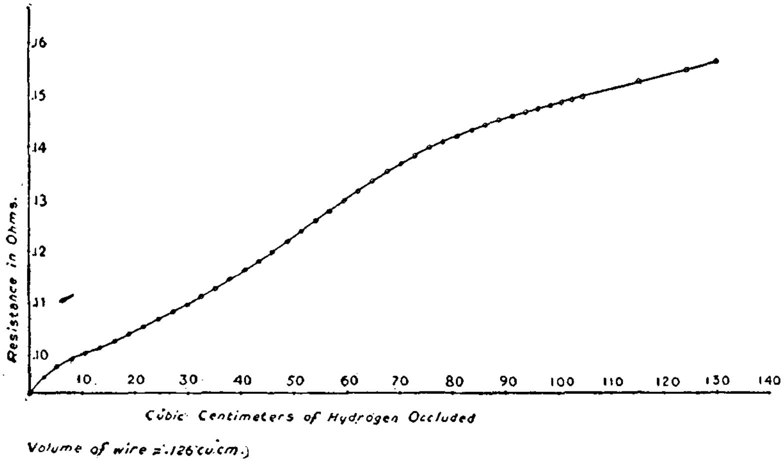

Figure 9.

tions of the second were made as nearly as possible like those of the first. The palladium wires were cut from the same piece and were subjected to practically the same treatment in preparation for their respective experiments. The curves are plotted with increase of resistance (above the resistance at start) for ordinates, and cubic centimeters of hydrogen occluded for abscissae. These curves should, then, be similar in form to curves whose coördinates are "proportional increase of resistance" and "proportional increase of mass" (see Fig. 1). The curves exhibited here show a decided increase in the resistance of the palladium from the very beginning of the experiment. In fact the first few volumes of occluded hydrogen seem to be more effective, in increasing resistance, than similar quantities added later in the charging process. The two curves are noticeably similar, and show a marked departure from the straight line. The writer hopes to repeat these experiments, varying the conditions somewhat, as regards rapidity of charging, temperature, etc., and, perhaps, using apparatus of different design, in the hope of fixing, more conclusively, the exact shape of the curve.

The maximum quantity of hydrogen occluded is, in the case illustrated by Fig. 8, 130 cu.cm. or about 1030 volumes. This increases the resistance from 0.0927 ohm to 0.1558 ohm, an advance of 68 per cent. It is believed that the last point of this curve corresponds, practically, to saturation of the wire; after the data for this point had been taken, the charging current was allowed to continue for twelve hours, and the resistance was measured at the end of the first five hours and again at the end of the twelve hours, but the results were practically those obtained for the last point of the curve. No attempt was made to measure the volumes of escaping hydrogen during this twelve-hour run. Knott found a maximum increase of resistance of about fifty-two per cent (0.1865 ohm to 0.285 ohm), corresponding to an occlusion of from 900 to 1000 volumes. Graham, in Poggendorff's Annalen for 1869, states that the conductivities of pure palladium and the hydrogen-charged palladium are as 5.99 to 8.10. He does not state the exact quantity of gas occluded in the hydrogen-charged wire, but elsewhere in the same paper describes experiments in which 900 volumes were absorbed.

The method employed in these experiments seems to be capable of results more accurate than those obtained by the investigators named above, and these results would seem to indicate that the resistance of the palladium wire, as hydrogen is absorbed, increases rapidly from the very start; it also appears that the increase of resistance is not strictly proportional to the volume of hydrogen occluded.

Grateful acknowledgment is made of many valuable suggestions and much assistance received from Professor E. H. Hall, of Harvard College.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY,
HARVARD COLLEGE.

EXPERIMENTS ON THE DEVIATIONS OF FALLING BODIES.

BY EDWIN H. HALL.

EXPERIMENTS ON THE DEVIATIONS OF FALLING BODIES.

By Edwin H. Hall.

Presented June 17, 1903. Received November 14, 1903.

This paper is a supplement to one recently published in the Physical Review * under the title, "Do Falling Bodies move South?" It will give some account of the preliminary experiments and variations of method which the author made in the course of his study of the behavior of falling bodies, and will discuss to some extent sources of possible error in the result arrived at; a result which is in favor, so far as it carries weight, of an affirmative answer to the question stated above. It is unlikely that such details as are here to be given will interest any large number of readers, but they may be of considerable value to any investigator who may hereafter occupy himself with this still unsettled question.

Cajori remarks that all experimenters in this research have used metal balls, and therefore I, desiring to vary the conditions, undertook at first to use spheres of ivory, about 2.54 cm. in diameter. They were made by a manufacturer of billiard balls, and were, presumably, good specimens of workmanship in ivory, but under the calipers they showed themselves not to be true spheres, differences of diameter along different axes being very perceptible, as large, perhaps, as 0.01 cm. A number of experiments were made in dropping these balls; but they were so erratic in their fall that I felt obliged to give up the hope of getting any valuable result from them, and I therefore resorted to balls of bell-metal, which, to my surprise, I found already in the hardware market, almost perfect in form.

It now seems probable that the unsatisfactory performance of the ivory balls was in part due to a cause which at first gave me much trouble in

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* Part I., Historical, in September, 1903; Part II., Methods and Results of the Author's work, October, 1903.

the use of the metal balls, an occasional slight adhesion of the ball to one side of the hole from the bottom of which it was released (see top of Fig. 1), due to the accidental presence there of a bit of wax intended for use on the suspending thread. This source of error was discovered April 17, 1902, and wax was thereafter avoided. Moreover, as each ball, in spite of considerable care given to making it clean before dropping, might possibly leave some small amount of grease adhering to its circular line of contact with the release plate, this part of the plate was, in all later work, wiped after the dropping of each ball. The usual instrument employed for this purpose was a button of cork fastened at the lower end of a wire.

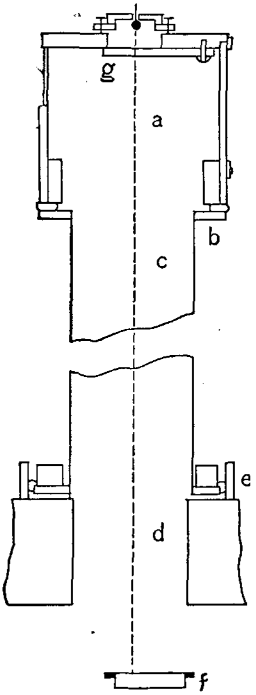

FIGURE 1.

Previously I had attributed the occasional very bad behavior of a ball to a jerk given by a stray fibre of the silk suspending thread catching on some little roughness of the neighboring metal; and it is probable that some of the irregularities observed were due to this cause. More than once I heard a sharp *clink* from the pull of a fibre when a suspended ball was slowly revolving. It was to prevent such complications that wax had been used on the thread. Fine copper wire was for a time used in place of silk thread for the suspension. It soon appeared, however, that the time required for the melting off of the wire, though short, was long enough to allow the ball, before full release, to swing off a trifle from the side on which the flame was applied to the wire. On April 23 and thereafter, fine cotton thread was used for the suspension, in the hope that its fibres would prove less troublesome than those of the silk; but, as an additional precaution, that part of the thread from which trouble was feared was singed, just before the ball was put into position, by passing it very quickly through a Bunsen flame.

After these changes of practice had been made, cases of exceedingly bad behavior on the part of the balls, such as occurred often before, were very infrequent, and I was henceforth able to make no great improvement in their performance. It has already been stated that considerable irregularity marked their behavior to the very end of the investigation. This irregularity was probably due to a variety of causes, among which the most obvious were, initial sidewise motion, due to vibrations of the tower, and variations of hardness in the arresting mixture. The method of release used, or, rather, the method of fixing and controlling the ball just before release, is evidently one which makes the ball share any accidental slight movements of the top of the tower, such as are maintained almost constantly during the day, and much of the time at night, by disturbances coming through the ground from machinery or street-cars or athletic exercises in the vicinity of the laboratory. Difficulty from this cause was reduced by working at night, and by releasing the ball at an instant when a mercury surface, in a glass vessel resting on the top of the box *a* (Fig. 1), gave an apparently undisturbed image of crossed threads attached to the side of the vessel.

At one time, about the middle of April, the experiment of releasing the ball from a condition of free suspension just beneath the upper plate was tried. This method required sharp scrutiny of the ball or of the suspending thread to determine whether pendulum movements of serious magnitude existed before release, and was therefore very trying to the nerves, besides being comparatively slow. Moreover, the pendulum movements never did quite die out, and the performance of balls dropped from a condition of free suspension seemed no better, on the whole, than that of balls dropped from the condition of suspension controlled by contact. Before April 26 I abandoned the method of free suspension.

April 18, I tried another experiment, using one of the bell-metal balls as a counterpoise for the similar ball to be dropped, and making the contact of the latter with the plate as light as it could well be made. This method of procedure also was tedious and not particularly successful, so that it was soon given up. Once or twice afterward, on May 5, for example, one of the balls was used as a counterpoise, no unusual attention being paid to the character of the contact, which of course must have been rather light; but I presently came to the conclusion that better results were obtained by means of the usual counterpoise, which weighed 93 gm., about 20 gm. more than the weight of a ball. This held the ball firmly in place till the thread was burned, and though the pressure was doubtless in many cases considerably greater on one side of the circle of contact than on the other, any lateral thrust from this source after the burning of the thread must have lasted an exceedingly short time, too short for any harm. The method of release to which I finally settled down, and which has now been sufficiently described, has the merit of being very expeditious, the twenty-four balls constituting an evening's tale being, as a rule, dropped in less than one hour and a half.

At the receiving end also of the apparatus there was considerable experimenting. At first the pans were not fastened in the supporting seat in such a way as to preclude the possibility of rotation under the shock of the balls, and it is quite possible that slight rotations did occur, sometimes in one direction and sometimes in the other, their net effect being presumably zero. A locking device for preventing accidental rotation was first used April 7; but until May 21, the screw then adopted as a part of the lock in Fig. 2, was not employed, a wedge being inserted in the lock beside the stud when it appeared to be needed.

Previous to April 16, but not later, the receiving pan (see Figs. 4 and 5), after catching ball No. 1, was turned counter-clockwise to take position for ball No. 2, in the same direction for ball No. 3, and so on. This practice dropped the 1st and also the 6th ball into a field symmetrical as to its north and south aspects; but it dropped the 2d, 3d, 4th, and 5th balls into a field having a ball already lodged in the southerly quarter, with no corresponding ball in the northerly quarter. The average clear distance between neighboring balls in the pan was probably rather more than 3.5 cm., and it seemed unlikely that, as a rule, the lodgment of a ball in the tallow would be appreciably affected by the lodgment of an earlier ball 3.5 cm. away; but in some cases the tallow was inclined to crumble for a considerable distance about each ball, and in such cases there was sometimes a crumbling all the way through from a ball to its nearest neighbor. Evidently the practice of turning the pan always in such a direction as to make the majority of the balls fall to the north of their nearest predecessors might make the crumbling more prevalent on the south than on the north of the balls at the critical instant of lodgment; and therefore the displacement of the balls due to this crumbling might show a balance toward the south. To avoid this possibility the practice was adopted of rotating the pan first in one direction and then in the other, so that, after April 16, the order of arrangement of the six balls in any pan corresponded to the order of arrangement of the six studs projecting from the pan, as numbered in Fig. 5.

Too little attention was paid, at first, to the telescopes with which observations on the plumb-line and on the point above the balls (see Fig. 4) were made. As the depth to which any ball would sink in the tallow was somewhat uncertain, it was not possible to adjust the telescopes in advance so that the tip of the plug above the ball would fall just at the

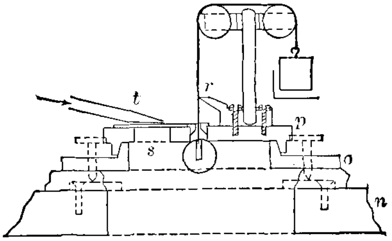

FIGURE 2.

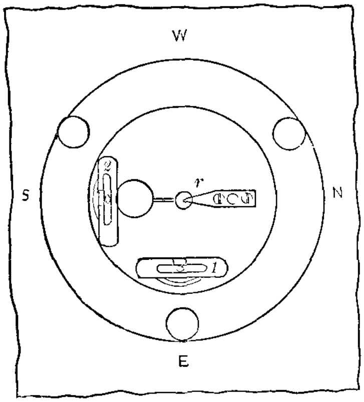

FIGURE 3.

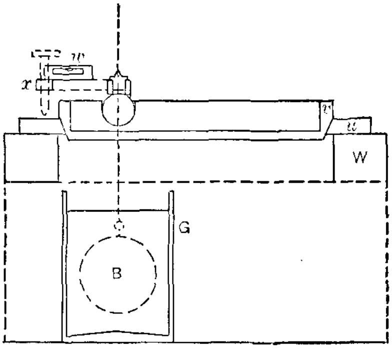

FIGURE 4.

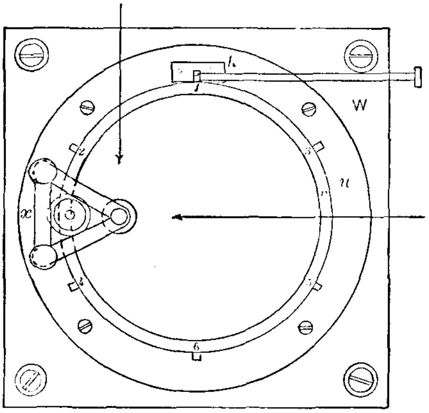
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FIGURE 5.

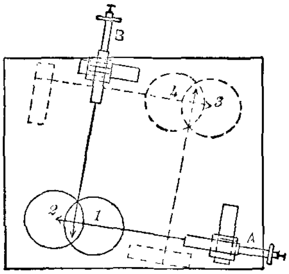

FIGURE 6.

level of the centre of the field of view; it might be two or three millimeters too high or too low. It would not do to readjust the telescope for each ball; and therefore it was highly important that the micrometer cross-hair, which in one part of the work was set on the plumb-line and in another part was set on the tip of the plug, should be strictly vertical. This condition was readily attained during the observations on the plumb-line, with which the work of each evening began; but the jar of the balls dropping into the pans had a tendency to loosen for a moment the fastening of the telescope, and during this instant the unbalanced weight of the micrometer screw, etc., sometimes caused a slight rotation of the instrument, with corresponding inclination of the cross-hair from the vertical. The full measure of the danger of error from this quarter was not realized before April 23; but after that date, and especially after May 23d, much care was given to the telescopes, and their performance was, on the whole, satisfactory.

It has already been intimated that the tallow mixture used in the receiving pans gave more or less trouble. At the temperature about 15° C., which prevailed in the base of the tower during the earlier weeks of the work, pure tallow was rather too hard, crumbling too much or cracking under the impact of the balls. To remedy this defect lard was added. After some weeks the temperature in this part of the tower rose a degree or two, and then the mixture of tallow and lard was too soft, so that the balls fell through it and struck the bottom of the pan.

Then beeswax, added to the tallow and lard, was used to give the right degree of hardness. The necessity of such adjustments was a little troublesome, but there were more serious difficulties. No mixture which was used appeared to be homogeneous when cold. All were more or less mottled, whiter in some places than in others, and the whiter spots appeared to be softer than the others. This condition of the arresting medium probably accounts in some measure for the irregularities of position of the arrested balls, the mixture giving way more readily on one side of a ball than on the other side. Such irregularities, however, must in the long run come to naught, if the variations of hardness are purely accidental in their distribution. They prolong the research, but do not affect the final result. Variation of hardness with distance from the edge of the pan has already (in a preceding paper) been mentioned, and the means taken for eliminating, in some measure, error from this source have been described. Such error would apparently not affect the southerly deviation of the balls; but it probably does affect to some extent even the final result along the east and west line; for the behavior of the tallow could not be kept uniform during all the changes of position of the pans indicated by Fig. 6. Accordingly, I regard the mean value found for the easterly deviation as entitled to considerably less confidence than the mean value found for the southerly deviation.

At times the tallow mixture adhered well to the bottom and side of the pan. At other times it would loose hold on the pan, during cooling, to such an extent as to be easily removable in a single cake. To prevent error from this condition it was the practice, whenever there was a gap between the tallow and the side of the pan, to force slips of paper on thin cardboard into the gap in such a way as to pack the tallow in place. Moreover the studs, which (see Fig. 5), on and after April 7, projected from the outside of the edge of the pan, were really the heads of screws which extended a considerable distance into the tallow, and these screws helped much to hold the tallow from rotation or sliding.

On July 10 soft wood (white wood) about 2 cm. thick was used in two of the pans instead of tallow. The mean of the range of balls in these two pans on this night was decidedly less, in both latitude and longitude, than the usual range in pans filled with tallow. Unfortunately, however, the impression made by a ball dropping upon wood is elliptical, because of the grain, so that when the ball is replaced in the hollow it has made it does not lie securely, but is likely to roll or rock a little, making determination of its proper position somewhat difficult. The results for the night of July 10, with wood, were omitted from the final calculations.

Two possible sources of error remain to be considered, — the action of air and the faults of levelling. Great care was taken from the first to prevent or reduce draughts. The description, previously given, of the apparatus shows that chinks at the releasing end were stuffed with cotton, if not otherwise closed. The cotton may have been slightly displaced at times, but certainly there was very little freedom of circulation through the top inclosure just before or just after a ball was dropped. The cloth tube (*c* in Fig. 1) was continuous, being woven in one piece, and at its lower end the chink about it was stuffed with cotton. Below this tube the balls fell about two meters through air inclosed only by the walls of the constant temperature room, which is about 6 m. long, 4 m. wide, and 3 m. high. The one door leading into this room was closed while the balls were falling. Two incandescent electric lamps of eight or sixteen candle power were available for lighting the room; but usually only one of them was in use at a time. These were suspended in such a way as to light up the plumb-line and the brass tip over the balls to advantage, one hanging nearly over the line of sight of telescope No. 1, the other nearly over the line of sight of telescope No. 2, the one nearest to the line of fall being, perhaps, 35 cm. distant from it. The question whether one of these lamps could by its heating effect maintain a circulation of air sufficiently brisk to affect the course of the descending balls appreciably was often in my mind. I found, however, by trial, that nowhere was the *horizontal* current caused by the lamp strong enough to show any effect on the most sensitive anemometer at my service, — an instrument made by Hicks, of London, for which the correction for friction is given as 30 feet per minute. For a short distance directly above the lamp the *upward* current was able to keep the vanes in motion; but it is evident that the disturbance of the air by the heat of the lamp is very slight along the line of fall of the balls, altogether too slight to produce in the very little time, perhaps 0.1 second, during which the ball is falling through the free air of the room, a horizontal deviation discernible in this investigation.

It seemed quite possible, however, that the circulation of air due to the lamp might affect perceptibly the position of the lower end of the plumb-line. The weight of the ivory plumb-ball being about 82 gms. in water, and the length of the suspension being 23 m., the horizontal force which, applied at the lower end of the suspending wire, would have kept it 1 mm. aside from its proper position, is less than 4 milligrams. Careful experiment showed, however, that the heat of a 16 c. p. lamp placed about 30 cm. from the wire, not far from its lower end, had so little effect on the position of the wire that I could not make sure of perceiving the deflection. It was, apparently, less than 0.001 cm.

But the very motion of the falling ball must have produced within the long cloth tube a very considerable disturbance of the air. Could such a disturbance affect the line of fall? If the tube were exactly cylindrical and the line of fall strictly along its axis, the symmetry of the conditions would answer this question in the negative. The tube was nearly cylindrical and the line of fall coincided nearly, but not perfectly, with its axis. How much deflection would be produced by a slight eccentricity of this line of fall? This question I found myself unable to answer without special experiments, which accordingly were undertaken. October 15, 1902, I dropped twelve balls with the spout displaced about 10 cm. toward the south, and twelve balls with the spout displaced about 10 cm. toward the north. The first set were deflected on the average about 0.18 cm. towards the south and the second set about 0.46 cm. toward the north, a mean deflection of 0.32 cm. away from the nearer side of the spout.* It is possible that in these trials, at some points of the fall, the clear distance between the ball and the nearest part of the spout was not more than 5 cm. On October 18 a set of twelve balls was dropped with the spout 5 cm. north from its original symmetrical position, and another set of twelve balls with the spout 5 cm. south from its symmetrical position. The first set fell about 0.004 cm. north and the second set the same distance south from the plumb-line, a mean deflection of 0.004 cm. away from the nearer side of the spout. On October 20 the experiments of October 18 were repeated with the same displacements of the spout, but on this last trial an average deflection of about 0.04 cm. away from the nearer side of the spout was observed. We have, then, according to these not very accurate tests: †

Eccentricity in spout.	Deflection due to eccentricity.
10 cm.	0.32 cm.
5 cm.	0.02 cm.

During the general course of the experiments the eccentricity of the line of fall was probably less than 1 cm., and, during the latter part of the time at least, it was in such a direction as to produce a tendency toward northerly deviations of the balls. This tendency must have been very slight indeed, however; too slight to deserve further consideration.

The possible errors from inaccurate levelling fall naturally into two classes: 1st, those which are made at the sending apparatus; 2d, those made at the receiving apparatus. The errors of the first class are due to the fact that, whereas the position of the *plumb-line* at its top is determined by the position of the "beak" (*r* in Fig. 2), the position of the *ball* just before release is determined by its contact with the lower end of the central hole in the release plate. The beak is about 4.5 cm. above the centre of the ball in position, and any considerable error in levelling would evidently put the tip of the beak a perceptible distance to one side from the vertical through the centre of the ball. Much care was given to this levelling; but even a careful observer may have the habit of setting the bubble a little to one side of its proper position, always to the right, let us say; and against error from such a habit the device of turning the plate through 180° in the course of the observations is no safeguard. If the observer could have moved about so as to be half the time on the east and half the time on the west of the apparatus, his trick of the eye could have been eliminated from the average conditions governing the experiments; but this was not the case. Some of the levellings were made by my assistant and some by myself, and accordingly it became necessary to study the "personal equation" of each of us in the use of the levels. This I did July 29, 1902, when it appeared that both had the habit of setting the bubble too far to the right, which would carry the plumb-line to the south of the release position of the balls and therefore tend to cause an apparent northerly deviation of the balls in falling. This effect, according to the observations of July 29, was probably somewhat more than 0.001 cm. It is not likely to have been as large as 0.002 cm.

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* Newton, in suggesting to Hooke a study of the course of falling spheres, warned him that "in a narrow well the bullet possibly may be apt to receive a ply from the straitened air neare the sides of the well, if in its fall it come nearer to one side than to another."

† In these experiments of October, 1902, lead was used instead of tallow as the receiving medium. The individual variations of position of the fallen balls were perhaps as great as when tallow was used; but with lead twelve balls could be received in one pan, and changes of temperature at the foot of the tower were no longer troublesome. In further experiments I should use lead.

As to faults of levelling at the receiving apparatus, the changes of position of this apparatus, and of the observer with respect to it, were such as possibly to eliminate error due to such faults from the final result; but whether it was perfectly eliminated, I have some doubt, as this levelling was less careful than that at the sending apparatus. There were two places for faults, one in levelling the ring which supported the receiving pan, the other in levelling the tripod which held the tip of brass above the imbedded ball.

The latter operation was performed for every ball, and as the observer was about half of the time south, and half of the time north, of the apparatus during the levelling, his errors in this particular should very nearly eliminate each other. The levelling of the supporting ring was not so often or so systematically done. On July 24, almost at the end of my main series of observations, I found the north side of the ring to be a little higher than the south side, the inclination being, perhaps, 5′. This would make the north side of the one inch ball fall through about 0.0036 cm. greater thickness of tallow than the south side. The effect of such a dissymmetry is rather difficult to estimate, but it seems unlikely that it would push the ball as much as 0.001 cm. toward the south.

The suggestion having been made that the copper wire used for the plumb-line might be sufficiently magnetic to suffer a slight deflection from the vertical toward the direction of the magnetic dip, I made such experiments as seemed likely to meet this suggestion. For this purpose I placed perhaps 10 cm. of the wire, not far above the ball as the whole hung suspended in place, in a horizontal magnetic field which probably averaged, for the 10 cm. mentioned, a strength more than three thousand times that of the earth's horizontal magnetic field. I then made observations on the position of the wire with *field off*, *field on* (*direction north*), *field on* (*direction south*), repeating the changes a number of times. I found no effect of which I could be sure, although the conditions were such as to make the magnetic deflecting action on the wire perhaps twenty or thirty times as great as the corresponding action which could have been exerted by the earth's magnetic field during the ordinary course of the experiments.

An attempt to measure approximately the magnetic deflection of an iron wire, substituted for the copper wire in the artificial magnetic field, failed, for the reason that a bit of thread, used to piece out the suspension at the top of the iron wire, by its continual untwisting kept the plumb-ball in a state of slow rotation, which produced a deflection unless the ball happened to be placed in a perfectly symmetrical position in the vessel containing the water surrounding it. The rotating ball acted as if slightly repelled by the nearer side of the vessel.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY,
HARVARD COLLEGE.

ON THE RELATION OF THE HALL EFFECT TO THE CURRENT DENSITY IN GOLD.

BY THOMAS C. MCKAY.

ON THE RELATION OF THE HALL EFFECT TO THE CURRENT DENSITY IN GOLD.

By Thomas C. McKay.

Presented by E. H. Hall, June 17, 1903. Received December 10, 1903.

This subject was suggested to the writer by Professor Hall. The work was carried on under his supervision in the Jefferson Physical Laboratory of Harvard University. The subject was taken up as a consequence of a paper of Moretto,* published about four years ago, in which measurements of the Hall effect over a very large current range were described, and results obtained at variance with the generally held theory of the phenomenon. The results of previous investigators are given first, with a general account of their work, in order that the subject of inquiry may be clearly set forth.

The first observations on this subject were made by Professor Hall,† on the plate with which he discovered the general law of the phenomenon. This was a strip of gold leaf, about 9 centimeters long and 2 centimeters wide, mounted on a piece of plate glass. Contact was made with the ends of the strip by pressing on them two pieces of polished brass. These were connected to the terminals of a Bunsen cell. The terminals of a high resistance Thomson galvanometer were connected to two opposite points on the edges of the strip about midway between the pieces of brass. The glass plate bearing the gold was cemented to the flat end of one pole of the electromagnet and the other pole brought to within 6 millimeters of the strip. The following results were obtained: —

C.	M.	c.	$\frac{C \times M}{c}$.
.0616	11420 H	2.32×10^{-9}	3.03×10^{11}
.0249	11240 H	0.85×10^{-9}	3.29×10^{11}
.0389	11060 H	1.35×10^{-9}	3.19×10^{11}
.0598	7670 H	1.47×10^{-9}	3.12×10^{11}
.0595	5700 H	1.04×10^{-9}	3.26×10^{11}

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* Moretto, Nuovo Cimento, Serie 4, **11**, 278–289 (1900).

† American Journal of Mathematics, **2**, 287–292 (1879).

H is the horizontal intensity of the earth's field, about 0.19; C is the strength of the primary current; M is the strength of the magnetic field; c is the current through the Thomson galvanometer.

This showed the general law of the phenomenon for a plate of a given thickness, since it showed $\frac{C \times M}{c}$ to be at least approximately a constant within the range of primary current used.

Righi * made a series of measurements maintaining the magnetizing current constant. He used gold, which he prepared according to the method of Professor Hall. He found the effect observed proportional to the current, except for weak currents, for which the effect was relatively greater. The differences were always very small and Righi thought that they might have been due to some unknown error. The tabulated results are not given in his paper.

Leduc † made a similar series of experiments on bismuth. The transverse difference of potential caused by the introduction of the magnetic field was measured with a capillary electrometer. The following table of results is given.

C.	I.	E.	$\frac{E}{C\,I} \times 10^6$.
5.77	5.127	0.01248	421
5.98	3.572	0.00895	419
5.97	2.119	0.00531	420
5.93	1.386	0.00343	417
5.92	0.823	0.00205	421
5.90	0.647	0.00159	417
5.87	0.330	0.00081	419
			Mean, 419

C is the intensity of the current in the electromagnet; I is the intensity of the primary current in the plate; E is the difference of potential at the secondary electrodes, caused by the magnetic field.

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* Memorie della Accademia delle Scienze di Bologna, p. 115 (1883).

† Comptes Rendus, **98**, 673–675.

The expression $\frac{E}{C\,I}$ is here seen to be practically constant over the range used.

Moreau[*] made a series of measurements in very thin silver and nickel plates. These metals were deposited on glass plates. The following results were obtained with a silver plate, in which the main current was varied while the magnetic field was kept constant:

E — 81 $\mu\mu$
H — 3300

I	70	57.2	50.4	41.5	37.3	27.2	24.5	18.2
Δ_1	127	103	92	74	67	48	44.5	32
$\frac{\Delta_1}{I}$	1.81	1.80	1.82	1.78	1.79	1.76	1.80	1.76

E is the thickness of the plate in millionths of a millimeter; I is the strength of the primary current in milliamperes; Δ_1 is the deviation of the galvanometer in the secondary circuit; H is the strength of the magnetic field.

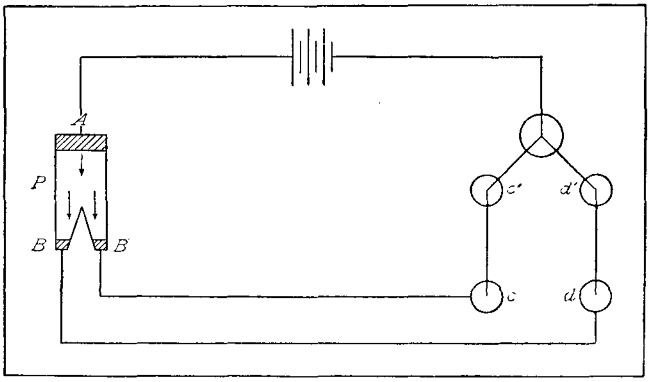

FIGURE 1.

The greatest range of primary current seems to have been obtained by Moretto.[†] The only metals which he mentions in his paper as having been worked upon are gold and bismuth. In preparing these for the measurement of the Hall effect he used the form of plate employed extensively by Righi; see P, Figure 1.

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[*] Journal de Physique, Serie 3, **10**, 478–493 (1901).

[†] Nuovo Cimento, Serie 4, **11**, 278–289.

[*] Moreau also investigated the influence of the thickness of the metal plate. He found that when the thickness of the plate was reduced below 44 $\mu\mu$ the Hall effect was less than the value calculated from measurements of thick plates.

It has been observed by various experimenters that the Hall effect in gold leaf is less than that of thick gold.

Thus the current entering the plate at *A* was divided into the two partial currents at *B* and *B*. On the introduction of the magnetic field one partial current was increased at the expense of the other above its normal value, but correspondingly decreased on the reversal of the field.

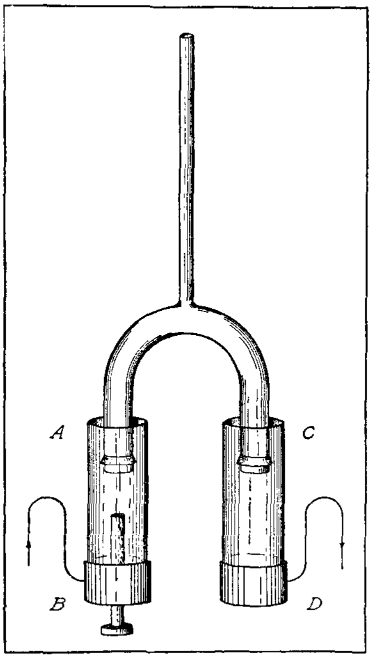

Figure 2.

At *c* and *c′*, *d* and *d′*, connection was made with the voltameters which were placed in each branch. The voltameters were of the following form. See Figure 2.

The jar *AB* contained a saturated solution of copper sulphate, the jar *CD* a solution of ten per cent by weight of sodium sulphate. The ends of the curved tube were covered with pieces of animal membrane. It was filled with a solution of sodium sulphate. The current was made to pass from the copper sulphate solution to the sodium sulphate solution. The amount of sodium hydroxide formed at the cathode was measured by a method of volumetric analysis suggested by Cardani,* for use in measuring the quantity of electricity in the discharge of a condenser.

Cardani added measured quantities of a $\frac{1}{200}$ normal oxalic acid solution to a nine per cent sodium chloride solution which had been around the cathode while the current was passing. Using phenolphtalein as indicator, he endeavored to obtain by titration the same degree of redness as was shown by the original weakly alkaline sodium chloride solution before the passage of the current. In Moretto's work the strengths of the two partial currents were measured, first without the magnetic field, again with the magnetic field in a given direction, then with the magnetic field in the opposite direction, and lastly without a field. A numerous series of determinations was made which gave results of the kind indicated by the following table, the only table of results bearing on our question in Moretto's paper.

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* Nuovo Cimento, Serie 4, **7**, 105 (1898); also Zeitschrift für physikalische Chemie, **27**, 378 (1898).

STRENGTH OF FIELD — 9600 C. G. S. UNITS.

I Amperes.	Without Field.	Field in a given Direction.	Field in opposite Direction.	w Amperes.	$\frac{w}{I}$
0.2820	q_1 4.300 q_2 4.480	4.315 4.465	4.285 4.495	0.00096	34×10^{-4}
0.1149	q_1 7.085 q_2 7.230	7.110 7.250	7.060 7.255	0.000401	34×10^{-4}
0.0459	q_1 8.420 q_2 8.755	8.450 8.725	8.390 8.785	0.000161	35×10^{-4}
0.0164	q_1 3.045 q_2 3.100	3.058 3.088	3.034 3.112	0.0000642	39×10^{-4}
0.0129	q_1 2.115 q_2 2.710	2.124 2.700	2.106 2.720	0.0000509	40×10^{-4}
0.00445	q_1 4.390 q_2 4.475	4.408 4.458	4.370 4.494	0.0000186	42×10^{-4}
0.000645	q_1 1.985 q_2 2.030	1.995 2.022	1.976 2.039	0.00000289	44×10^{-4}
0.000616	q_1 1.880 q_2 1.955	1.890 1.948	1.872 1.964	0.00000273	45×10^{-4}
0.0000285	q_1 2.115 q_2 2.435	0.916 1.038	0.862 1.005	0.000000155	54×10^{-4}

q_1 and q_2 are the quantities in cubic centimeters of a $\frac{1}{200}$th normal solution of sodium hydrate, formed by the passage of the two partial currents respectively; I is the main current, and equal to the sum of the two partial currents; w is obtained by taking the difference made in each partial current by reversing the field, summing these differences, and dividing by 2. Moretto does not state the metal used in obtaining this series of results. It was probably bismuth.

The measurement of w depended on a small difference of the partial currents, which were not measurable with the highest accuracy. An accuracy of one-tenth of a per cent in the measurement of four cubic centimeters of a $\frac{1}{200}$ normal solution by volumetric analysis would be very good work. In the opinion of the two instructors in chemistry consulted by the writer, it was considered very improbable that so great accuracy could be obtained. Taking the set of observations above in which the main current was 0.2820 amperes, we find that an error of one-tenth of one per cent would mean possible errors of 0.004 in each measurement of the q's. The difference obtained on reversal of the magnetic field is 0.030. The error in w might then be 25 per cent. The possible error would be greater in the case of some sets of observations where the amount of sodium hydroxide formed was less than in the above case. In general in the above table smaller values of q correspond to smaller values of I. The error in the measurement of w would therefore in general be greater for the series in which I is small. Accordingly, the values of w obtained from the above table for different values of I seem to the present writer to be not very reliable, particularly in the case of the smallest values of I. There is an obvious error in the record of the results given for the primary current 0.0000285.

The present writer used in his measurements the same general method as that adopted by Professor Hall and described in the account of the latter's work at the beginning of this paper.

In order to measure the transverse current with more accuracy, the present writer made a sensitive Thomson galvanometer of low resistance.

Description of Astatic Galvanometer.

The general plan followed in constructing this galvanometer was that which Paschen describes in the Zeitschrift für Instrumentenkunde, 1893. In order to make the suspension nearly astatic the more easily, about sixty small magnets were used in the two systems. These magnets were made of fine hair-spring. They were 2 to 3 mm. long and about 0.2 mm. wide. The distance between the systems was 5 cm. A mirror 8 mm. long, 2 mm. wide, and 0.4 mm. thick, was made from glass which had been ground plane on both sides by Zeiss of Jena. A very fine quartz fibre was used to suspend the system, which system as finally used had in the earth's field a period of eight seconds. Number 22 copper wire, Brown and Sharp gauge, was wound on a cone of 1.4 cm. height, the radius of the base being 1.9 cm. This was wound nearly full with wire, so that the coil and cone made a cylinder 1.5 cm. in height and nearly 1.9 cm. in radius. Four of these coils were used in two branches of two each. The resistance of this arrangement was 2.22 ohms at 16.3° C. By means of a control magnet placed about half-way between the upper and lower systems and 20 to 30 cm. away, the period was increased to give the required sensitiveness. A second control magnet placed nearly two meters away sufficed to direct the system without producing any considerable change of sensitiveness.

With the needle 1.7 m. from the scale, and a period of complete vibration of the needle of 12 seconds, the deflection was 1 mm. for 7.7×10^{-10} amperes.

Considerable difficulty was experienced on account of induced currents from outside electric disturbances, such as those caused by the passage of electric cars in the neighborhood. In reading the deflection of the galvanometer needle, an effort was made to get the mean position of the needle at times when it did not seem to be influenced by these induced currents. To make less the error caused by the wandering of the needle the difference of position on reversal was made large, in nearly every case greater than 18 cm. on the scale. The vibration of the needle when the reading was taken was often as much as 2 cm. or 3 cm. on the scale. To lessen the error, five or seven or sometimes more reversals were made in a single set of observations with given conditions. At first it was attempted, before and after each measurement of the transverse effect, to measure also the direct effect of the electromagnet, about 45 m. away, and of the lead-wires of the magnetizing current, which came within 3 meters of the galvanometer. This effect was small in comparison with the transverse current deflection and could not be measured with certainty.

Resistance of Transverse Circuit.

The resistance of the galvanometer was measured on the Carey Foster bridge in comparison with coils of manganin wire of temperature-coefficient 0.00002 per degree centigrade. These coils were made by Mr. Persons and by Mr. Coulson of the Jefferson Physical Laboratory. Some of them were made by direct comparison with standard coils made by Wolff of Berlin and tested at the Physikalisch-Technische Reichsanstalt. The other coils were made by indirect comparison with these standard coils. All such coils of manganin wire made in the above way by Messrs. Persons and Coulson are referred to in this paper as standard coils.

Two measurements of the resistance of the galvanometer were taken with an interval of four months. The first gave 2.243 ohms at 17.2° C.; the second, 2.223 ohms at 16.3°C. The difference between these values when correction is made for difference of temperature is 0.011 ohms. The second value was used, that having been taken during the period when the determinations of the Hall effect were made. During these determinations a thermometer was kept with its bulb in the interior of the galvanometer case, and the temperature recorded. In making correction for the temperature, the value 0.0039 was used as the temperature coefficient of the copper wire of the coils.

The resistance of the remainder of the secondary circuit was measured frequently, and, in the case where the effect in gold *leaf* with weak currents was being determined, before and after each measurement of the transverse current. The measurement of this resistance was also made on the Carey Foster bridge. With gold of thickness of 0.0002 cm. and above, the variation of the resistance of the gold and of the connecting wires outside the galvanometer did not amount to more than 0.03 ohms. The variations of the resistance in the gold *leaf* will be referred to below. In these resistance measurements much use was made of a box of coils made by Morris E. Leeds and Co. of Philadelphia. The temperature of these coils was noted. They were compared with standard coils. A measurement of the temperature coefficient of the Leeds coils gave + 0.03 per cent per degree centigrade. The other coils used were manganin coils, whose temperature coefficient was given by Messrs. Persons and Coulson as 0.00002 per degree centigrade.

MEASUREMENT OF SENSITIVENESS OF GALVANOMETER.

The sensitiveness of the astatic galvanometer was in almost every case determined before and after each set of consecutive observations of the transverse effect. The following diagram (Figure 3) will explain the arrangement for the measurement of the sensitiveness.

The resistances whose values are given in this figure were standard resistances. Connections were made between them by means of mercury cups set in heavy bronze blocks. A double commutator was used to reverse the current from the storage cell and that from the Carhart-Clark cell simultaneously.

The certificate of the Carhart-Clark cell used as a standard dated from 1896. A similar cell, which had been put up in the same case with it, at the same time, agreed with it to about one-tenth of one per cent. On comparison with a cadmium cell and another Carhart-Clark

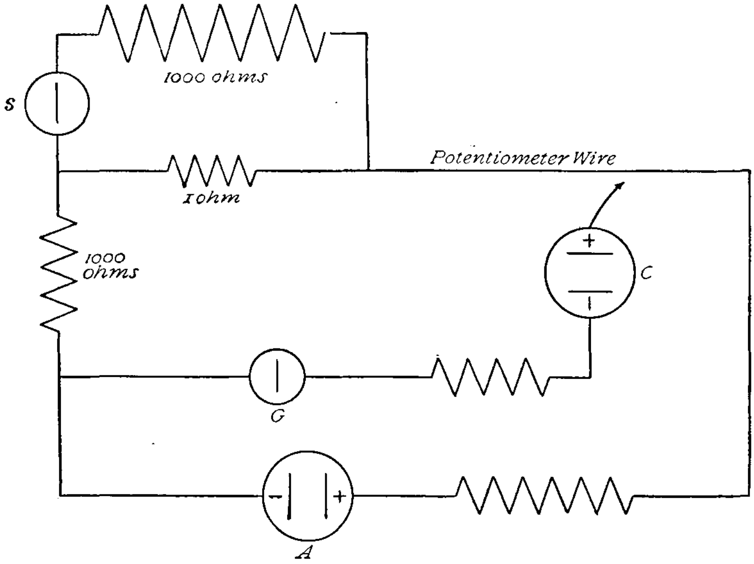

FIGURE 3.

S is the galvanometer whose sensitiveness is to be measured.
C, a Carhart-Clark cell. *A*, storage cell. *G*, a sensitive galvanometer.

cell, both recently put up, the Carhart-Clark cell, which had been used by the writer, was found to have an electromotive force of 1.426 volts at 21°C., instead of 1.437 volts, which was the value derived from the certificate. The value of the *e. m. f.* as corrected by the cadmium cell was used in the calculations below.

MEASUREMENT OF PRIMARY CURRENT.

The primary current, when above 0.01 ampere, was measured with a commercial ammeter which had been corrected at various points as described below. At first a Siemens and Halske ammeter was used; but in most of the measurements in which currents of the above order were used, a Weston ammeter was used in series with the Siemens and Halske ammeter.

For currents from 0.025 ampere up to 0.72 ampere the ammeters were compared with a tangent galvanometer which is described below under the account of the measurement of *H*. In this comparison the 22 turns of wire for which the constant of the galvanometer had been calculated were used. To correct the ammeters for currents from 0.72 ampere to 2.2 amperes, four turns of wire were wound on the original layer of the tangent galvanometer. As these turns did not fill the breadth of the channels, they were kept in position by wedges of cork placed at frequent distances around the circumferences. Since they could not be made to lie as evenly as the 22 turns which filled the breadth of the channel, the constant for these four turns was determined from the value of H, measured as described below, by passing a current through two copper voltameters in series with the galvanometer. Two measurements of this constant, made with the copper voltameter, using the value 0.1630 for H, gave 1.1920 and 1.1926 respectively. An attempt was made to determine the constant for the four turns from the constant for the 22 turns, as calculated from the dimensions, as follows. A current was sent through the 22 turns and the four turns in series, first in such a way that their effects on the needle agreed, and second in such a way that they were opposed. If G_1 and G_2 be the respective constants,

$$H \tan \theta = \gamma\,(G_1 + G_2) \text{ for the first case,}$$

and

$$H \tan \theta' = \gamma\,(G_1 - G_2) \text{ for the second case,}$$

where θ and θ' are the respective deflections, and γ is the current in c. g. s. units.

Hence
$$G_2 = G_1 \times \left\{ \frac{\tan\theta - \tan\theta'}{\tan\theta + \tan\theta'} \right\}$$

The values of θ and θ' were 47° 6′ and 36° 13′. G_1, calculated as described below for the 22 turns, was 6.294. Hence $G_2 = 1.196$, which agrees well with the value for G_2 given above, considering the small difference between θ and θ' and the possible error in the value of H in the previous determinations.

This constant having been determined, the ammeters could be calibrated for currents from 0.72 amperes up to 1.65 amperes for the Siemens and Halske ammeter, and to 2.2 amperes for the Weston. The Weston ammeter was provided with three shunts, the upper limit of the strength of current which could be measured being 0.1 ampere, 1 ampere, and 10 amperes in the three cases. The Siemens and Halske instrument was a milliamperemeter of 1 ohm resistance, reading up to 150 milliamperes. A shunt, of 0.1 ohm resistance, was made of manganin wire. The lead-wires were soldered on the terminals of this shunt coil. By means of this shunt, therefore, the range was extended to 1.65 amperes.

The ammeters were also compared directly with copper voltameters for positions on the scale in the neighborhood of 1.2 amperes, this portion of the scale being very frequently used. In preparing the voltameters, the directions followed were in general those found desirable by Gray, as given in Holman's "Laboratory Notes for Use in the Massachusetts Institute of Technology." In accordance with the advice of Ostwald and Luther, a stream of carbon dioxide was kept passing through the solutions during the run. The weights used in measuring the gain of copper were compared with each other and the value of one of them was kindly determined by Professor T. W. Richards in terms of the Washington standard. A table of corrections was made out, these corrections not amounting to more than four hundredths of a milligram, except in the case of the five and ten gram weights, for which the corrections were 0.1 and 0.2 milligrams respectively.

The following results of direct comparison were obtained in the neighborhood of 1.20 amperes.

Current from Copper Deposit.	Reading from Siemens and Halske Ammeter by Shunt Arrangement.	Weston Ammeter.
1.1698	1.177	1.220
1.2064	1.212	1.256

For the Siemens and Halske instrument this was about three-fourths of the full scale reading, for the Weston about one-eighth of the full scale reading.

Primary currents below 0.010 ampere were measured by a potentiometer method. (See Figure 4.) The same Carhart-Clark cell and German silver potentiometer wire were used as in the determination of the sensitiveness of the astatic galvanometer.

The ammeters were also calibrated in the neighborhood of 0.01 ampere to 0.02 ampere by the potentiometer method, as they were sometimes used for measuring primary currents of about this strength. For this calibration they were placed in series in the position corresponding to that of the gold plate in the figure above.

STRENGTH OF THE MAGNETIC FIELD.

The magnetic field was obtained between the poles of a large electromagnet. The pole-pieces were plane and parallel, about 4.2 cm. in diameter and 0.5 cm. from each other. For measuring the strength of the magnetic field, a coil consisting of a single turn of fine wire was jerked out of the field by means of a spring, which was released by means of a long cord pulled by the observer at a ballistic galvanometer. The coil was jerked quickly by the spring to a distance of 34 cm. from the pole-pieces. No appreciable effect was obtained by jerking it from this position to a point 2 meters farther away.

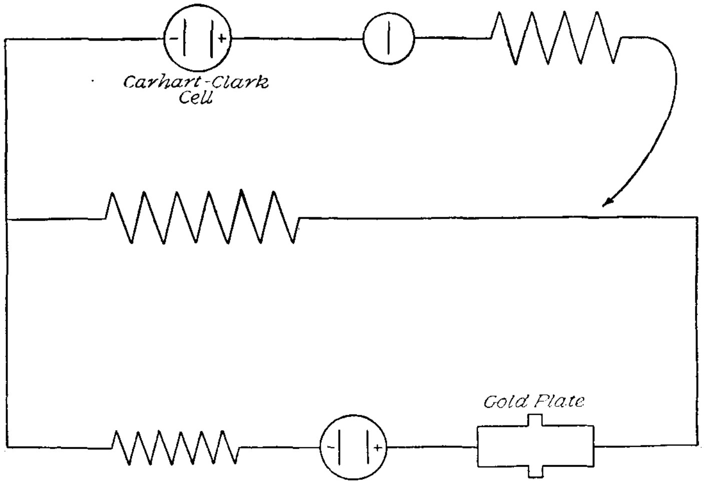

FIGURE 4.

The ballistic galvanometer was situated 8 or 9 meters from the electromagnet. The suspension of this instrument was made of hollow bell-shaped magnets with hard-rubber cores, to give considerable moment of inertia with small damping·effect. The mirror was made of a narrow strip of heavy glass. No vane was placed in the suspension, which was supported by a fine quartz fibre. An earth-inductor, in series with the one-turn test-coil mentioned above, was 10 or 11 meters from the electromagnet. It had been wound with six sets of turns by Professor Hall, whose values for the areas of these turns were taken. By making use of one or more of the six sets of turns in series with the ballistic galvanometer and test-coil, it was possible to get nearly the same deflection by turning the earth-inductor through 180° as by jerking the test-coil from the field, and this was done. The earth-inductor was mounted with its plane vertical and its axis north and south.

DETERMINATION OF H.

The horizontal intensity of the earth's magnetism at the position of the earth-inductor was determined by two methods. In the first method the tangent galvanometer, already referred to, belonging to Professor Hall, was used. This was of the Helmholtz form. There were 11 turns of wire wrapped on each circle. The terms G_2, G_3, in Maxwell's* formula were found to be negligible and the constant was given by the expression

$$\frac{2\,\Pi\,A^2\,n}{(A^2+B^2)^{\frac{3}{2}}}$$

where A is the radius of the coils, B one-half the mean distance between the coils, and n the number of turns in the two circles. The constant so obtained was 6.294.

A current was passed through two copper voltameters and the tangent galvanometer in series. The galvanometer deflection was 41°. The value given by Ostwald and Luther, 0.000329 gm., was used as the amount of copper deposited by the passage of one coulomb of electricity. This method gave for H 0.1628 c. g. s. units. H was measured also by Gauss's method,† which gave 0.1632.

PREPARATION AND USE OF GOLD STRIPS.

In the following account of the writer's observations, most of the preliminary work is given, as well as that to which more weight is attached. The various improvements which increased the reliability of the measurements are briefly mentioned. The dates on which the work was performed (in the case of the first and fourth sets only approximate) are given.

At first an attempt was made to measure the Hall effect in thicker gold than had been previously used for such measurements. A strip of gold about 0.25 mm. thick was obtained from Drew, a gold-beater in Boston. It was stated by him to be very pure. Its density was determined by weighing in and out of water at a known temperature, and was found by this method to be 19.27 relatively to water at 4° C. From this gold two crosses, called hereafter Plate 1 and Plate 2, were cut about 3 cm. long and 1.1 cm. wide. Contact was made at both the primary and the secondary electrodes by electroplating with copper. The gold was fastened on a glass plate with a mixture of resin and beeswax. The plate was placed in a tank through which a current of water was kept flowing. A thermometer was kept in this tank, and the temperature noted.

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* Maxwell's "Electricity and Magnetism," Vol. **2**, § 700.

† The writer is indebted to Professor B. O. Pierce for the apparatus for the measurement by Gauss's method and for the means of comparison of the resistances used with standard resistances.

The currents, the resistances, and strengths of the magnetic fields were measured as described above. The last mentioned measurement was taken before and after each series of measurements of the Hall effect with Plate 1. The strength of the magnetizing current was not read, however, during the measurements of the transverse effect in the case of this plate.

The thickness of the plates was estimated by weighing, the value of the density mentioned above being used, and was also measured with the calipers. The effective thickness was taken as the thickness of the plates between the secondary electrodes. The ratio of this thickness to the average thickness was obtained, therefore, by means of the calipers, while the average thickness was obtained by weighing.

In the following tables,

i is the transverse current in amperes,
R is the resistance of the transverse circuit in ohms,
e is the transverse electromotive force in volts,
I is the main current in amperes,
M is the strength of the field in c. g. s. units.

The value of $\frac{e\,d}{I\,M}$ is given in c. g. s. units.

APRIL 23. PLATE 1; THICKNESS 0.0238 CM.

i.	R.	e.	I.	M.	$\frac{e\,d}{I\,M}$ (c.g.s. units).
6.4×10^{-8}	3.93	2.5×10^{-7}	1.01	7970	0.00074
5.1×10^{-8}	3.94	1.99×10^{-7}	0.822	7910	0.00073
6.8×10^{-8}	3.93	2.66×10^{-7}	1.30	7680	0.00063
1.77×10^{-8}	3.93	6.9×10^{-8}	0.280	7450	0.00079

This gives for $\frac{e\,d}{I M}$ 0.000722.

In these measurements the strength of the magnetic field fell off rapidly at the last. There was also a possibility that these first measurements were affected a few per cent by short circuiting of a part of the potentiometer wire.

Another series of measurements was taken with Plate 2 of the same gold. In this series the magnetizing current was read at frequent intervals during the time of measurement of the transverse effect. This series gave the following results:

May 16. Plate 2; thickness 0.0234 cm.

i.	R.	e.	I.	M.	$\frac{e\,d}{I M}$ (c.g.s. units)
8.28×10^{-8}	3.96	3.28×10^{-7}	1.535	6990	0.000716

The series of measurements on the second plate is believed to be the more reliable, because of the method employed to measure the magnetic field, and because of the steadier conditions of the galvanometer.

The values obtained above agree well with the value 0.00071 obtained by Von Ettingshausen and Nernst * in what they considered their best measurement. In the paper referred to the thickest gold used was 0.00392 cm. thick. The strength of the main current is not given by them.

In the remaining experiments, which were made with thinner gold, it was attempted to get as great a range of primary current as possible. Observations of the Hall effect were taken with weak and strong primary currents alternately, except in the first set of observations given below.

Plate No. 3 was made of gold supplied by Nickolds, of Brooklyn, N. Y., and said by him to be pure ("1000 fine"). By weighing, the average thickness was estimated to be about 0.00018 cm. A plate was made of the same form as Plates 1 and 2, and cemented on the glass plate with a mixture of resin and beeswax. The connection with the electrodes was made as before by electroplating with copper. In doing this, the surface of the gold was accidentally punctured at one or two points, which fact, together with the uncertainty as to local thickness in so thin a sheet, made the plate unfit for measurement of the absolute value of the Hall effect, but did not injure it for a test of the variation of

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* Akademie der Wissenschaften, Wien, 94, **2**, 602 (1886).

the transverse effect with the primary current. The observations were taken in the same manner as with Plate 1. The first set of observations, in which the main current, I, was increased by steps from 0.02 ampere to 1.59 amperes, gave the following results:

APRIL 30. PLATE 3; THICKNESS 0.000184 CM.

i.	R.	e.	I.	M.	$\frac{e\,d}{I\,M}$ (c.g.s units).
1.11×10^{-7}	3.965	4.42×10^{-7}	0.0200	6210	0.000655
1.50×10^{-7}	6.965	1.05×10^{-6}	0.0504	6200	0.000617
1.31×10^{-7}	43.96	5.77×10^{-6}	0.277	6196	0.000618
2.14×10^{-7}	103.77	2.22×10^{-5}	1.064	6210	0.000615
1.72×10^{-7}	203.8	3.50×10^{-5}	1.591	6180	0.000656

As these results were somewhat irregular, it was determined to make measurements with weak and strong currents alternately. In the following set of measurements the readings of the ammeters for the weak current were taken at about one-sixth of the full scale reading for the Weston ammeter, and about one-tenth of the full scale reading for the Siemens and Halske ammeter.

MAY 7. PLATE 3.

i.	R.	e.	I.	M.	$\frac{e\,d}{I\,M}$ (c.g.s. units).	
9.94×10^{-8}	3.958	3.93×10^{-7}	0.0158	7320		0.000626
1.37×10^{-7}	204.0	2.79×10^{-5}	1.130	7330	0.000621	
9.70×10^{-8}	3.99	3.87×10^{-7}	0.0157	7230		0.000629
		Means			0.00062	0.00063

On account of the position of the ammeter scale readings for weak currents, these measurements are not considered as accurate as the succeeding measurements.

Plate No. 4 was very kindly prepared by Professor Hall. The method adopted was the same as that in preparation of the plate in which the phenomenon was discovered. A glass plate was covered with a thin layer of shellac dissolved in alcohol. The alcohol was allowed to evaporate. Then the plate was heated and pressed on the thin gold leaf. A strip about 7 cm. long and 1.5 cm. wide was cut from this, with two short arms for the secondary electrodes as described at the beginning of this paper. Contact with the electrodes was made by pressing polished brass pieces on tin-foil in contact with the gold. The brass pieces were kept in place by a cement of resin and beeswax, which also prevented the water from reaching the electrodes. The gold was then covered with a thin layer of paraffin.

The first two sets of observations gave the following results:—

MAY 9. PLATE 4.

i.	R.	e.	I.	M.	$\frac{e\,d}{I\,M}$ (c.g.s. units.)	
6.04×10^{-8}	12.30	7.43×10^{-7}	0.00142	7240		72.2
1.52×10^{-7}	3009.3	4.58×10^{-4}	0.780	7390	79.5	

MAY 11. PLATE. 4.

i.	R.	e.	I.	M.	$\frac{e\,d}{I\,M}$ (c.g.s. units).	
4.74×10^{-8}	8.85	4.20×10^{-7}	0.000712	7440		79.2
6.59×10^{-8}	10015	6.59×10^{-4}	1.21	7440	73.4	
4.96×10^{-8}	8.857	4.40×10^{-7}	0.000712	7440		82.9
Means					73.4	81.0

Hitherto in almost every case the magnetic field had been measured before and after each measurement of the transverse effect for a given strength of primary current. This required the removal of the plate from the field at every change from weak to strong or strong to weak in the primary current. Hereafter direct determinations of the strength of the magnetic field by means of the test coil were made only at the beginning or end, generally at both the beginning and end, of each period of work, which included one or more changes from strong to weak or weak to strong of primary current.

MAY 12. PLATE 4.

i.	R.	e.	I.	M.	$\frac{e}{IM}$.	
8.51×10^{-8}	8.92	7.59×10^{-7}	0.00142	7260		73.8
1.29×10^{-7}	5011	6.46×10^{-4}	1.21	7260	73.6	
9.02×10^{-8}	8.80	7.94×10^{-7}	0.00142	7260		77.1
1.38×10^{-7}	5010	6.90×10^{-4}	1.21	7260	78.6	
Means					76.1	75.5
9.49×10^{-8}	8.88	8.43×10^{-7}	0.00142	7180		82.9
1.32×10^{-7}	5010	6.60×10^{-4}	1.20	7190	76.5	
9.33×10^{-8}	8.91	8.32×10^{-7}	0.00142	7210		81.3
Means					76.5	82.1

In the next two sets the magnetizing current was read at intervals during the measurement of the transverse effect, and corrections for change of field were made accordingly. A measurement of the resistance of the transverse circuit was made before and after each measurement of the transverse effect when the weak values of I were used.

In passing from one set to another the gold was in nearly every case removed from the bath, and remained out many hours.

MAY 13. PLATE 4.

i.	R.	e.	I.	M.	$\frac{e}{IM}$.	
8.89×10^{-8}	8.88	7.89×10^{-7}	0.00142	7360		75.6
1.30×10^{-7}	5010	6.50×10^{-4}	1.19	7330	74.4	
8.96×10^{-8}	8.88	7.95×10^{-7}	0.00142	7270		77.2
1.31×10^{-7}	5010	6.55×10^{-4}	1.178	7250	76.7	
9.24×10^{-8}	8.93	8.25×10^{-7}	0.00142	7260		80.3
Means					75.6	77.7

May 14. Plate 4.

9.26×10^{-8}		8.85	8.19×10^{-7}	0.00142	7240		79.9
1.30×10^{-7}	5010		6.54×10^{-7}	1.174	7200	77.3	
9.70×10^{-8}		8.82	8.56×10^{-7}	0.00142	7160		84.4
Means						77.3	82.2

In the case of the gold *leaf* no value for the thickness has been given. When a calculation of the thickness is made from the dimensions of the strip (6.2 cm. long and 1.5 cm. wide) and its resistance (3.8 ohms), one obtains 2.3×10^{-6} cm. as a result. This is based on the value 2.15×10^{-6} ohms as the specific resistance of gold. This value is probably not quite accurate, since it was the custom of the gold-beater from whom the leaf was obtained to use an alloy containing about 96 per cent of gold, 2 per cent of silver, and 2 per cent of copper. It was not at all uniform in thickness, as could be seen even with examination by the naked eye. Again, in very thin sheets of metal the electrical resistance increases very rapidly as the thickness is diminished.* The above value of the thickness is therefore not given as in any sense accurate. When it is used to estimate the Hall effect in the case of Plate 4, the value 0.000181 is obtained, that is about one fourth the value obtained in the case of the thicker gold plates 1, 2, and 3.

It is also to be remembered that Moreau, quoted above, found in the case of silver (and nickel) that the Hall effect diminished rapidly with thicknesses below 50 millionths of a millimeter. It seems probable that the Hall effect in gold diminishes in the same way.

Two additional sets of observations were made on Plate 3, the potentiometer method (see Fig. 4) being adopted to measure the main current. As in the case of the later measurements on Plate 4 the plate was kept in the same position during a set of consecutive measurements, including changes from weak to strong main current, or *vice versa;* but frequent readings of the magnetizing current were taken, and thus the changes of the magnetic field observed.

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* Ettingshausen, Sitzungberichte Akad. Wien, **81**, 446 (1880); A. C. Longden, Physical Review, **11**, 84–94 (1900); J. J. Thompson, Cambridge Proc. (2), **9**, 120–122 (1901).

June 20. Plate 3.

i.	R.	e.	I.	M.	$\frac{e\,d}{I\,M}$ (c.g.s. units).	
1.09×10^{-7}	3.99	4.34×10^{-7}	0.0168	7360		0.00065
1.05×10^{-7}	304.0	3.20×10^{-5}	1.23	7360	0.00065	
1.11×10^{-7}	3.99	4.43×10^{-7}	0.0168	7350		0.00066
1.01×10^{-7}	304.0	3.08×10^{-5}	1.23	7350	0.00063	
Means					0.00064	0.000655

The following set differs from all the other measurements in this paper in the method of reading the galvanometer. Instead of attempting to pick out times when the galvanometer needle was not affected by outside disturbances, the experimenter determined each position of the index by taking the mean of several (usually six) instantaneous observations made at regular intervals, about 8 seconds, as measured by the metronome. Five such determinations of the position of the index were made in a single test, three with the magnetic field in one direction and two with the field in the opposite direction. The following results were obtained.

June 23. Plate 3.

i.	R.	e.	I.	M.	$\frac{e\,d}{I\,M}$ c.g.s. units.	
1.13×10^{-7}	3.96	4.46×10^{-7}	0.0168	7390		0.00066
1.10×10^{-7}	304.0	3.34×10^{-5}	1.23	7370	0.00068	
1.10×10^{-7}	3.96	4.37×10^{-7}	0.0168	7330		0.00065
1.05×10^{-7}	304.0	3.18×10^{-5}	1.23	7300	0.00065	
1.24×10^{-7}	3.96	4.92×10^{-7}	0.0168	7320		0.00074
1.07×10^{-7}	304.0	3.25×10^{-5}	1.23	7300	0.00067	
1.22×10^{-7}	3.96	4.82×10^{-7}	0.0168	7290		0.00073
1.15×10^{-7}	304.0	3.51×10^{-5}	1.22	7290	0.00073	
Means					0.00068	0.000695

The larger absolute values of the Hall effect obtained here may have been due to a smaller air-gap between the magnet poles than in the preceding measurements. The strength of the magnetic field was measured before this last set, but not, as had been previously the custom, after it also. As the wedging, intended to keep a constant width of the gap, was generally changed during the removal of the plate from the gap, some differences between the sets of measurements might be expected from this cause. In the case of all measurements later than May 7 the wedging was left unchanged by the experimenter during a set of consecutive measurements involving changes from weak to strong main current or *vice versa*.

The following table gives the values of the current density used in each set of observations on each plate, and the corresponding values of the Hall effect. In the case of Plate 4, as explained above, the value of the thickness given, being calculated from the resistance, is not considered accurate. The current density is given in c.g.s. electromagnetic units of current per square centimeter of cross section of plate. The Hall effect is given as above in c.g.s. units.

		Current Density.	Hall Effect.	
Plate 1	April 23	3.9	0.00074	
		3.1	0.00073	
		5.0	0.00063	
		1.07	0.00079	
Plate 2	May 16	6.0	0.00071	
Plate 3	April 30	10.9	0.00066	
		27.4	0.00062	
		151.	0.00062	
		580.	0.00062	
		860.	0.00066	
Plate 3	May 7	8.6		0.00063
		610	0.00062	
	June 20	9.1		0.000655
		670	0.00064	
	June 23	9.1		0.000695
		670	0.00068	
			0.000647	0.000660

		Current Density.	Hall Effect.	
Plate 4	May 9	40		0.000169
		22000	0.000186	
	May 11	20		0.000190
		34000	0.000172	
	May 12	40		0.000176
		34000	0.000178	
		40		0.000192
		34000	0.000179	
	May 13	40		0.000182
		34000	0.000177	
	May 14	40		0.000192
		34000	0.000181	
			0.000179	0.000184

CONCLUSIONS.

Moretto's method is defective and subject to large errors. His conclusion as to the effect of current density is given apparently as a general property of metals. This is an unwarranted inference from the evidence given in his paper, which does not specify the metals worked upon in obtaining the table reproduced above.

The writer's work on gold appears to indicate a very small increase in the Hall effect for the weaker values of the main current, as in a decided majority of the sets of observations the Hall effect comes out a little larger for the weaker values of the main current. The difference is perhaps within the limits of error. It may possibly be a result of the large stresses in thin gold due to the expansion and contraction of the paraffin and cements. Under the influence of the heating due to the strong currents, these stresses might be of a different order of strength from the stresses when a very weak current is flowing. There might therefore be some difference in the properties of the metals in the two cases.

The writer proposes to continue the investigation, using also some other metals.

The writer desires to express his gratitude to Professor Hall for constant advice and personal aid, and to him and to the other professors of the physical department for the kindly loan of the apparatus for the work.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY,
HARVARD COLLEGE.

ON GENERALIZED SPACE DIFFERENTIATION OF THE SECOND ORDER.

By B. O. Peirce.

ON GENERALIZED SPACE DIFFERENTIATION OF THE SECOND ORDER.

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If one has to investigate the strength of a field of force defined by a given scalar potential function, or to study the flow of electricity in a massive conductor under given conditions, or to apply Green's Theorem to given functions in the space bounded by a given closed surface, or, indeed, to treat any one of a large number of problems in Mathematical Physics or in Analysis, one often needs to find the numerical value at a point, of the derivative of a point function taken in a given direction. This has given rise to the familiar idea of simple space differentiation and of the normal derivative of one scalar function with respect to another; indeed the properties of the first and of the higher space derivatives of a function of n variables taken with respect to any *fixed* direction in n dimensional space, have been treated very clearly and exhaustively by Czuber.*

It is sometimes desirable to use also the conception of general space derivatives of the second order. This is the case, for instance, when one is determining the rate of change of the intensity of a conservative field of force at a point which is moving, either along a curved line of force or on a curved surface related to such lines in a prescribed manner. It is easy to define the general space derivative of any order of a given function.

This paper discusses very briefly a few elementary facts with regard to generalized space differentiation of the second order, and treats first, for the sake of simplicity, differentiation of functions of two variables, in the plane of those variables.

Plane Differentiation.

Let there be in the xy plane two independent families of curves ($u = c$, $v = k$) such that in the domain, R, one and only one curve of each family passes through every point and no curve of either family has anywhere a multiple point. At every point, P, in the domain, the two curves (one of the u family and one of the v family) which pass through the point indicate two directions, s_1, s_2, and if the sense of each of these be determined by any convenient convention, they may be defined by pairs of direction cosines (l_1, m_1), (l_2, m_2), where l_1, m_1, l_2, m_2 are given scalar functions such that at every point

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* E. Czuber, Wienerberichte, p. 1417 (1892).

$$l_1^2 + m_1^2 = 1, \quad l_2^2 + m_2^2 = 1. \tag{1}$$

If Ω is any scalar function of the coördinates which within R has finite derivatives of the first and second orders with respect to these coördinates, the derivative of Ω at P in the direction s_1 is the value at the point of the quantity

$$l_1 \cdot \frac{\partial \Omega}{\partial x} + m_1 \cdot \frac{\partial \Omega}{\partial y} \tag{2}$$

and this new scalar function of x and y may be conveniently indicated by the expression $[D_{s_1} \Omega]_P$. If P' is a point on the u curve which passes through P, taken near P and in the sense of the direction s_1, $[D_{s_1} \Omega]_P$ is the limit, as P' approaches P, of $\dfrac{\Omega_{P'} - \Omega_P}{\overline{PP'}}$.

If on the curve of the second (or v) family which passes through P, a point Q be taken near P and in the sense of the direction s_2, the limit, as Q approaches P, of the quantity

$$\frac{[D_{s_1} \Omega]_Q - [D_{s_1} \Omega]_P}{\overline{PQ}} \tag{3}$$

may be indicated by the expression $[D_{s_2} D_{s_1} \Omega]_P$, and this is the second derivative of Ω at P taken with respect to the directions s_1 and s_2 in the order given.

Thus, if

$$\Omega = 2x^2 - y^2, \qquad l_1 = \frac{2x}{\sqrt{4x^2+1}}, \qquad m_1 = \frac{1}{\sqrt{4x^2+1}},$$

$$l_2 = \frac{y}{\sqrt{x^2+y^2}}, \quad m_2 = \frac{-x}{\sqrt{x^2+y^2}}; \quad D_{s_1} \Omega = \frac{2(4x^2-y)}{\sqrt{4x^2+1}},$$

$$D_{s_2} D_{s_1} \Omega = \frac{x(32x^2y + 8y^2 + 16y + 8x^2 + 2)}{\sqrt{x^2+y^2} \cdot (4x^2+1)^{\frac{3}{2}}}.$$

It is evident from the definition just given that

$$D_{s_2}D_{s_1}\Omega = l_1 \cdot l_2 \cdot \frac{\partial^2\Omega}{\partial x^2} + (l_1 \cdot m_2 + l_2 \cdot m_1)\frac{\partial^2\Omega}{\partial x \cdot \partial y} + m_1 \cdot m_2 \cdot \frac{\partial^2\Omega}{\partial y^2}$$
$$+ \left(l_2 \cdot \frac{\partial l_1}{\partial x} + m_2 \cdot \frac{\partial l_1}{\partial y}\right) \cdot \frac{\partial\Omega}{\partial x} + \left(l_2 \cdot \frac{\partial m_1}{\partial x} + m_2 \cdot \frac{\partial m_1}{\partial y}\right)\frac{\partial\Omega}{\partial y}, \quad (4)$$

and that $D_{s_1} D_{s_2} \Omega$ is quite different in general from $D_{s_2} D_{s_1} \Omega$: the order of the two differentiations is material.

If the u curves happen to be a family of parallel straight lines and the v curves another family of parallel straight lines,

$$D_{s_2} D_{s_1} \Omega = l_1 l_2 \cdot \frac{\partial^2\Omega}{\partial x^2} + (l_1 m_2 + l_2 m_1) \cdot \frac{\partial^2\Omega}{\partial x \cdot \partial y} + m_1 \cdot m_2 \cdot \frac{\partial^2\Omega}{\partial y^2}, \quad (5)$$

and the coefficients in this expression are constants.

If the u curves and the v curves are identical and are a family of straight parallel lines, we have

$$D_{s_1}{}^2\Omega = l_1{}^2 \cdot \frac{\partial^2\Omega}{\partial x^2} + 2\, l_1 m_1 \cdot \frac{\partial^2\Omega}{\partial x \cdot \partial y} + m_1{}^2 \cdot \frac{\partial^2\Omega}{\partial y^2}, \quad (6)$$

the familiar form of the second derivative of Ω along the *fixed direction* s_1, which often appears in work involving the transformation of Cartesian coördinates. Simple special cases of this formula are obtained by putting l equal to 1, 0, and m.

Since $l_1{}^2 + m_1{}^2 = 1$,

$$\frac{\partial l_1}{\partial x} \cdot \frac{\partial m_1}{\partial y} = \frac{\partial m_1}{\partial x} \cdot \frac{\partial l_1}{\partial y},$$

and if at any point s_1 and s_2 are such as to make the coefficient of $\frac{\partial\Omega}{\partial x}$ in (4) vanish, the coefficient of $\frac{\partial\Omega}{\partial y}$ will vanish also. Such points as this lie, in general, on a definite curve, the equation of which is to be found by equating one of these coefficients to zero. If s_1 is a fixed direction so that l_1 and m_1 are constants, (4) takes the form (5), but the coefficients are not constants unless s_2 also is fixed.

If the two variable directions s_1, s_2 coincide, (4) becomes the second derivative of the function Ω taken with respect to the direction s_1; that is,

$$D_{s_1}{}^2\,\Omega = l_1{}^2 \cdot \frac{\partial^2\Omega}{\partial x^2} + 2\,l_1\,m_1 \cdot \frac{\partial^2\Omega}{\partial x \cdot \partial y} + m^2 \cdot \frac{\partial^2\Omega}{\partial y^2}$$
$$+ \left(l_1 \cdot \frac{\partial l_1}{\partial x} + m_1 \cdot \frac{\partial l}{\partial y}\right)\frac{\partial\Omega}{\partial x} + \left(l_1 \cdot \frac{\partial m_1}{\partial x} + m_1 \cdot \frac{\partial m_1}{\partial y}\right)\frac{\partial\Omega}{\partial y}. \tag{7}$$

If the direction cosines of a plane curve at a point on it are l and m, the curvature of the curve at P has the same absolute value as have the expressions

$$\frac{1}{m}\left(l \cdot \frac{\partial l}{\partial x} + m \cdot \frac{\partial l}{\partial y}\right), \qquad \frac{1}{l}\left(l \cdot \frac{\partial m}{\partial x} + m \cdot \frac{\partial m}{\partial y}\right). \tag{8}$$

If, therefore, two directions, s_1, s_3, are defined by two curves which, at a point, P, common to both, have a common tangent and equal curvatures, the second derivatives at P of a function Ω taken with respect to the two directions are equal.

If at any point the curvature of the curve of the u family which defines the direction s_1 is zero, the coefficients of $\partial\Omega/\partial x$ and $\partial\Omega/\partial y$ in the expression for $D_{s_1}{}^2\,\Omega$ at the point vanish. If the u curves are a family of straight lines, the last two terms of (7) disappear, but the coefficients of the other terms are, in general, not constant.

If there is no point in the region R at which both the quantities $\partial\Omega / \partial x$, $\partial\Omega / \partial y$ vanish together, and if the direction s is at every point of R that in which Ω increases most rapidly, $D_s\,\Omega = h$, where h is the gradient of Ω, that is, the tensor of the gradient vector. Now h is itself, in general, a scalar point function, which, when equated to a parameter, yields a family of curves the directions of which are usually quite different from those of the lines of the gradient-vector. The normal at any point P to the curve of this h family which passes through the point, has the direction cosines

$$\frac{\partial h}{\partial x}\Big/ h', \quad \frac{\partial h}{\partial y}\Big/ h',$$

where h' is the gradient of h. The angle between the direction, s, of the gradient vector of Ω and the normal to the h curve has at every point the value

$$\cos\,(\Omega,\,h) = \left[\frac{\partial h}{\partial x} \cdot \frac{\partial\Omega}{\partial x} + \frac{\partial h}{\partial y} \cdot \frac{\partial\Omega}{\partial y}\right] \Big/ h\,h', \tag{9}$$

and the second derivative of Ω with respect to the direction s is, therefore,

$$D_s{}^2\,\Omega = \left[\frac{\partial h}{\partial x} \cdot \frac{\partial\Omega}{\partial x} + \frac{\partial h}{\partial y} \cdot \frac{\partial\Omega}{\partial y}\right] \Big/ h = h' \cdot \cos\,(\Omega,\,h). \tag{10}$$

Let the *normal derivative*,* at any point P, of a point function V, taken with respect to another point function W, be the limit, as PQ approaches zero, of the ratio of $V_Q - V_P$ to $W_Q - W_P$, where Q is a point so chosen on the normal at P to the surface of constant W which passes through P, that $W_Q - W_P$ is positive: if, then, (V, W) denotes the angle between the directions in which V and W increase most rapidly, the normal derivatives of V with respect to W, and of W with respect to V, may be written

$$[D_W V] = h_V \cdot \cos(V, W)/h_W, \quad [D_V W] = h_W \cdot \cos(V, W)/h_V: \tag{11}$$

if $h_V = h_W$, these derivatives are equal.

With this notation (10) may be rewritten in the form

$$D_s^2 \Omega = h \cdot [D_\Omega h.] \tag{12}$$

If at any point $D_s^2 \Omega$ vanishes, it is easy to see from (10) that either the gradient (h') of h vanishes at the point, or else the h and Ω surfaces cut each other there orthogonally. This latter case is exemplified in the familiar instance of the electrostatic field due to two long parallel straight wires of the same diameter, charged to equal and opposite potentials: if the wires cut the xy plane normally at P_1, P_2, and if the line joining these intersections be taken for x axis with the point midway between them for origin, the potential function is of the form $V = A \log r_1/r_2$, where $r_1^2 = (x-a)^2 + y^2$, $r_2^2 = (x+a)^2 + y^2$. The intensity of the field, in absolute value, is $h = 2\,a\,A/r_1 r_2$, and the second derivative of V taken along the line of force (that is, the rate at which the intensity of the field changes) is numerically equal to $\dfrac{-4\,a\,A\,x}{r_1^2 \cdot r_2^2}$.

$D_s^2 V$ taken along a line of force vanishes, therefore, at all points on the y axis, and at all such points the curve of constant V $(r_1/r_2 = b)$ cuts the curves of constant h $(r_1 r_2 = k)$ orthogonally. At points on the y axis the direction of the lines of force is parallel to the x axis, and the second derivative of V with respect to the fixed direction x happens to vanish here also where $l = 1$, $\dfrac{\partial l}{\partial x} = 0$, $m = 0$, $\dfrac{\partial V}{\partial y} = 0$. The quantity h' does not vanish at any finite point.

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* Peirce, The Newtonian Potential Function, p. 116. A Short Table of Integrals, p. 106.

The example just discussed is in contrast with the case where the Ω family are a set of parallel curves of any kind, and h in consequence (if not constant) is a function of Ω alone, so that the h curves and the Ω curves coincide, and if $D_s^2\,\Omega$ vanishes anywhere, it must be where h' vanishes. A simple example of this is furnished by the field of attraction within a very long cylinder of revolution, the density of which is a function of the distance from the axis alone.

If the directions s_1 and s_2 are everywhere perpendicular to each other, we may without loss of generality write $l_2 = -m_1$, $m_2 = l_1$; in which case the coefficients of $\partial\Omega/\partial x$, $\partial\Omega/\partial y$ in (4) become

$$\left(l_2 \cdot \frac{\partial m_2}{\partial x} + m_2 \cdot \frac{\partial m_2}{\partial y}\right) \text{ and } -\left(l_2 \cdot \frac{\partial l_2}{\partial x} + m_2 \cdot \frac{\partial l_2}{\partial y}\right): \qquad (13)$$

these vanish if the v curves form a family of straight lines, or the u curves a family of straight or curved parallels. The order of differentiation with respect to the orthogonal directions s_1, s_2 is immaterial if both the u and the v curves are straight lines, that is, if the directions are fixed.

If s_1 is the direction in which Ω increases most rapidly, and s_2 the direction of constant Ω,

$$D_{s_2} D_{s_1} \Omega = D_{s_2} h = \left[\frac{\partial \Omega}{\partial x} \cdot \frac{\partial h}{\partial y} - \frac{\partial \Omega}{\partial y} \cdot \frac{\partial h}{\partial x}\right] \Big/ h$$

$$= \left\{ \frac{\partial^2 \Omega}{\partial x \cdot \partial y}\left[\left(\frac{\partial \Omega}{\partial x}\right)^2 - \left(\frac{\partial \Omega}{\partial y}\right)^2\right] + \frac{\partial \Omega}{\partial x} \cdot \frac{\partial \Omega}{\partial y}\left[\frac{\partial^2 \Omega}{\partial y^2} - \frac{\partial^2 \Omega}{\partial x^2}\right] \right\} \Big/ h. \qquad (14)$$

Now the direction cosines and the slope of the line of the gradient vector at any point are

$$\frac{1}{h} \cdot \frac{\partial \Omega}{\partial x}, \quad \frac{1}{h} \cdot \frac{\partial \Omega}{\partial y}, \quad \text{and} \quad \frac{\partial \Omega}{\partial y} \Big/ \frac{\partial \Omega}{\partial x}.$$

So that the curvature of the line is

$$\frac{1}{\rho} = \frac{\dfrac{d}{dx}\left[\dfrac{\partial \Omega}{\partial y} \Big/ \dfrac{\partial \Omega}{\partial x}\right]}{\left[1 + \left(\dfrac{\partial \Omega}{\partial y} \Big/ \dfrac{\partial \Omega}{\partial x}\right)^2\right]^{\frac{3}{2}}}$$

$$= \left\{ \frac{\partial^2 \Omega}{\partial x \cdot \partial y}\left[\left(\frac{\partial \Omega}{\partial x}\right)^2 - \left(\frac{\partial \Omega}{\partial y}\right)^2\right] + \frac{\partial \Omega}{\partial x} \cdot \frac{\partial \Omega}{\partial y}\left[\frac{\partial^2 \Omega}{\partial y^2} - \frac{\partial^2 \Omega}{\partial x^2}\right] \right\} \Big/ h^3 \qquad (15)$$

and we may write in this case

$$D_{s_2} D_{s_1} \Omega = h/\rho.$$

This expression gives the rate at which the maximum slope of the surface the coördinates of which are (x, y, Ω), changes as one goes along a line of level.*

When s_1 and s_2 are perpendicular to each other, we have in general

$$\begin{aligned} D_{s_2}{}^2 \Omega = {} & m_1{}^2 \cdot \frac{\partial^2\Omega}{\partial x^2} - 2 l_1 m_1 \cdot \frac{\partial^2 \Omega}{\partial x \cdot \partial y} + l_1{}^2 \cdot \frac{\partial^2\Omega}{\partial y^2} \\ & + \left(m_1 \cdot \frac{\partial m_1}{\partial x} - l_1 \cdot \frac{\partial m_1}{\partial y} \right) \frac{\partial \Omega}{\partial x} + \left(l_1 \cdot \frac{\partial l_1}{\partial y} - m_1 \cdot \frac{\partial l_1}{\partial x} \right) \frac{\partial \Omega}{\partial y}, \end{aligned} \tag{16}$$

and since $l_1{}^2 + m_1{}^2 = 1$,

$$l_1 \cdot \frac{\partial l_1}{\partial x} + m_1 \cdot \frac{\partial m_1}{\partial x} = 0, \qquad l_1 \cdot \frac{\partial l_1}{\partial y} + m_1 \cdot \frac{\partial m_1}{\partial y} = 0.$$

So that if we add together (7) and (16) we shall get

$$\begin{aligned} D_{s_1}{}^2 \Omega + D_{s_2}{}^2 \Omega & = \frac{\partial^2\Omega}{\partial x^2} + \frac{\partial^2\Omega}{\partial y^2} + \frac{1}{m_1} \cdot \frac{\partial l_1}{\partial y} \cdot \frac{\partial \Omega}{\partial x} + \frac{1}{l_1} \cdot \frac{\partial m_1}{\partial x} \cdot \frac{\partial \Omega}{\partial y} \\ & = \frac{\partial^2\Omega}{\partial x^2} + \frac{\partial^2\Omega}{\partial y^2} - \frac{1}{l_1} \cdot \frac{\partial m_1}{\partial y} \cdot \frac{\partial \Omega}{\partial x} - \frac{1}{m_1} \cdot \frac{\partial l_1}{\partial x} \cdot \frac{\partial \Omega}{\partial y}. \end{aligned} \tag{16'}$$

It is evident that the values of the space derivatives defined above are wholly independent of the particular system of rectangular coördinates which may be used.

SPACE DIFFERENTIATION.

At every point of the space domain, R, let two independent directions (s_1, s_2) be defined by the direction cosines (l_1, m_1, n_1), (l_2, m_2, n_2), where l_1, m_1, n_1, l_2, m_2, n_2 are any six single-valued point functions which satisfy the identities

$$l_1{}^2 + m_1{}^2 + n_1{}^2 = 1, \qquad l_2{}^2 + m_2{}^2 + n_2{}^2 = 1, \tag{17}$$

and have finite derivatives of the first order with respect to the coördinates x, y, z. If, then, Ω is any single-valued function of the coördinates which within R has finite derivatives of the first and second orders with respect to these coördinates, the derivative of Ω at the point P, in the direction s_1, is the value at P of the quantity

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* Boussinesq, Cours d'Analyse Infinitésimale, T. 1, f. 2, p. 236.

$$D_{s_1}\Omega \equiv l_1 \cdot \frac{\partial \Omega}{\partial x} + m_1 \cdot \frac{\partial \Omega}{\partial y} + n_1 \cdot \frac{\partial \Omega}{\partial z}. \tag{18}$$

Through the point P passes a curve of the family defined by the equations

$$\frac{dx}{l_2} = \frac{dy}{m_2} = \frac{dz}{n_2}, \tag{19}$$

and this curve indicates the direction s_2. If on this curve a point Q be taken near P and in the sense of the direction s_2, the limit, as Q approaches P, of the quantity

$$\frac{[D_{s_1}\Omega]_Q - [D_{s_1}\Omega]_P}{\overline{PQ}} \tag{20}$$

may be represented by $[D_{s_2} D_{s_1} \Omega]_P$ and this is the second directional derivative at P of Ω taken with respect to the directions s_1 and s_2 in the order given. It is evident that

$$\begin{aligned} D_{s_2} D_{s_1} \Omega = {} & l_1 l_2 \cdot \frac{\partial^2 \Omega}{\partial x^2} + m_1 m_2 \cdot \frac{\partial^2 \Omega}{\partial y^2} + n_1 n_2 \cdot \frac{\partial^2 \Omega}{\partial z^2} \\ & + (l_1 m_2 + l_2 m_1)\frac{\partial^2 \Omega}{\partial x \cdot \partial y} + (m_1 n_2 + m_2 n_1)\frac{\partial^2 \Omega}{\partial y \cdot \partial z} + (n_1 l_2 + n_2 l_1)\frac{\partial^2 \Omega}{\partial z \cdot \partial x} \\ & + \left(l_2 \cdot \frac{\partial l_1}{\partial x} + m_2 \cdot \frac{\partial l_1}{\partial y} + n_2 \cdot \frac{\partial l_1}{\partial z} \right) \frac{\partial \Omega}{\partial x} \\ & + \left(l_2 \cdot \frac{\partial m_1}{\partial x} + m_2 \cdot \frac{\partial m_1}{\partial y} + n_2 \cdot \frac{\partial m_1}{\partial z} \right) \frac{\partial \Omega}{\partial y} \\ & + \left(l_2 \cdot \frac{\partial n_1}{\partial x} + m_2 \cdot \frac{\partial n_1}{\partial y} + n_2 \cdot \frac{\partial n_1}{\partial z} \right) \frac{\partial \Omega}{\partial z}, \end{aligned} \tag{21}$$

and that this is not equal to $D_{s_1} D_{s_2} \Omega$.

If the directions s_1, s_2 are fixed, the six direction cosines are constants, the last three terms of (21) disappear, and the coefficients of the other six terms are constant. If the fixed directions s_1, s_2 coincide, (21) reduces to the familiar form

$$D_{s_1}{}^2\,\Omega = l_1{}^2\cdot\frac{\partial^2\Omega}{\partial x^2} + m_1{}^2\cdot\frac{\partial^2\Omega}{\partial y^2} + n_1{}^2\cdot\frac{\partial^2\Omega}{\partial z^2}$$
$$+ 2\,l_1\,m_1\cdot\frac{\partial^2\Omega}{\partial x\cdot\partial y} + 2\,m_1\,n_1\cdot\frac{\partial^2\Omega}{\partial y\cdot\partial z} + 2\,l_1\,n_1\cdot\frac{\partial^2\Omega}{\partial x\cdot\partial z}, \qquad (22)$$

whereas, if s_1 is not fixed,

$$D_{s_1}{}^2\,\Omega = l_1{}^2\cdot\frac{\partial^2\Omega}{\partial x^2} + m_1{}^2\cdot\frac{\partial^2\Omega}{\partial y^2} + n_1{}^2\cdot\frac{\partial^2\Omega}{\partial z^2} + 2\,l_1\,m_1\cdot\frac{\partial^2\Omega}{\partial x\cdot\partial y} + 2\,m_1\,n_1\cdot\frac{\partial^2\Omega}{\partial y\cdot\partial z}$$
$$+ 2\,l_1\,n_1\cdot\frac{\partial^2\Omega}{\partial x\cdot\partial z} + \left(l_1\cdot\frac{\partial l_1}{\partial x} + m_1\cdot\frac{\partial l_1}{\partial y} + n_1\cdot\frac{\partial l_1}{\partial z}\right)\frac{\partial\Omega}{\partial x}$$
$$+ \left(l_1\cdot\frac{\partial m_1}{\partial x} + m_1\cdot\frac{\partial m_1}{\partial y} + n_1\cdot\frac{\partial m_1}{\partial z}\right)\frac{\partial\Omega}{\partial y}$$
$$+ \left(l_1\cdot\frac{\partial n_1}{\partial x} + m_1\cdot\frac{\partial n_1}{\partial y} + n_1\cdot\frac{\partial n_1}{\partial z}\right)\frac{\partial\Omega}{\partial z}. \qquad (23)$$

All the coefficients in (22) are constants; all those of (23) are in general variable. If s_1 is defined by any infinite system of straight lines of which just one passes through every point of space, and if the direction s_1 at all points of any one of the lines is that of the line itself, the coefficients of $\partial\Omega / \partial x$, $\partial\Omega / \partial y$, $\partial\Omega / \partial z$ in (23) vanish. In particular, if the direction s_1 is that of the radius vector from a fixed point (a, b, c), (23) takes the form of (22) though the remaining coefficients are not constants. In any case if the coefficients of two of the three quantities $\partial\Omega / \partial x$, $\partial\Omega / \partial y$, $\partial\Omega / \partial z$ vanish, the third must vanish also.

If the gradient, h, of Ω does not vanish at any point of R and if s is the direction in which Ω increases most rapidly,

$$D_s\,\Omega = h,$$
$$D_s^2\,\Omega = \left[\frac{\partial h}{\partial x}\cdot\frac{\partial\Omega}{\partial x} + \frac{\partial h}{\partial y}\cdot\frac{\partial\Omega}{\partial y} + \frac{\partial h}{\partial z}\cdot\frac{\partial\Omega}{\partial z}\right]\Big/ h. \qquad (24)$$

If h' is the gradient of the scalar point function which gives the value of h, and if (Ω, h) represents the angle between the directions in which the point functions Ω and h increase most rapidly,

$$\cos(\Omega, h) = \left[\frac{\partial h}{\partial x}\cdot\frac{\partial\Omega}{\partial x} + \frac{\partial h}{\partial y}\cdot\frac{\partial\Omega}{\partial y} + \frac{\partial h}{\partial z}\cdot\frac{\partial\Omega}{\partial z}\right]\Big/ h\cdot h' \qquad (25)$$

and

$$D_s^2\,\Omega = h'\cdot\cos(\Omega, h)\text{, or } h\,[D_\Omega\,h] \qquad (26)$$

where $[D_\Omega h]$ represents the normal derivative of h with respect to Ω.

If the equation $\Omega = k$ happens to represent a set of parallel surfaces, h, if not constant, is a function of Ω alone, so that the h and Ω surfaces are coincident: in this case $\cos(\Omega, h) = 1$ and $D_s^2\,\Omega$ can vanish only where h' vanishes. In general, $D_s^2\,\Omega$ vanishes when the h and Ω surfaces cut each other at right angles.

If s_1, s_2, s_3 are any three mutually perpendicular directions,

$$l_1^2 + l_2^2 + l_3^2 = m_1^2 + m_2^2 + m_3^2 = n_1^2 + n_2^2 + n_3^2 = 1,$$

$$l_1 m_1 + l_2 m_2 + l_3 m_3 = m_1 n_1 + m_2 n_2 + m_3 n_3 = l_1 n_1 + l_2 n_2 + l_3 n_3 = 0,$$

and

$$D_{s_1}^{\;2}\,\Omega + D_{s_2}^{\;2}\,\Omega + D_{s_3}^{\;2}\,\Omega = \frac{\partial^2\Omega}{\partial x^2} + \frac{\partial^2\Omega}{\partial y^2} + \frac{\partial^2\Omega}{\partial z^2}$$

$$+ \frac{\partial\Omega}{\partial x}\left[m_1\cdot\frac{\partial l_1}{\partial y} + m_2\cdot\frac{\partial l_2}{\partial y} + m_3\cdot\frac{\partial l_3}{\partial y} + n_1\cdot\frac{\partial l_1}{\partial z} + n_2\cdot\frac{\partial l_2}{\partial z} + n_3\cdot\frac{\partial l_3}{\partial z}\right]$$

$$+ \frac{\partial\Omega}{\partial y}\left[n_1\cdot\frac{\partial m_1}{\partial z} + n_2\cdot\frac{\partial m_2}{\partial z} + n_3\cdot\frac{\partial m_3}{\partial z} + l_1\cdot\frac{\partial m_1}{\partial x} + l_2\cdot\frac{\partial m_2}{\partial x} + l_3\cdot\frac{\partial m_3}{\partial x}\right]$$

$$+ \frac{\partial\Omega}{\partial z}\left[l_1\cdot\frac{\partial n_1}{\partial x} + l_2\cdot\frac{\partial n_2}{\partial x} + l_3\cdot\frac{\partial n_3}{\partial x} + m_1\cdot\frac{\partial n_1}{\partial y} + m_2\cdot\frac{\partial n_2}{\partial y} + m_8\cdot\frac{\partial n_3}{\partial y}\right]. \tag{27}$$

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY.

ON THE COOPER HEWITT MERCURY INTERRUPTER.

By George W. Pierce.

With Three Plates.

ON THE COOPER HEWITT MERCURY INTERRUPTER.

By George W. Pierce.

Presented by John Trowbridge June 17, 1903. Received January 6, 1904.

I. Introduction.

Mr. Peter Cooper Hewitt has devised a new form of interrupter designed to take the place of the spark-gap in the wireless transmission of signals and in the production of high potential discharges with the Tesla transformer. Mr. Hewitt's interrupter employs the discharge between mercury electrodes in an exhausted bulb instead of the usual spark in air between solid metallic terminals. Figure I is a diagram of the usual form of Cooper-Hewitt interrupter. At the bottom of an exhausted bulb 15 or 20 cm. in diameter are two deep depressions containing pools of mercury, between which the discharge is made to pass. Short pieces of platinum wire (1.5 mm. in diameter) fused into the glass serve to lead the current into the bulb. To prevent unequal heating of the sealed-in wires the two protuberances may be dipped into tumblers of mercury to which the connections are made.

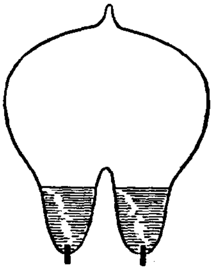
Figure I.

In the preparation of the interrupters employed in these experiments the mercury for the bulbs was carefully purified by distillation and by washing through a nitric-acid filter; when the mercury is slightly contaminated the interrupter soon becomes inoperative. The bulbs were attached to a condenser and the source of current while being pumped, and their vacuum was tested from time to time during the exhaustion. In these tests care must be taken not to send the discharge through the interrupter when it contains too much air, as oxygen under the action of the discharge contaminates the mercury. When the vacuum was too low, the discharge through the bulb showed striations, and when the vacuum was too high, the discharge could not readily be started. The proper vacuum could be distinguished by the appearance of the bulb or by the crackling sound it emitted; or, better, the proper vacuum could be determined by using the bulb as interrupter for a Tesla coil during the exhaustion.

Figure II is a diagram of the arrangement of the circuits as they are employed with the Tesla coil. C is a condenser charged from the secondary of a step-up transformer P S actuated by the 110-volt alternating light circuit. At intervals this condenser discharges through the interrupter I and through the primary P′ of the Tesla coil, as follows: When the potential of C becomes high enough to start the discharge, the resistance of the interrupter drops to a fraction of an ohm, and the electricity from the condenser surges back and forth through C P′ I. These rapid oscillations in the primary of the Tesla coil induce high potentials in the secondary S′. When the current density through the interrupter becomes small, the bulb ceases to be conducting, the condensers are again charged by the transformer P S, and the series of oscillations is repeated. To get good results the secondary S′ of the Tesla coil must be in resonance with the primary circuit C P′. With the Cooper Hewitt interrupter at I the Tesla coil gives a much longer and a much more uniform discharge than when a zinc or an iridium spark-gap is employed as interrupter.

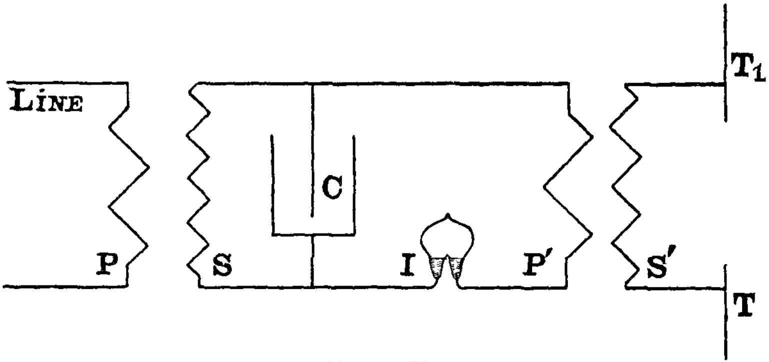

FIGURE II.

In one form of wireless telegraph circuit first suggested by Braun * and

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* F. Braun, Phys. Zeit., **3**, 143 (1901). Drahtlose Telegraphie durch Wasser und Luft, Leipzig, 1901. Simon and Reich, Phys. Zeit., **4**, 365 (1903). M. Wien, Ann. der Phys., **3**, 686 (1902).

See also the controversy between Braun and Slaby as to priority, in various recent numbers of Ann. der Physik.

now employed also by Marconi and by Slaby, the arrangement at the sending station is essentially the same as in the Tesla coil, with one of the terminals, T, put to earth and the other, T_1, attached to one or more vertical wires carried by masts. One object of this research is thus the attempt to discover in what way and to what extent the Hewitt interrupter is superior to the spark-gap in the wireless transmission of signals.

The study of the mercury interrupter is also of interest in its relation to the theories of electric conduction in gases and in its relation to the phenomena of electro-luminescence.

The present paper comprises:

II. Quantitative measurements of the induction between circuits with the two forms of interrupter in the sending circuit.

III. Resonance between such circuits.

IV. Photographs of the oscillations in the Hewitt interrupter with the aid of the revolving mirror.

V. Photographs showing the rapidity of recovery of the Hewitt interrupter.

VI. Calorimetric measurement of the ohmic resistance of the Hewitt interrupter.

VII. Determination of the proper vacuum for the Hewitt interrupter.

II. Quantitative Measurement of the Induction between Loops.

In order to obtain a direct comparison of the mercury interrupter with the spark in air between solid metallic terminals, I have measured the intensity of signals obtained in a receiving circuit with the two forms of interrupter respectively in the sending circuit. For a receiving instrument recourse was had to a form of oscillating current galvanometer devised by Fleming and employed in 1897 by Northrup, Pierce, and Reichmann* in an experiment on induction between distant circuits. Figure III is a diagram of this instrument. In the centre of the figure, between S and N, is a carefully insulated coil usually of about one hundred turns of fine wire. The coil has an internal diameter of about 1 cm., and is put in series with the receiving circuit by means of binding posts outside of the enclosing vulcanite box. For the purpose of varying the sensitiveness of the instrument, the coil may be removed and replaced by another of any desired number of turns. Within the coil is suspended a thin circular disc of silver foil about 6 mm. in diameter, to which a mirror is attached by a slender rod of glass. For sensitiveness the disc and mirror should be as light as possible. The suspension is hung by a fine quartz fibre so that the disc makes an angle of about 45° with the plane of the coil. By the oscillations in the receiving circuit, which pass also through the coil, oscillations are induced in the disc, which is thus repelled and tends to set itself at right angles to the coil. The vulcanite box, which encloses the suspension, is provided with a glass face through which the deflections may be read by a telescope and scale. The *period of the instrument* is five seconds, and the suspension is so light that its deflections are practically dead-beat, rendering unnecessary the damping magnet S N that was employed in the earlier experiments.

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* Electrical World, Dec. 18 and 25, 1897.

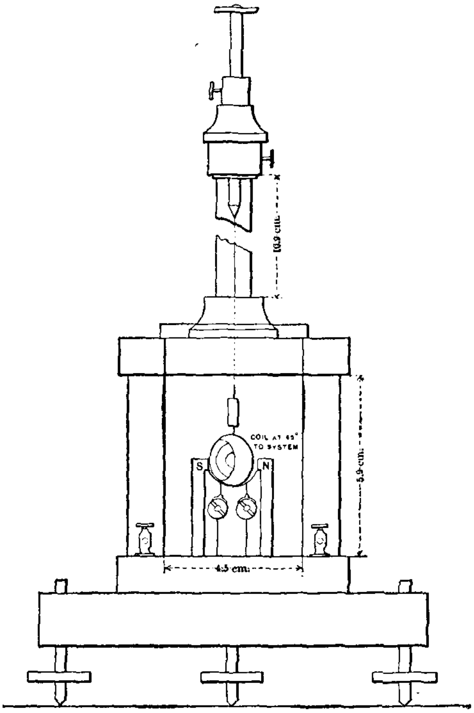

Figure III.

When used in ordinary wireless telegraph circuits this instrument shows great sensitiveness. In making the present comparisons, the data are obtained not from ordinary wireless telegraph circuits of the "open" type, but from circuits consisting of closed loops. These circuits are of the same form as those previously employed by Northrup, Reichmann, and the author, and are represented in Figure IV. The sending circuit A consists of a glass condenser *a* in series with the spark-gap or interrupter *b* and a rectangular loop of wire four meters by three meters. About the spark-gap or interrupter are connected the terminals of the secondary of a step-up transformer, actuated by the alternating electric light circuit. The spark-gap is made of small pieces of iridium set in heavy brass balls. By throwing a switch the Hewitt interrupter may be put in the place of the iridium spark-gap.

The receiving circuit B is at a distance of twenty meters from the transmitting circuit. The receiving circuit consists of a variable air-condenser in series with the receiving instrument and a closed rectangular loop (2 m. × 1 m.) of wire in a plane parallel to the sending circuit. When the receiving circuit is brought to approximate resonance with the sending circuit, large deflections of the instrument are obtained. In taking readings the discharge was kept up during the period of swing of the instrument, and throws were read.

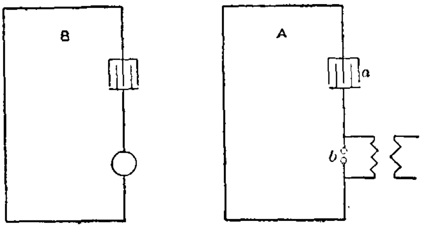

FIGURE IV.

The following sets of readings (Table I) are given to show the uniformity and effectiveness of the mercury interrupter in comparison with the iridium spark-gap. The columns of readings were taken in the order given in the table. A rheostat in the primary circuit of the transformer was adjusted so that the mercury interrupter functioned vigorously; the set of readings in the first column were taken, then without changing the receiving circuit of the rheostat, the iridium spark was put in the place of the mercury interrupter, and was adjusted in length to give its best deflection. With this adjustment readings in the second column were taken. The third column shows another set of readings with the mercury interrupter. The fourth column is with the spark-gap. The readings are in centimeters with a scale distance of 60 cm.

Whence it is seen that the deflections with the mercury interrupter are about four times as large as the deflections that could be obtained under similar conditions with the iridium spark-gap. Other comparisons with various capacities and inductances in the circuits gave likewise substantially larger deflections with the mercury interrupter than with the spark-gap. With this interrupter the spark at the secondary of a Tesla coil was also four or five times as long as that obtainable with the iridium spark-gap. The increased spark length with the Tesla coil did not, however, indicate an increase of integral effect in the secondary, as the sparks may have been fewer in number. Hence, the experiment with induction between loops was made. The result shows that the total energy communicated between the circuits was about four times as great with the mercury interrupter as with the spark-gap. The results here given pertain, of course, only to the particular form of circuits and the particular interrupter employed. I have made some experiments with forms of open circuit of the type used in wireless telegraphy, but the results are not yet ready for presentation.

TABLE I.

C. H.	Spark.	C. H.	Spark.
25.9	6.4	26.4	7.7
25.8	6.5	26.3	6.8
26.5	6.4	26.0	6.4
25.8	6.7	26.6	6.4
26.3	6.7	25.9	6.5
27.1	6.7	25.8	6.4
27.0	6.8	27.7	6.3
25.2	6.4	26.1	6.8
26.2	6.7	27.5	6.5
26.2	6.8	26.9	6.3
26.3	6.62	26.5	6.62 mean.

III. Resonance.

On account of the regularity of the mercury interrupter, it can be employed advantageously in the study of resonance between circuits of high frequency. For example, the *closed loops* used in this experiment can be quite accurately tuned with the aid of this interrupter and the receiving instruments described above. With a fixed sending circuit the accompanying curve (Figure V) was obtained by a single set of readings when the air condenser in the receiving circuit was varied from six plates up to twenty-one plates. The capacities are represented as abscissas, while the deflections in centimeters are the ordinates. The capacity for each plate of the air condenser is 248 cm. The capacity at resonance can easily be located within one or two per cent. I have not been able to get quite such smooth resonance curves when the spark-gap is employed as interrupter in the sending circuit. Experiments are now in progress with commercial wireless telegraph circuits, instead of the closed loops.

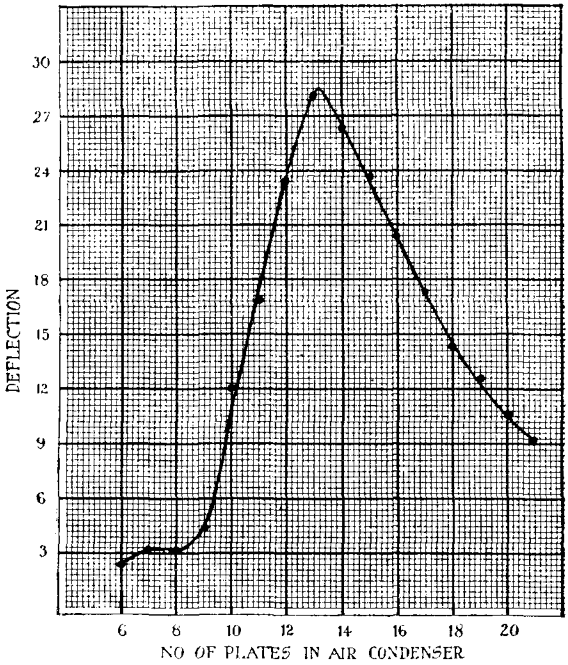

Figure V.

IV. Photographs with the Revolving Mirror.

In the search for an explanation of the greater uniformity and effectiveness of the mercury interrupter in producing inductive action between circuits, I have taken a series of photographs of condenser discharges through the interrupter. For this purpose the familiar revolving-mirror apparatus was employed (Figure VI). The concave mirror M has a focal length of 1.52 meters, and is driven by a battery motor at a speed of twenty to seventy revolutions per second. The interrupter, mirror, and plate are in a light-tight box, of which the end carrying the photographic plate projects into a dark room. Neither side of the sensitive plate is covered, so that the observer, who charges the condenser from a step-up transformer by operating a switch in the dark room, can see a flash upon the plate when the plate is struck. In fact, he can see quite plainly each of the oscillations, though they persist in some cases for only a fraction of a millionth of a second.

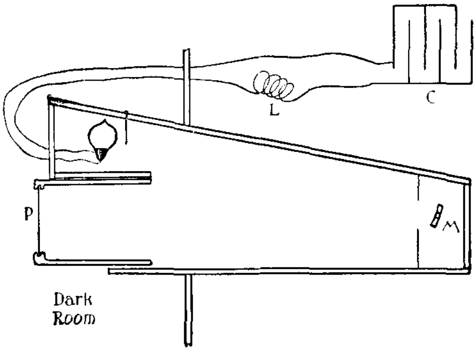

FIGURE VI.

In order to measure the time of the discharge, the speed of the mirror was obtained by the aid of a stroboscopic device (Figure VII) as follows: A small aluminum disc D, marked off in alternate black and white sectors, is attached to the axis of the mirror. The disc is illuminated periodically by flashes in a Geissler tube G, connected to the secondary of an induction coil C, of which the primary is interrupted by an electrically driven tuning-fork T. The tuning-fork makes 256 vibrations per second. The disc contains 12 black sectors; so that if the disc makes $\frac{1}{12}$ of a revolution between two consecutive flashes in the tube, the disc will appear to stand still. Thus by observing the disc (by a telescope through the wall of the dark room) and adjusting the resistance in the field of the motor that drives the mirror, the disc is brought to an apparently stationary condition. The mirror is then making $\frac{1}{12}$ of $256 = 21.33$ revolutions per second. Other apparently stationary conditions of the disc correspond to 42.66 and 64 revolutions per second. It is not difficult to set on these speeds with an accuracy of one or two tenths of one per cent.

The mercury interrupters of which the accompanying photographs were taken were so constructed that the mercury surfaces were brought near to each other (about 1 mm.), so that the image of both electrodes fell near together on the plate.

The revolving-mirror photographs of the mercury interrupter and of the ordinary spark in air between cadmium terminals are shown in Plate I.

When the mercury interrupter in action is viewed directly by the eye at rest, without the intermediation of the revolving mirror, it shows an intense luminescence throughout the bulb, while brilliant flashes are thrown up from both electrodes all around the line of contact of the mercury with the glass. It looks as if a great many of these little fountains of fire occur simultaneously. The revolving mirror shows, however, that their occurrence is usually successive, — each little flash going through its series of oscillations and dying out before another flash appears. It is thus not difficult to make the exposure so short that only one fountain with its oscillations appears on the plate. By diaphragming the bulb and adjusting the position of the sensitive plate the pictures of Plate I, Figures 1, 3, and 4, were made to take in only the illumination of the nearer regions of the electrodes. The mirror was turning in the direction from the bottom of the cut towards the top, and the two vertical lines of impressions in Figures 1, 3, and 4 are respectively the oscillations at the two electrodes. The picture of Figure 3, which is clearer, shows that a bright point of light appeared first on the electrode to the left, and that this spot persisted for a time sufficient for the mirror to turn through a distance indicated by the length of the bright spot on the plate. During this time the point of light widened a little and then died out. Shortly after the extinction of the illumination at the left-hand electrode, a bright spot appeared on the electrode at the right. After this disappeared a second illumination occurred on the left, and so on for a series of oscillations whose number depends on the self-inductance, capacity, and resistance of the circuit. Figure 1 (Plate I) shows four oscillations at a single electrode. Figures 2 and 5 of Plate I are the

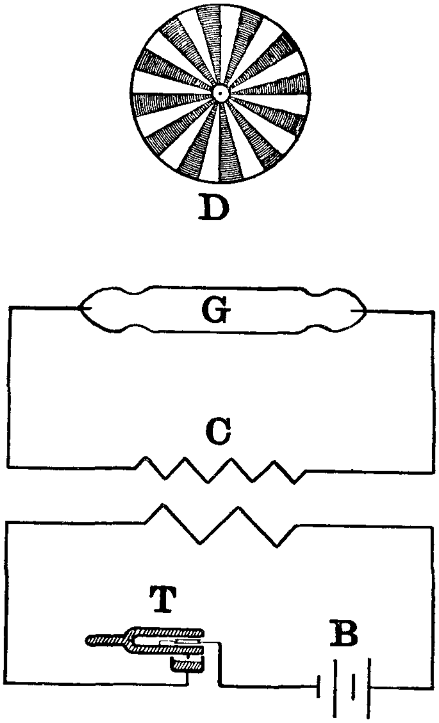

Figure VII.

familiar photographs taken in the same way with the discharge in air between cadmium terminals. The two terminals become successively luminous in an oscillatory fashion.

In Figures 1, 3, and 4 of Plate I light bright enough to affect the photographic plate is obtained only from one of the electrodes, the cathode. The exposure is too short to show what happens at the anode. By the use of a larger capacity and a larger inductance the period of oscillation can be increased so that the exposure is long enough to get a faint impression from the anode. This was done in a case where the capacity was .117 microfarads, the inductance .0071 henries, and the time of an oscillation, therefore, .000178 second. The picture obtained, which is not here reproduced, showed that, during the existance of the bright point of light at the cathode, there is also a weak glow spread over the entire surface of the anode.

These results indicate that the current whose action is here photographed is exactly of the same character as the mercury arc, except that the current is reversed several times during the condenser discharge; for in the mercury arc with direct current the anode is covered with a faint glow over its entire surface, while the cathode region is dark except for a very small spot of light of extreme brilliancy.

The fact that the several pictures of Plate I show different frequencies and different damping is of no significance, as they were made with various inductances, capacities, and resistances in the circuit of discharge.

One important fact about the pictures obtained with the mercury interrupter is that the impressions are so sharp (in the negative) that their distance apart (cf. Figure 1, or the distance from the first to the last on the left-hand electrode of Figure 3) can be measured with great accuracy, which makes this form of interrupter useful in the photographic measurement of the time of a condenser discharge. For example, taking the stroboscopic determination of the speed of the mirror and measuring the distances between impressions on various photographic plates, the values given in Table II were obtained for the time in seconds of a double oscillation.

The measurements in the last column were made with a cadmium spark-gap instead of the mercury interrupter, and show larger variations than the columns obtained with the interrupter.

Measurements similar to those of Table II have been utilized in the determination of the capacity of the condensers and the inductance of the leads,— quantities that are needed in the discussions of section VI of this paper. The determination of these quantities was made as follows.

TABLE II.

23 PLATES IN GLASS CONDENSER.

Inductance.	Coil I. and Leads.	Leads.	Coil II. and Leads.
Time in sec.	2.30×10^{-5}	$.588 \times 10^{-5}$	7.90×10^{-5}
	2.30	.585	8.03
	2.28	.584	8.10
	2.29	.585	8.12
	2.30	.584	7.90
	2.30		8.12
	2.29		8.03
Mean	2.295	.585	8.03
Mean error	.3%	.2%	.9%

Coil I and coil II are accurately wound, and their inductances, L_1 and L_2 have been calculated from their geometrical dimensions.

$$L_1 = 1.06 \times 10^5 \text{ magnetic units,}$$
$$L_2 = 14.1 \times 10^5 \quad \text{"} \quad \text{"} \ .$$

Let L = the unknown inductance of the leads, then by Thomson's formula, we have from Table II,

$$2\pi\sqrt{(L_1 + L)C} = 2.295 \times 10^{-5} \qquad (1)$$
$$2\pi\sqrt{LC} = .585 \times 10^{-5} \qquad (2)$$
$$2\pi\sqrt{(L_2 + L)C} = 8.03 \times 10^{-5} \qquad (3)$$

Eliminating C from (1) and (2), we have $L = .073 \times 10^{-5}$, which, substituted in (2), gives

$$C = .1175 \times 10^{-15} \text{ magnetic units,}$$
$$= 1.05 \times 10^5 \text{ cm.}$$

Likewise, from equation (3)

$$C = 1.04 \times 10^5 \text{ cm.}$$

When we remember that the percentage error of the time is doubled in the calculation of capacity, it is seen that these two computed values of C agree with an error not greater than the error of observation, or the errors possibly made in the computations of L_1 and L_2.

In this way the inductance of the leads and the capacities of the condensers were determined in a number of cases (Table III) to be used in the discussion of the results obtained in section VI for the resistance of the interrupter.

TABLE III.

n = the number of plates in condenser,
T = period in millionths of a second for the discharge through the leads alone.
T' = period through leads and .000106 henries in series.

n.	T.	T'.	Inductance of Leads in Henries (calc.)	Capacity in Microfarads (calc.)
3	2.39	7.76	.0000111	.0130
7	3.78	12.1	.0000116	.0313
19	6.14	18.6	.0000130	.0730
24	7.48	23.5	.0000120	.1170

In putting in additional condensers up to nineteen plates the inductance of the leads had to be increased, hence the progression in the first three values of column four. To get twenty-four plates, plates were introduced back nearer to the interrupter by leads whose direction was such as to diminish the inductance, giving the smaller value of inductance in the last line. The capacities are correct within about 1%. The inductances having been determined as a difference, may contain an error as great as 2%.

V. Photographs Showing Rapidity of Recovery of the Mercury Interrupter.

The revolving-mirror photographs of the mercury interrupter show that when the condenser in series with the interrupter is charged to a sufficiently high potential the matter in the globe in some way becomes conducting, and that this conductivity continues during a series of oscillations. In this respect no difference is apparent between the action of the mercury interrupter and the ordinary discharge of the condenser between metallic terminals in air. It thus occurs to one that the advantage of the mercury interrupter may lie in the rapidity of recovery of the nonconducting character of such an interrupter after the discharge has passed, so that the condenser which is connected to the transformer may again be charged to a high potential and may thus again quickly accomplish a strong series of oscillations.

To test this point, a number of photographs (Plates II and III) of the mercury interrupter and the spark in air were taken to ascertain how many times distinct series of oscillations occur during a single cycle of the charging transformer. The pictures were taken upon films or sheets of bromide paper attached to a rapidly rotating disc. The image of the interrupter was focussed upon the sensitive paper or film by a lens of short focal length. To avoid overlapping of the pictures on the film when the disc made more than one revolution during the exposure, the lens was mounted in a pendulum, and the exposure was made by swinging the lens behind a diaphragm in front of the revolving film. In this way the image, instead of moving in a circle, was made to trace out a spiral on the film. With such an apparatus the motion was too slow to resolve the discharge into its separate oscillations. On the other hand, each *series* of oscillations constituting a *complete discharge* made an impression upon the plate. The speed of the motor could be varied so as to take from one to four cycles of the transformer per revolution. The period of the transformer, $\frac{1}{60}$ second, served as a measure of the speed of the motor.

In order to obtain an understanding of these pictures it must be remembered that the secondary of the transformer was attached permanently to the condensers; the primary of the transformer was closed, and the condensers were allowed to charge directly and to discharge through the interrupter. During this action of the interrupter, the pendulum containing the lens was allowed to drop, thus making the exposure.

Simon and Reich * have already shown by a photographic method that several discharges may occur during a single half-cycle of the transformer. Evidently, the number of such complete discharges, each with its series of oscillations, will depend on the capacity of the condenser, the inductance of the secondary of the transformer, and the potential at the terminals of the secondary of the transformer. For a given secondary, as we increase the charging potential by increasing the current in the primary of the transformer, the number of charges and discharges will increase. Their number will also increase with decreasing condenser capacity.

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* Phys. Zeit., **4**, 361 (1903).

With a large capacity the number of discharges occurring during the half-cycle is small. In Figure 6, Plate II ($C = .117$ microfarads, $V = 15{,}000$ volts), only one, or sometimes two, discharges occur during the half-cycle of the transformer. In this picture the whole of the mercury-vapor bulb together with the bright surfaces of the electrodes are shown, while the legs of the interrupter, of course, make no impression on the film. The images are arranged in the form of a spiral on the film. Both electrodes are bright, for several oscillations occur during each discharge.

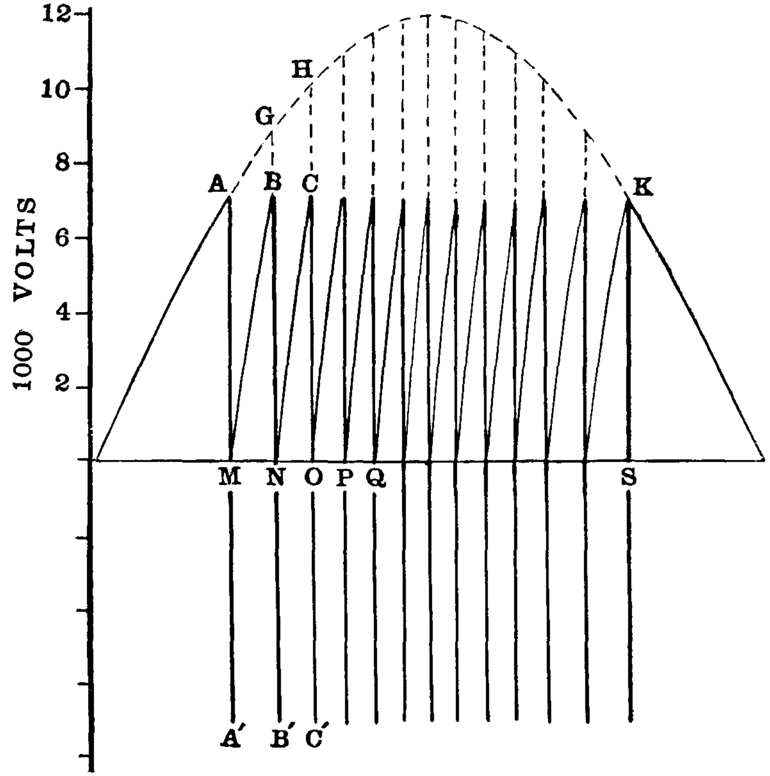

Figure VIII.

By keeping the voltage of the transformer the same and reducing the capacity about thirty-fold, Figure 7, Plate II was obtained. In taking this photograph, to prevent confusion, the greater part of the bulb was covered by a paper screen, leaving only the electrodes and the lower part of the bulb to be photographed. In this case ($C = .0043$ microfarads, $V = 15,000$ volts) about sixty complete discharges occur during the half-cycle of the transformer, which is $\frac{1}{120}$ second.

By lowering further the capacity of the condensers and raising the potential of the transformer, the number of discharges may be greatly increased and the period of rest at the reversal of the cycle can be made small, so that the attempt of Simon and Reich to make the mercury interrupter operate on a direct current can be approximately realized with the transformer as source of current.

In order to obtain an understanding of the succession of charges and discharges through the interrupter let us examine the photograph of Figure 8, Plate III, which is a *negative*. In this case the capacity was .0130 microfarads, the maximum potential of the transformer cycle 12,000 volts. The numbers of complete charges and discharges during successive half-cycles of the transformer are seen to be 12, 11, 12, 13, 13, 9, 11, 12, 11, 12, 12, 16, 13, 13, 12, 11, 13, 11, 14, 12, 12. By a separate experiment it has been shown that with the particular interrupter the condensers *begin to discharge* when their potential is 7070 volts, whatever the capacity of the condensers.

If the condensers always discharge at 7070 volts throughout each series, and if we neglect the reactance of the discharges on the potential of the transformer, the diagram of Figure VIII would represent approximately the manner in which the discharges occur. The sine-curve of Figure VIII represents the potential of the open-circuited secondary of the transformer with condenser in series. This curve is plotted with voltage as ordinates and epoch as abscissas. When about 7070 volts is reached, for this particular interrupter, the condenser discharges with a series of oscillations up and down the line A M A′. The condenser again charges along the practically straight line M B, and discharges again along the line B N B′, and so on. The rapidity with which the condenser charges, after any given series of oscillations, is approximately proportional to the potential of the transformer during the charge, so that the areas M A G N, N G H O, . . . should be equal; thus a division of the area M A G H K S into equal n smaller areas M A G N, N G H O, . . . ought to give the distribution of the discharges M, N, O, . . . S.

The construction of Figure VIII is slightly erroneous, for the discharge is never complete, but with the present interrupter always leaves the condenser charged to about 1600 volts. This residual voltage chances sometimes to be positive and sometimes negative, which is a possible explanation of the irregularities apparent in the distribution of the images in the actual photographs. The similarity of their distribution at the beginning of a series and their distribution at the end of a series shows that *the accumulated effect of a number of discharges does not render the bulb conducting so as to weaken succeeding discharges.*

By the use of a small Leyden jar as capacity and a charging potential of 15,000 volts, I have been able to obtain over 200 complete discharges, each with its series of oscillations, during one-half cycle ($\frac{1}{120}$ second) of the charging transformer. These complete discharges, comprising each many oscillations, were separated by an interval of time of about $\frac{1}{100,000}$ of a second, yet every discharge was sharp, definite, and regular, and showed that even after a long operation of the interrupter at this frequency of charging the bulb did not become filled with conducting vapor or conducting ions so as to lower materially the potential of succeeding discharges. This seems to me to be a very important part of the whole advantage that the Cooper-Hewitt mercury interrupter has over the spark in air.

Figure 9, Plate III, is a typical example of the behavior of a spark in air when produced by a high potential. This is a picture of a spark between zinc terminals taken with a rotating film. Each spot is the image of a complete discharge with its series of oscillations. It is seen that these discharges at the beginning of a cycle are strong, but throughout the cycle become spasmodically strong and weak, showing that the spark-gap often retains its conducting character long enough to prevent the proper subsequent charging of the condenser. This discussion may apply only to the case in which the condenser is charged by some persistent source of current like the transformer. I have not ascertained whether a similar result is to be found when the condensers are charged by an induction coil or a static electric machine.

VI. THE RESISTANCE OF THE MERCURY INTERRUPTER.

In measuring the mean resistance of the mercury interrupter, I have made use of a calorimetric method similar to that employed by Battelli and Magri[*] in their determination of the resistance of a spark-gap in a series of measurements on condenser discharges. A calorimeter containing the mercury interrupter was put in series with a calorimeter containing a known resistance. The discharge from the condenser while connected to the transformer was allowed to pass for a suitable time through the known resistance and the interrupter in series. Since the heating of both resistances was produced by the same current, the heat developed in the two calorimeters was proportional to their respective resistances. The calorimeters are shown in Figures IX and X. Figure IX is the calorimeter about the mercury interrupter (calorimeter "A"). In its construction a glass bell-jar was put down over the interrupter and cemented with fish-glue and plaster of Paris to a thick glass plate provided with two holes, through which the leads to the

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[*] Phys. Zeit, **3**, 539 (1901–1902).

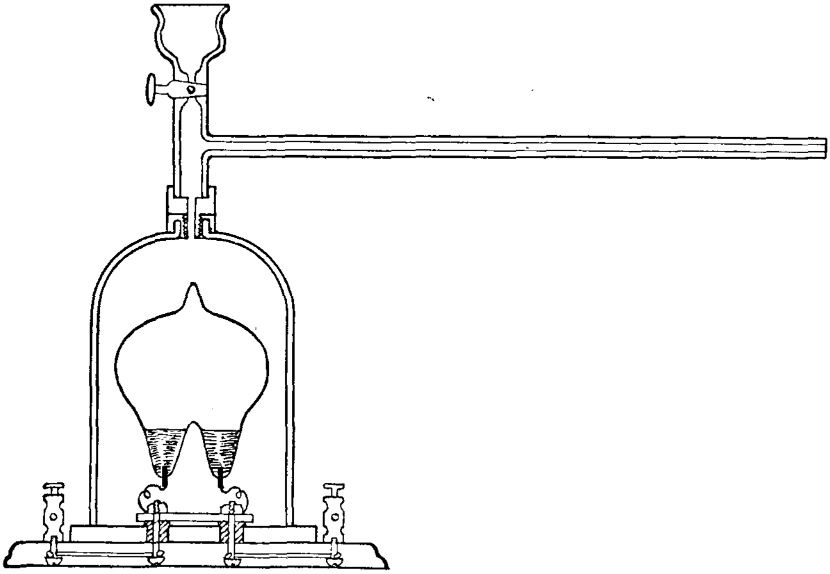

FIGURE IX. CALORIMETER A.

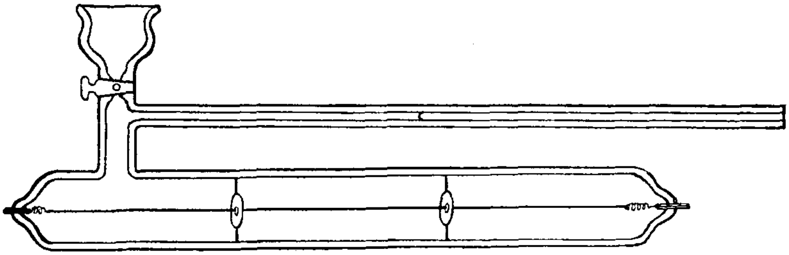

FIGURE X. CALORIMETER B.

interrupter were admitted. Into the neck of the bell-jar, by means of a screw-connector, was fitted a funnel-tube closed above by a stop-cock and communicating with a horizontal glass capillary to be used as an index. The bell-jar, funnel-tube, and a part of the capillary were filled with a light transformer oil. The expansion of the oil, read on a scale attached to the capillary, served as a measure of the heat developed in the jar. Within the oil, around the interrupter, black paper was loosely wrapped to prevent radiation.

For comparison with calorimeter A, two calorimeters "B" and "C' of different resistances were constructed of the form shown in Figure X. The resistance consisted of a straight manganine wire connected by springs of copper to thick pieces of platinum sealed into the ends of a glass tube 8 cm. in diameter. This tube was also filled with oil and provided with a capillary index similar to that of calorimeter A. The manganine wire was supported axially within the tube by circular discs of glass. About these discs of glass, within the oil, half-way between the wire and the walls of the tube, a cylinder of black paper was wound to prevent radiation. The resistance of the manganine wires of calorimeters B and C was measured on a Wheatstone bridge. Computed by Rayleigh's formula, the correction for surface travel when these wires should be used with the oscillatory current, was found to be negligibly small on account of the high specific resistance of the material of the wires. The calorimeters B and C had the following constants:

Calorimeter.	Length of Wire in cm.	Diameter of Wire in cm.	Resistance in Ohms.
B	66	.078	1.025
C	48	.108	.258

To determine what amount of heat corresponded to 1 cm. of expansion of the oil, the three calorimeters were calibrated by a *direct dynamo current* through the bulb of the mercury interrupter and the two manganine wires in series. The current was started, as in the Aaron's lamp, by tipping the bulb so that the mercury of the two electrodes came momentarily together. The energy expended in the three calorimeters by the calibrating current could now be calculated from the current, the voltage about the interrupter, the resistances of the two manganine wires, and the time.

Having in this way obtained the amount of heat required to give one centimeter of expansion on the scale of each of the calorimeters, the resistance of the mercury interrupter for the oscillatory discharge was measured for various inductances and capacities of the discharge circuit. The resistance of the interrupter was found to vary with the capacity and inductance. When this resistance was large, the calorimeter B was used in the comparison; when it was small, C was used. To make sure that the two comparison calorimeters were consistent, the resistance of the interrupter was occasionally measured by both B and C and found to give concordant results.

In making the final measurements with the oscillatory current the mercury interrupter in the calorimeter A was put in the discharge circuit in series with the known resistance of one of the comparison calorimeters. The secondary of the transformer was connected to the condenser, a switch in the primary of the transformer was closed, and the condenser was allowed to charge and discharge for a time varying from twenty to sixty seconds. The expansion of the oil of the two calorimeters was read, and then observations on the cooling of the calorimeters were taken for four minutes, so that the correction for cooling could be estimated.

At the end of the series of resistance measurements, the inductances and capacities of the various circuits were measured with an accuracy of about one per cent by the photographic measurement of the time of the condenser discharge. A sketch of the method of the computations for this purpose is given in section IV, p. 399 of this paper. The advantage of this method of determining the constants of the circuit is that it gives these constants for the required frequency.

The following tables (Tables IV, V, and VI) give a series of results for the resistance of the interrupter for various capacities and inductances.

From these tables it is seen that the resistance of the mercury interrupter decreases with increasing capacity of the condensers, and decreases with decreasing inductance of the discharge circuit.

These facts might perhaps be anticipated from the relation between the voltage and the current in the mercury arc with direct current, and the relation between the capacity, inductance, and current in the condenser discharge.

For direct currents greater than 3 amperes the voltage about the mercury arc is practically constant, and equal to 16 volts for the particular bulb here employed; therefore,

$$i R = \text{constant} = 16. \qquad (1)$$

TABLE IV.

INDUCTANCE = .0000110 HENRY. DISCHARGE POTENTIAL, 7070 VOLTS.

Capacity in Microfarads	.0130	.0313	.0730	.117
Period, Millionths of Sec.	2.39	3.78	6.14	7.48
Resistance of Mercury Interrupter in Ohms .	.29 .29 .23 .31 .27 .30 .29	.23 .21 .22 .21 .23 .22 .21	.16 .14 .13 .13 .14 .13 .13	.12 .11 .14 .13 .13 .13 .13
Average	.284	.219	.137	.127
$R\sqrt{C} \times 10$	.32	.38	.37	.43

TABLE V.

INDUCTANCE = .000117 HENRY. DISCHARGE POTENTIAL, 7070 VOLTS.

Capacity in Microfarads	.0130	.0313	.0730	.117
Period, Millionths of Sec.	7.76	12.1	18.6	23.5
Resistance of Mercury Interrupter in Ohms .	.69 .66 .68 .63 .64 .69 .68	.45 .48 .45 .48 .43 .43 .40	.25 .23 .23 .24 .23 .24 .23	.20 .20 .20 .18 .22 .20 .20
Average	.667	.444	.236	.20
$R\sqrt{C} \times 10$	.76	.78	.64	.68

TABLE VI.

INDUCTANCE = .00142 HENRY. DISCHARGE POTENTIAL, 7070 VOLTS.

Capacity in Microfarads			.0730	
Period, Millionths of Sec.			64.7	
Resistance of Mercury Interrupter in Ohms .			.60 .60 .59 .63 .60 .59 .58	
Average			.598	
$R\sqrt{C} \times 10$			1.62	

Now the current for the simplest case of a condenser discharge is given by the equation

$$i = \frac{2\,E\,C}{\sqrt{4\,L\,C - R^2\,C^2}}\,e^{\frac{-Rt}{2L}} \sin \omega\, t. \tag{2}$$

If the resistance is negligible in comparison with $2\sqrt{\frac{L}{C}}$ the square root of the mean square value of i becomes (neglecting damping)

$$I = \frac{1}{\sqrt{2}}\,\frac{\sqrt{C}}{\sqrt{L}}\,E. \tag{3}$$

If the relations (1) and (3) were exact, we should have for a given inductance

$$\sqrt{C}\,R = \text{constant}, \tag{4}$$

and for different values of the inductance

$$\frac{\sqrt{C}}{\sqrt{L}} \times R = \text{constant}. \tag{5}$$

We should not expect the relations (4) and (5) to be exact, because, first, equation (2) is obtained on the assumption that the resistance in the discharge circuit is independent of the current, which is a contradiction of (1), and, second, equation (1) is not true for small values of the current. Especially is (5) inaccurate because we have neglected the effect of the inductance on the damping.

An examination of the experimental data of Table IV, V, and VI shows that the inductance relation (5) is not verified. On the other hand, with a constant inductance $\sqrt{C} \times R$, for an eight-fold variation of C, is near enough to a constant to be of use, perhaps, in certain cases where only a rough approximation is required.*

For comparison with the resistance of the mercury interrupter, as obtained in these experiments, the following tables VII and VIII for the resistance of the ordinary spark in air are taken from the researches respectively of Lindemann † and Battelli and Magri.‡ These experimenters, by a bolometric method and a calorimetric method respectively, have measured the resistance of the spark in air between solid metallic terminals. For a proper comparison we should take the resistance for those spark-lengths that require the same potential to start the discharge as is required by the mercury interrupter (7070 volts).

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* This result is not to be confused with the apparently more exact relation found by Lindemann for the dependence of spark-energy on capacity.

† Lindemann, Ann. der Phys., **12**, 1012 (1903).

‡ Battelli and Magri, Phys. Zeit., **3**, 539 (1901–1902), and **4**, 181 (1903–4).

TABLE VII.

RESISTANCE OF SPARK. R. LINDEMANN.

Inductance Henries.	Capacity Microfarads.	Spark Length mm.	Disch. Pot. Volts.	Resistance of Spark in Ohms.
.00000560	.00496	.67	3300	2.72
.00000560	.00496	1.16	5100	2.59
.00000560	.00496	1.58	6500	2.44
.00000560	.00496	2.24	8550	1.78
.00000560	.00496	.40	2224	2.18
.00000560	.00982	.40	2224	1.42
.00000560	.01593	.40	2224	1.055
.00000560	.02131	.40	2224	1.255

TABLE VIII.

RESISTANCE OF SPARK. BATTELLI AND MAGRI.

Inductance Henries.	Capacity Microfarads.	Spark Length mm.	Disch. Pot. Volts.	Resistance of Spark in Ohms.
.0000741	.0158	2	8301	.323
.0000295	.0158	2	8085	.341
	.00797	2	8220	.551
	.00397	2	8190	.723
.0000175	.00397	2	8103	.564
.00000367	.00397	2	7635	.290

Among the results given by Lindemann only the fourth value of Table VII. is appropriate for these comparisons. Lindemann's value is

1.78 ohms. A single observation that I made with the mercury interrupter and with capacity and inductance about equal to those of Lindemann in this case gave .60 ohms. Extrapolations from the other values obtained by Lindemann indicate that for the same discharge potential, and corresponding capacities and inductances, his method gives for the resistance of the spark in air, values perhaps three or four times as large as the resistance of the mercury interrupter.

On the other hand, it is seen from Table VIII that the values obtained by Battelli and Magri for the resistance of a 2 mm. spark in air between solid metallic terminals * are of about the same magnitude as the values I obtained for the resistance of the particular mercury interrupter. The discharge potentials in the two cases were approximately the same.

Lindemann's observations were made with a single discharge, while those of Battelli and Magri and those of the author were made with a great number of discharges following each other in rapid succession. This latter arrangement would be the condition under which the resistance would be least. I found, however, that a considerable change in the rapidity of charging made no appreciable change in the resistance of the mercury interrupter.

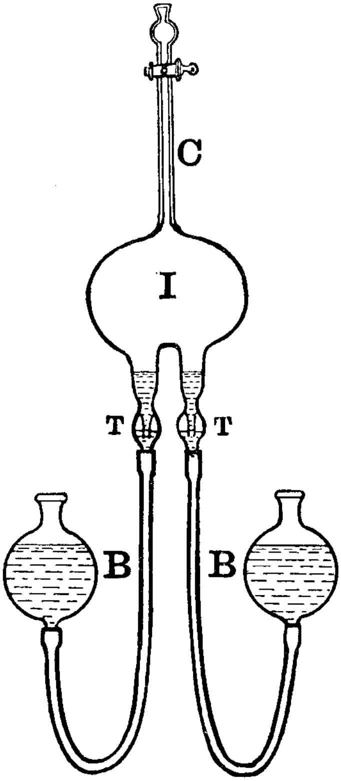

FIGURE XI.

VII. THE VACUUM OF THE COOPER-HEWITT INTERRUPTER.

In order to be able rapidly to change the vacuum in the mercury interrupter and to measure the pressure appropriate for use with a transformer capable of giving about 15,000 volts, the apparatus of

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* Battelli and Magri found the same resistance whether they employed platinum-iridium or cadmium terminals.

Figure XI was devised. The protuberances of the bulb I, instead of being provided with platinum terminals, were left open, and to them rubber tubes were connected. The other ends of the rubber tubes were connected to reservoirs containing mercury. To the top of the bulb I was fused a capillary tube of uniform bore closed above by a stop-cock. The vessels B B were stationary, and the bulb I could be lowered with stop-cock open so as to fill with mercury. The bulb I was then raised with stop-cock closed to a height greater than the barometric column, and was thus exhausted. Traps T T in the protuberances of I prevented air bubbles, which escaped from the rubber tube, from entering the bulb. The bulb, which could thus be exhausted to any desired degree, was connected through the mercury columns in series with the primary of a Tesla coil, of which the discharge at the secondary served as a test of the correctness of the vacuum. The vacuum could be measured by lowering I with stop-cock closed, and bringing the residual gas under atmospheric pressure into the capillary C. It was found that with the particular voltage at my disposal (15,000 volts), the Tesla coil gave its best action when the pressure in the cold bulb, before the discharge, was about .02 mm. When the pressure was two or three times this amount the bulb gave a brilliant arc, while the spark at the terminals of the secondary was feeble. For pressures lower than .02 mm. (cold) the bulb showed pale green luminescence resembling somewhat the glow in a Roentgen tube. Under these circumstances the condensers seemed not to discharge.

Jefferson Physical Laboratory, Harvard University,
Cambridge, Mass., Dec. 20, 1903.

Fig. 1.

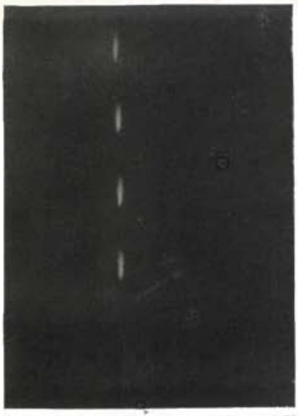

Fig. 2

Fig. 3.

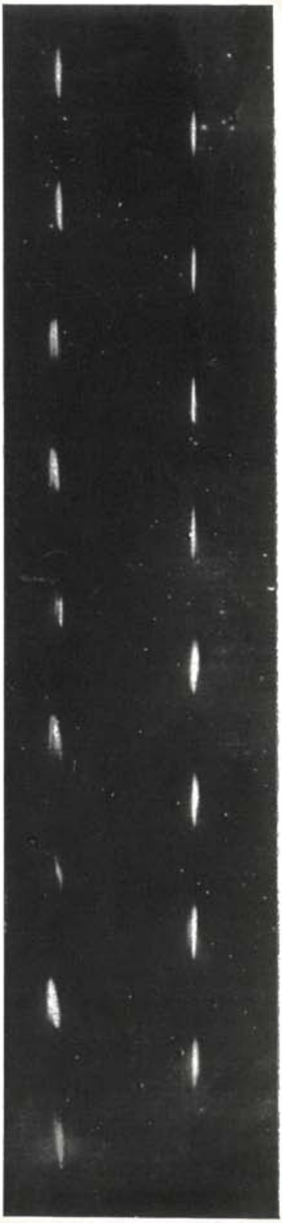

Fig. 4.

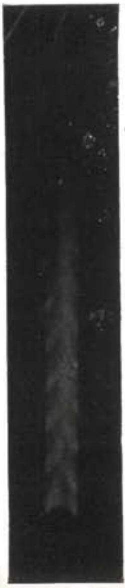

Fig. 5.

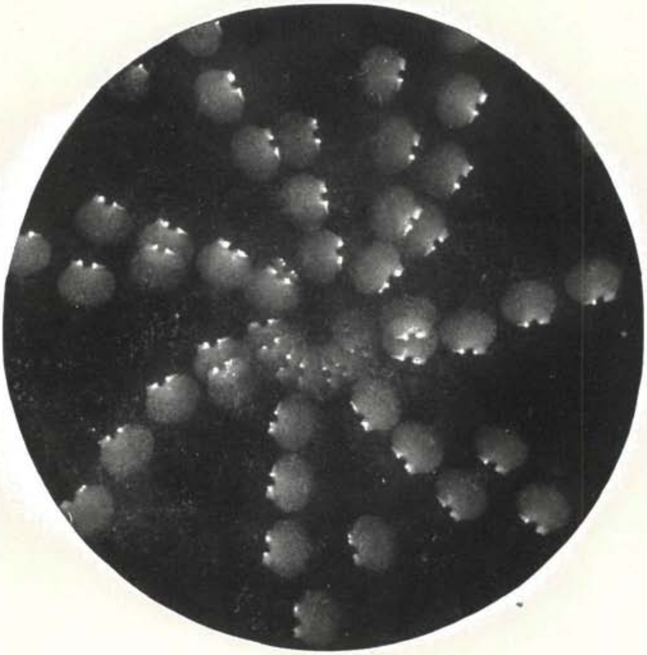

Fig. 6.

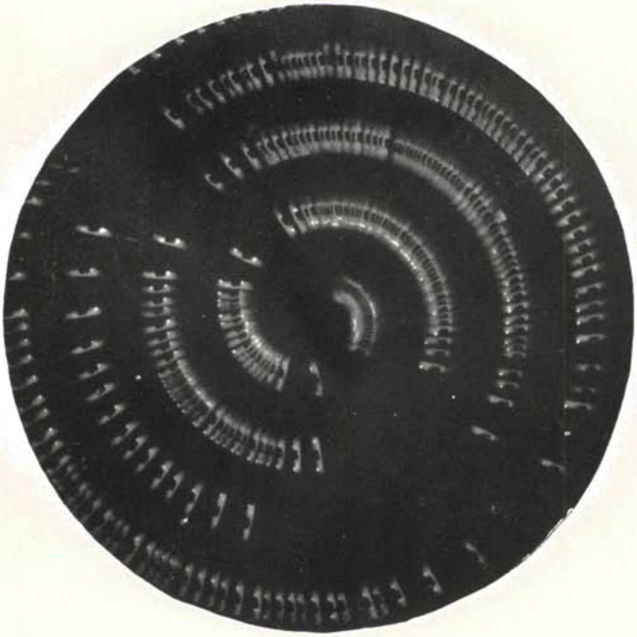

Fig. 7.

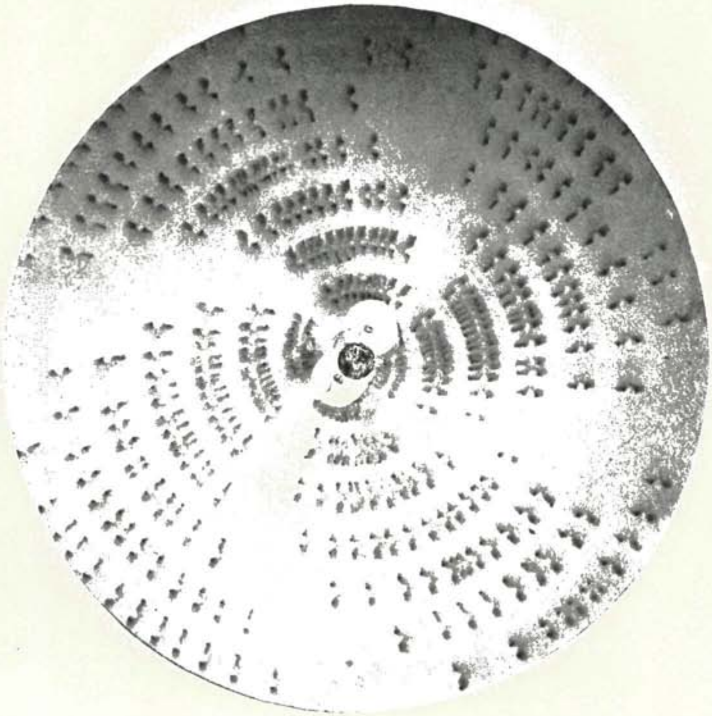

Fig. 8

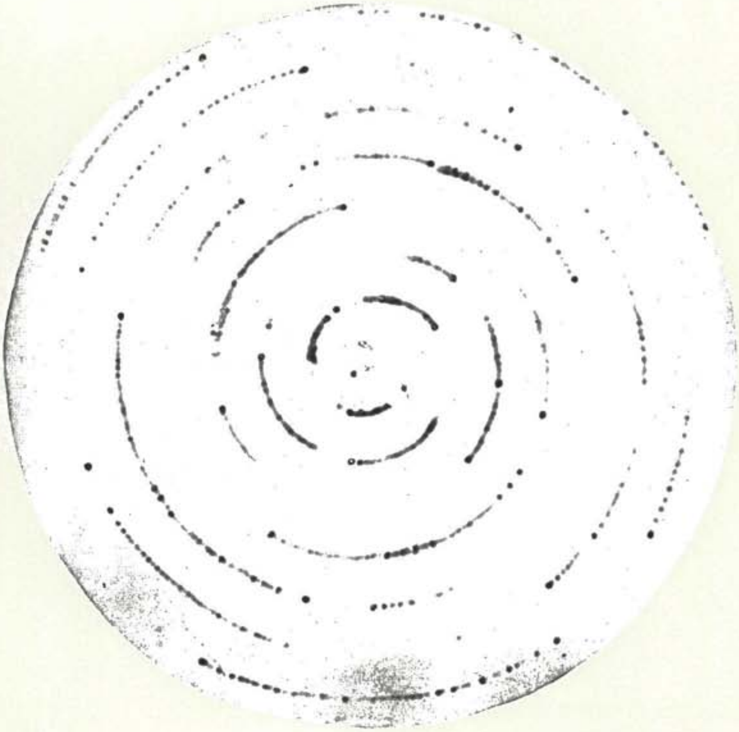

Fig. 9.

Proceedings of the American Academy of Arts and Sciences.

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EDGE CORRECTIONS IN THE CALCULATION OF THE ABSOLUTE CAPACITY OF CONDENSERS.

By J. G. Coffin.

EDGE CORRECTIONS IN THE CALCULATION OF THE ABSOLUTE CAPACITY OF CONDENSERS BY THE SCHWARZIAN TRANSFORMATION.

By J. G. Coffin.

Presented by A. G. Webster, December 9, 1903. Received December 11, 1903.

In the construction of an absolute condenser, consisting of two silvered plane circular glass plates, it was found necessary to compute the edge corrections for many cases which have not yet been determined.

In the following pages will be found a number of such cases worked out by means of the general conformal transformation due to Cristoffel[*] and Schwarz.[†]

A very clear résumé of the method will be found in the chapter on conjugate functions of J. J. Thomson's Recent Researches in Electricity and Magnetism.

The method has been applied to problems of different kinds by Kirchhoff,[‡] Potier,[§] Michell,[||] and Love.[¶]

It will be noticed that in all the cases here presented, there is a variable parameter which allows the correction for any relative position or size of the edges to be given from the general solution.

It then follows that a number of cases already solved by J. J. Thomson are but special cases of these the more general solutions.

This is a great advantage, as it affords an excellent check upon the accuracy of the more complicated results.

In a number of cases the problem was solved in two distinct manners, the more elegant being here presented, but the results in every case agreeing with each other.

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[*] Annali de Math., **1**, 89 (1867).

[†] Crelle, **70**, 105–120 (1869).

[‡] Gesam. Abhand., p. 101.

[§] French Trans. of Maxwell, Appendix.

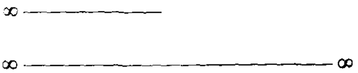
[||] Phil. Trans., A, 389 (1890).

[¶] Proc. Camb. Phil. Soc., **7**, 175 (1891).

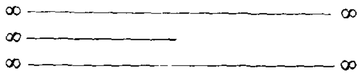
Professor J. J. Thomson solves the following problems, which are most concisely stated by diagrams. The sign ∞ at the end of a line, denotes that the line is to be considered to extend to infinity in that direction.

Every line represents the intersection with the plane of the paper of planes perpendicular to the paper and extending to infinity in both directions, above and below. So that figure (a), for example, represents a semi-infinite plane, near and parallel to another infinite plane.

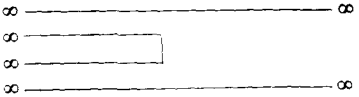
In all the diagrams these planes are to be considered conductors of electricity.



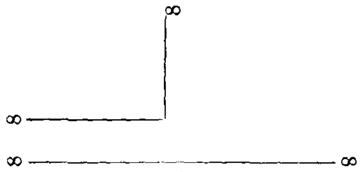
(a)



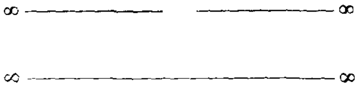
(b)



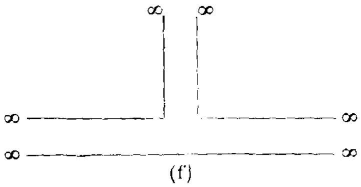
(c)



(d)



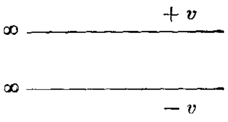
(e)



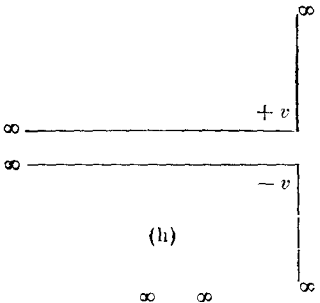
(f)

The last two cases have special reference to the correction for the slit in a guard ring electrometer.

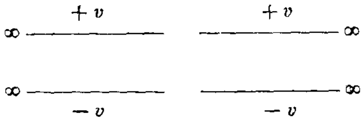
For further work on the electrometer from this point of view see the French translation of Maxwell, Vol. II, p. 568, et seq., where M. Potier has worked out more general cases. We can immediately extend these results of Thomson's by the method of electrical images. We thus obtain the solutions for the following cases, where v represents the potential:



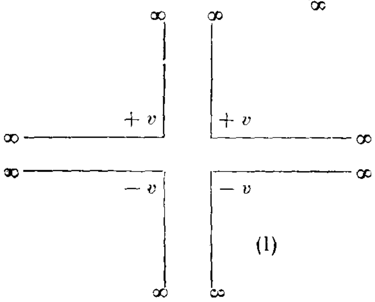
(g)



(h)



(k)

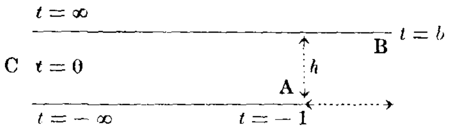


(l)

Problem I.

Consider two semi-infinite planes one of whose edges extends beyond the other.

At A $t_1 = -1$, $\alpha_1 = 2\pi$
C $t_2 = 0$, $\alpha_2 = 0$
B $t_3 = b$, $\alpha_3 = 2\pi$



Hence the Schwarzian transformation for this case is, in the general case of the three values $a\ b\ c$ for $t_1\ t_2\ t_3$, respectively,

$$\frac{dz}{-C} = \frac{(t-a)\ (t-b)}{t-c}\,dt = \left((c-a) + (t-b) + \frac{(c-a)\ (c-b)}{t-c}\right) dt$$

and

$$\frac{z}{-C} = (c-a)\ t + \frac{(t-b)^2}{2} + (c-a)\ (c-b) \log\ (t-c) + \Gamma.$$

Putting $a = -1$, $b = b$, $c = 0$, we obtain

$$\frac{z}{-C} = t + \frac{(t-b)^2}{2} - b \log t + \Gamma.$$

If we make the point $t = -1$ correspond to the origin in the z-plane we shall have then for $t = -1$

$$o + o\,i = -1 + \frac{(b+1)^2}{2} - b \log(-1) + \Gamma.$$

Therefore the constant of integration is:

$$\Gamma = 1 - \frac{(b+1)^2}{2} + b \log(-1).$$

When t is very small the principal term is:

$$\frac{z}{C} = -b \log t + \cdots \qquad b \geqq 1.$$

Now as long as t is negative and its absolute value very small, the imaginary part must be equal to zero. But when t passes through the value zero from negative to positive, z increases by hi.

Hence $$z = -C\,(-b \log t - b\,\pi\,i)$$

which fulfils the above conditions,

and $$C\,b\,\pi = h \qquad C > 0 \text{ and real.}$$

$$C = \frac{h}{b\,\pi}$$

Also when $$t = b \qquad z = g + h\,i,$$
then

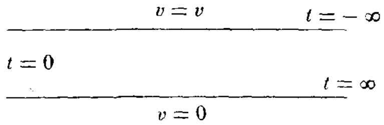
$$g + h\,i = -C\left(b - b \log b + 1 - \frac{(b+1)^2}{2} - b\,\pi\,i\right);$$

$$h = C\,b\,\pi$$

again, and which gives:

$$g = -C\left(b\,(1 - \log b) + 1 - \frac{(b+1)^2}{2}\right).$$

In the w-plane the diagram is:



and the corresponding transformation is, since the polygon has one zero angle at $t = 0$,

$$\frac{dw}{D} = \frac{dt}{t}. \qquad \therefore\; w = D \log t = \phi + i\,\psi.$$

When t is positive, $\psi = 0$;

when t is negative, $\psi = v$.

$$\therefore \quad D i \pi = v i, \qquad \text{or} \qquad D = \frac{v}{\pi},$$

and the final expression is

$$w = \frac{v}{\pi} \log t. \tag{a}$$

Now to find the amount of electricity on the under side of the upper plate, from the edge ($t = b$) to a point so far in ($z = -x + h i$) that the charge is uniform, we have, since

$$q = \frac{1}{4\pi} (\phi_{t=P} - \phi_{t=b})$$

or using eq. (a)

$$q = \frac{v}{4\pi^2} (\log t_P - \log b).$$

When t is very small and positive

$$z = -x + hi = -C\left(\frac{b^2}{2} - b \log t_P + 1 - \frac{(b+1)^2}{2} - b\pi i\right),$$

$$-x = -C\left(\frac{1-2b}{2} - b \log t_P\right)$$

$$\therefore \quad \log t_P = \frac{-x}{Cb} + \frac{1-2b}{2b},$$

from which

$$q = \frac{v}{4\pi^2}\left(\frac{-x}{Cb} + \frac{1-2b}{2b} - \log b\right),$$

and since $C = \frac{-h}{b\pi}$,

$$= \frac{-v}{4\pi h}\left[x + \frac{h}{\pi}\left(\frac{2b-1}{2b} + \log b\right)\right].$$

When b is unity, which brings the two edges symmetrically above one another, the correction becomes

$$q = \frac{-v}{4\pi h}\left(x + \frac{h}{2\pi}\right).$$

When t is very small and negative, $z = -x + o i$, and the quantity of electricity on the upper side of the lower plate is

$$q = \frac{-1}{4\pi}(\phi_{t=P} - \phi_{t=-1}) = \frac{v}{4\pi^2}\log(-t_P).$$

$$\therefore \quad q = \frac{-v}{4\pi^2}\left(\frac{-x}{Cb} + \frac{1-2b}{2b}\right)$$

$$= \frac{+v}{4\pi h}\left[x + \frac{h}{\pi}\left(\frac{2b-1}{2b}\right)\right].$$

If b is infinite this becomes

$$q = \frac{+v}{4\pi h}\left(x + \frac{h}{\pi}\right).$$

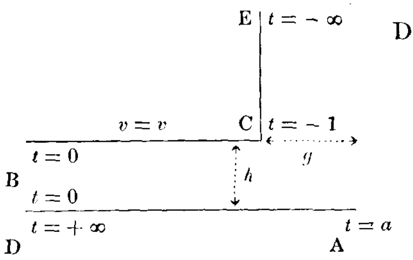
If b is unity the equation becomes

$$q = \frac{v}{4\pi h}\left(x + \frac{h}{2\pi}\right),$$

which agree with the known special cases already worked out.

Problem II.

Consider the problem of a square edge facing a semi-infinite plane, which may extend beyond the edge as per diagram.



The polygon consists of the sides DABCE.

At A	$t_1 = a,$	$\alpha_1 = 2\pi.$
B	$t_2 = c,$	$\alpha_2 = 0.$
C	$t_3 = b,$	$\alpha_3 = \frac{3\pi}{2},$

The transformation for this case is

$$\frac{1}{C}\frac{dz}{dt} = \frac{\sqrt{t-b}\,(t-b)}{t-c} = \sqrt{t-b}\left(1 + \frac{(c-a)}{t-c}\right),$$

which integrates into

$$\frac{z}{C} = \tfrac{2}{3}(t-b)^{\frac{3}{2}} + 2(c-a)\sqrt{t-b} + (c-a)\sqrt{c-b}\log\frac{\sqrt{t-b} - \sqrt{c-b}}{\sqrt{t-b} + \sqrt{c-b}} + \Gamma.$$

Let us put $a = a, \quad b = -1, \quad c = 0;$ then

$$\frac{z}{C} = \tfrac{2}{3}(t+1)^{\frac{3}{2}} - 2a\sqrt{t+1} + a\log\frac{\sqrt{t+1}+1}{\sqrt{t+1}-1} + \Gamma.$$

Let the origin be at $t = -1$. Then

$$\frac{o + o\,i}{C} = a \log \frac{1}{-1} + \text{const.} \qquad \therefore \text{ const} = \Gamma = a\,\pi\,i.$$

When $t = a$, $z = g + h\,i$; hence

$$g + h\,i = C\left(\tfrac{2}{3}\,(a+1)^{\frac{3}{2}} - 2\,a\sqrt{a+1} + a \log \frac{\sqrt{a+1}+1}{\sqrt{a+1}-1} + a\,\pi\,i\right).$$

$$h = C\,a\,\pi, \qquad C = \frac{h}{a\,\pi}.$$

From which

$$\text{(d)} \qquad g = \frac{h}{a\,\pi}\left(\tfrac{2}{3}(a+1)^{\frac{3}{2}} - 2\,a\sqrt{a+1} + a \log \frac{\sqrt{a+1}+1}{\sqrt{a+1}-1}\right).$$

The diagram in the w-plane consists of two infinite lines with one zero angle, the same as in the preceding problem.

The transformation for which is

$$\frac{dw}{D} = \frac{dt}{t}, \quad \text{or} \quad w = D \log t.$$

When t is negative, $\psi = v$. $\therefore\ v = D\,\pi$, and $w = \frac{v}{\pi} \log t$.

The equation for the electricity on the lower face of the square corner from the corner $t = -1$ to a point so far in, that the distribution may be considered uniform is

$$q = \frac{v}{4\,\pi^2}\left(\log t_P - \log\,(-1)\right)$$

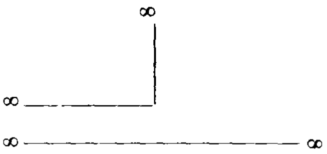
We must now find a value of t or of $\log t$ for a point at which t is negative and very small.

Putting $t = 0$ except in the terms which become large under such circumstances: —

$$-x + o\,i = C\left(\tfrac{2}{3} - 2\,a + 2\,a \log 2 - a \log t + a\,\pi\,i\right)$$

$$\log t = \frac{x}{a\,C} - \frac{2}{3\,a} + 2\,(1 - \log 2) - \pi\,i$$

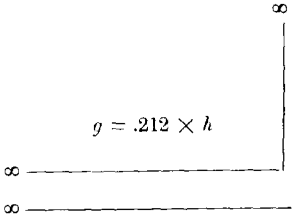
$$\therefore \quad q = \frac{v}{4\,\pi\,h}\left[x + \frac{2\,h}{\pi}\left(1 - \log 2 - \frac{1}{3\,a}\right)\right]$$



When a is infinite we have the case which agrees with that already worked out.

It is seen that when a is in the vicinity of unity the correction undergoes large variations. The correction becomes zero when:

$$1 = \log 2 + \frac{1}{3a} \quad \text{or } a = 1.086$$

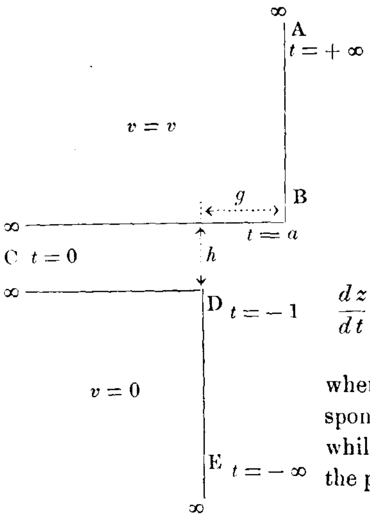


The value of g corresponding is found by substituting in formula (d) this value of (a); which makes

$$g = .212 \times h$$

or approximately as drawn.

Problem III.



Find the capacity per unit length of the following system of conductors.

The dimensions perpendicular to the paper are all infinite. The Schwarzian transformation for this case is:

$$\frac{dz}{dt} = C(t-t_1)^{\frac{\alpha_1}{\pi}-1}(t-t_2)^{\frac{\alpha_2}{\pi}-1}(t-t_3)^{\frac{\alpha_3}{\pi}-1},$$

where t_1, t_2, t_3 are the values of t, corresponding to the three corners D, C, and B, while α_1, α_2, α_3 are the internal angles of the polygon EDCBA — E at D, C, and B.

At D let $t_1 = b \qquad \alpha_1 = \frac{3\pi}{2}$

C $\qquad t_2 = c \qquad \alpha_2 = 0$

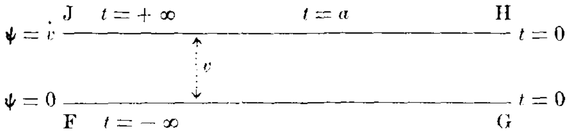
B $\qquad t_3 = a \qquad \alpha_3 = \frac{3\pi}{2}$

$$\therefore \quad \frac{dz}{dt} = \frac{C\sqrt{(t-a)(t-b)}}{t-c}$$

By making the substitution $u^2 = \frac{t-b}{t-a}$ or $t = \frac{a u^2 - b}{u^2 - 1}$ this expression becomes rationalized, and integrates into

$$z = C\left(\sqrt{(t-a)(t-b)} + \frac{2c-a-b}{2}\log\frac{\sqrt{t-a}+\sqrt{t-b}}{\sqrt{t-a}-\sqrt{t-b}}\right.$$
$$\left. -\sqrt{(b-c)(a-c)}\log\frac{\sqrt{(c-b)(t-a)}+\sqrt{(c-a)(t-b)}}{\sqrt{(c-b)(t-a)}-\sqrt{(c-a)(t-b)}} + \Gamma\right) \qquad \text{(a)}$$

where Γ is the constant of integration.



The w-diagram consists of two straight lines parallel to the ϕ-axis, at a distance v apart.

At $G \quad t = 0, \quad \alpha = 0,$ and the transforming function is obtained from $\frac{dw}{dt} = B\frac{1}{t}$ where B is a constant.

From this equation we obtain the integrated form

$$w = B(\log t + \text{const.}) = \phi + i\psi$$

As t runs from $-\infty$ to 0 and from 0 to $+\infty$, the value of $\log t$ diminishes by $i\pi$ as t passes through zero.

$$\therefore \quad w = B(\log t - i\pi) = \phi + i\psi$$

and as t passes through zero w increases by $i v$

$$\therefore \quad v = -B\pi \quad \text{or } B = -\frac{v}{\pi}.$$

Hence finally

$$w = \phi + i\psi = \frac{v}{\pi}(i\pi - \log t). \tag{b}$$

Take now in equation (a)

$$\begin{aligned} a &= a > 1 \\ b &= -1 \\ c &= 0 \end{aligned}$$

which becomes

$$z = C\left(\sqrt{(t-a)\,(t+1)} + \frac{1-a}{2}\log\frac{\sqrt{t-a}+\sqrt{t+1}}{\sqrt{t-a}-\sqrt{t+1}}\right.$$

$$\left. - i\sqrt{a}\log\frac{\sqrt{t-a}+i\sqrt{a\,(t+1)}}{\sqrt{t-a}-i\sqrt{a\,(t+1)}} + \Gamma\right). \tag{c}$$

Make the origin in the z-plane correspond to $t = -1$ and let $C = -i\,D$, then

$$z = o + o\,i = -i\,D\left(\frac{1-a}{2}\log 1 - i\sqrt{a}\log 1 + \Gamma\right)$$

from which follows $\Gamma = 0$.

When $\qquad t = a \qquad z = g + hi,$

then

$$z = g + hi = -Di\left(\frac{1-a}{2}\log(-1) - i\sqrt{a}\log(-1)\right)$$

$$= -Di\left(\frac{1-a}{2}\pi i - i\sqrt{a}\,\pi i\right).$$

From which follows, equating real and imaginary parts,

$$\begin{aligned} h &= -D\sqrt{a}\,\pi \\ g &= +D\frac{1-a}{2}\pi \end{aligned} \qquad \text{and } D = \frac{-h}{\sqrt{a}\,\pi}.$$

Equation (c) now becomes:

$$z = \frac{h}{\sqrt{a}\,\pi}\,i\left(\sqrt{(t-a)\,(t+1)} + \frac{1-a}{2}\log\frac{\sqrt{t-a}+\sqrt{t+1}}{\sqrt{t-a}-\sqrt{t+1}}\right.$$

$$\left. + \,i\,\sqrt{a}\log\frac{\sqrt{t-a}+i\,\sqrt{a\,(t+1)}}{\sqrt{t-a}-i\,\sqrt{a\,(t+1)}}\right). \qquad \text{(d)}$$

From the values for h and g above we find, if we put

$$\frac{g}{h} = b = \frac{a-1}{2\,\sqrt{a}}$$

$$a^2 - 2\,a\,(1 + 2\,b^2) + 1 = 0$$

whose solution is

$$a = 1 + 2\,b^2 \pm 2\,b\,\sqrt{1+b^2}.$$

We should expect *a priori* that a should be a function of $b = \frac{g}{h}$, as it is their ratio alone which determines the shape and relative position of the conductors. Since a must be greater than 1, we must take the larger root.

Suppose we take a point on one of the conductors so near to the value $t = 0$ that we may neglect t in comparison with 1 and *a fortiori* with a. We obtain by suitable transformations of the logarithms using the formula

$$\log\frac{x+i\,y}{x-i\,y} = 2\,i\tan^{-1}\frac{y}{x} + 2\,n\,\pi\,i.$$

Hence in formula (d), neglecting t and retaining 1 and a, we obtain

$$z = \frac{h}{\sqrt{a}\,\pi}\left(-\sqrt{a} + i\,\frac{1-a}{2}\,(2\,i\tan^{-1}\sqrt{a} - i\,\pi)\right.$$

$$\left. + \sqrt{a}\,(\log\frac{4\,a}{a+1} + i\,\pi + \log t_P)\right).$$

The imaginary part of this expression is $i\,y = \frac{h}{\sqrt{a}\,\pi}\sqrt{a}\,i\,\pi = i\,h$, which is a true result.

The real part is

$$-x = \frac{h}{\sqrt{a}\,\pi}\Big[-\sqrt{a} - (1-a)\left(\tan^{-1}\sqrt{a} - \frac{\pi}{2}\right) + \sqrt{a}\log\frac{4\,a}{a-1} + \sqrt{a}\log t_{P_0}\Big]$$

and

$$\log t_{P_0} = \frac{-x\,\pi}{h} - \left[1 + \frac{1-a}{\sqrt{a}}\left(\tan^{-1}\sqrt{a} - \frac{\pi}{2}\right) + \log\frac{a+1}{4\,a}\right].$$

Substituting this value of $\log t_P$ in the expression for the quantity of electricity on the conductor,

$$q = \frac{v}{4\,\pi^2}\Big(\log t_P - \log(-1)\Big)$$

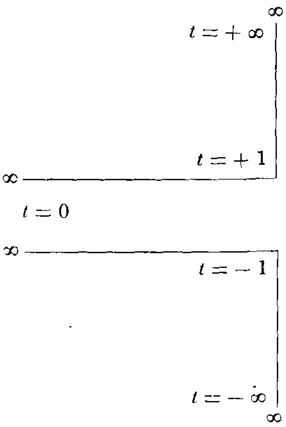
from $t = -1$ to $t = P$, we obtain the final expression for that quantity

$$\text{(e)}\quad q = -\frac{v}{4\,\pi\,h}\left\{x + \frac{h}{\pi}\left[\frac{1-a}{\sqrt{a}}\left(\tan^{-1}\sqrt{a} - \frac{\pi}{2}\right) + 1 - \log\frac{4\,a}{a+1}\right]\right\}.$$

When $v = 1$ this expression is the capacity of a strip of unit depth, from the point D to a point P_0 at a distance x towards the left; and we see that there is a correction of amount

$$x_c = \frac{h}{\pi}\left[\frac{1-a}{\sqrt{a}}\left(\tan^{-1}\sqrt{a} - \frac{\pi}{2}\right) + 1 - \log\frac{4\,a}{a+1}\right]$$

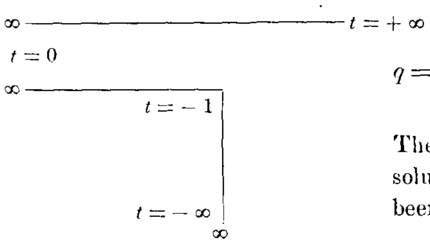
to be added to x, to take into account the concentration of the electricity at the corner. We may now calculate the amount of electricity on the face $D\,P$, assuming the distribution to be uniform, only instead of x we must use the value $x + x_c$.



When $a = 1$, or when we consider the case of two square corners facing one another, as in the figure, the equation (e) reduces to

$$q = \frac{-v}{4\pi h}\left(x + \frac{h}{\pi}(1 - \log 2)\right).$$

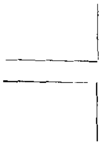
When we let a become infinite the expression (e) becomes indeterminate, but by determining its true value we find for the case of a square corner facing an infinite plane, the expression



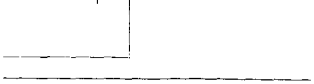
$$q = -\frac{v}{4\pi h}\left(x + \frac{2h}{\pi}(1 - \log 2)\right).$$

These two results check the general solution, as these special cases have been worked out by J. J. Thomson.*

As an approximate expression for these cases we may write:

$$q = -\frac{v}{4\pi h}\left(x + \frac{h}{10}\right)$$ for  I

and

$$q = -\frac{v}{4\pi h}\left(x + \frac{h}{5}\right)$$ for  II

where $\frac{h}{10}$ is used in place of $h \times .0978$.

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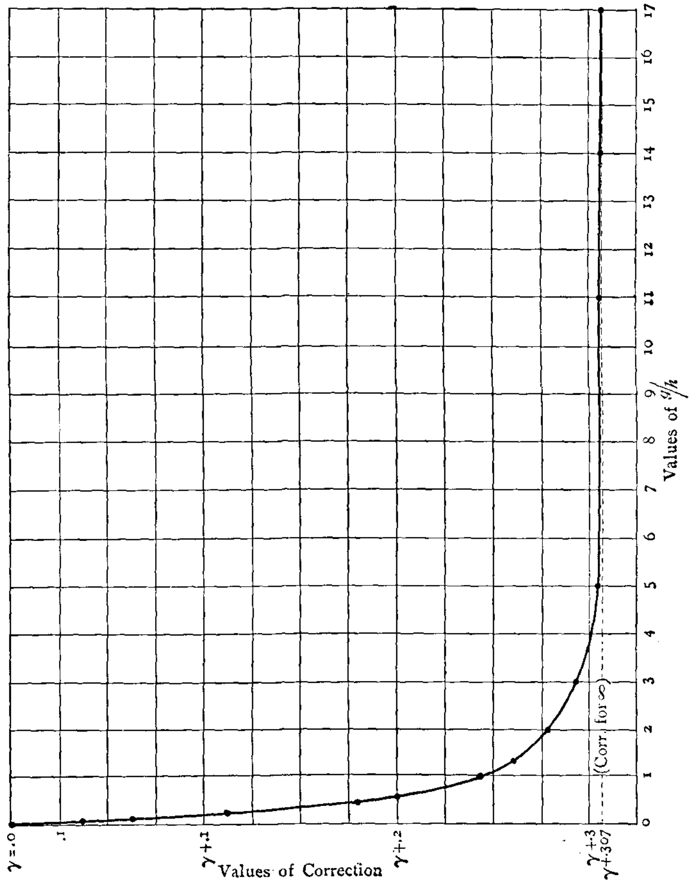
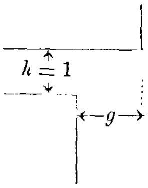
* Recent Researches in El. & Mag., Chapt. on Conjugate Functions.



Curve showing relation between the correction factor and g, the projection of the larger corner from the smaller.



The case where $g = 0$ has for correction factor $\frac{h}{\pi}(1 - \log 2)$

The case where $g = \infty$ has for correction factor $\frac{2h}{\pi}(1 - \log 2)$

The general case is

$$\frac{h}{\pi}\left\{\frac{a-1}{\sqrt{a}}\left(\frac{\pi}{2} - \tan^{-1}\sqrt{a}\right) + 1 - \log\frac{4a}{a+1}\right\}$$

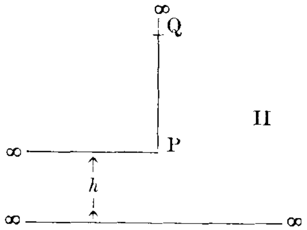
$$= \frac{h}{\pi}\left\{\underbrace{\frac{a-1}{\sqrt{a}}\left(\frac{\pi}{2} - \tan^{-1}\sqrt{a}\right) - \log\frac{2a}{a+1}} + (1 - \log 2)\right\}$$

where $\frac{g}{h} = \frac{a-1}{2\sqrt{a}}$ and the quantities plotted are $\frac{g}{h}$ and $\frac{a-1}{\sqrt{a}}\left(\frac{\pi}{2} - \tan^{-1}\sqrt{a}\right) - \log\frac{2a}{a+1}$.

Looking at the plot, we notice the remarkable rapidity with which the projecting edge acts as if it were an infinite plane. In fact, for all purposes a projection of ten or twenty times the distance apart acts as nearly as possible as an infinite plane, as far as the electricity on the inside face is concerned.

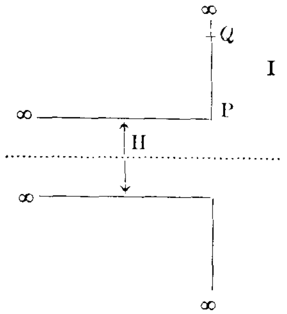
J. J. Thomson has found for the amount of electricity on the side PQ the expression:

$$q_{PQ} = \frac{v}{2\,\pi^2} \log \left(\frac{\pi\, PQ}{2\,h}\right).$$



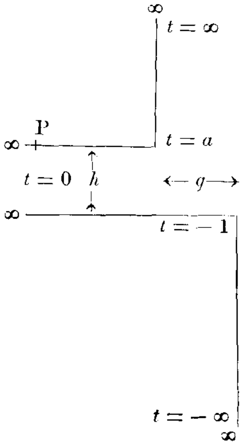
By the method of electrical images it is easily seen that for the case I, the expression is:

$$q = \frac{v}{\pi^2} \log \left(\frac{\pi\, Pq}{h}\right).$$



This problem was also completely solved with the following coördinates for the corners, where $a \leqq 1$. The solution for the quantity of electricity on the smaller plate being

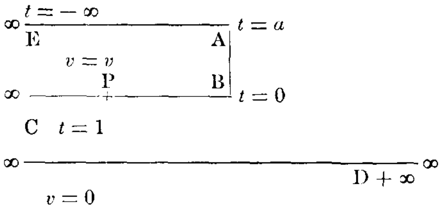
$$q = \frac{-v}{4\,\pi\,h} \left\{\kappa + \frac{h}{\pi} \left[\frac{1-a}{\sqrt{a}} \left(\tan^{-1}\frac{\pi}{2} + \frac{\pi}{2}\right) + 1 - \log\frac{4}{a+1}\right]\right\}.$$



This reduces to the correct expression for the case $a = 1$; but to obtain that for a rectangular corner near an infinite plane it cannot be applied, hence the change to the above notation.

Problem IV.

To find the distribution at the edge of a plate of finite width, which is at potential v, near to and parallel to an infinite plate at potential zero.



For generality put at

A $t_1 = a$, $\alpha_1 = \frac{3\pi}{2}$,

B $t_2 = b$, $\alpha_2 = \frac{3\pi}{2}$,

C $t_3 = c$, $\alpha_3 = 0$.

The Schwarzian transformation then becomes:

$$\frac{dz}{C} = \frac{\sqrt{(t-a)(t-c)}}{t-c}\,dt. \tag{a}$$

When $a = b$ this assumes the form

$$\frac{dz}{c} = \frac{t-a}{t-c}\,dt = \left(1 + \frac{c-a}{t-c}\right)dt.$$

$$\therefore \quad \frac{z}{c} = [t + (c-a)\log(t-c) + \text{const.}]. \tag{b}$$

To integrate the above expression (a) make the substitution $t - c = v$,

then $\quad t - a = v + c - a = v + d, \qquad d = c - a,$

$\quad t - b = v + c - b = v + e, \qquad e = c - b,$

and $\quad d + e = 2c - (a + b), \qquad de = (c - a)(c - b)$

(a) now becomes

$$\frac{dz}{C} = \frac{\sqrt{(v+d)(v+e)}}{v}\,dv$$

and is of the form under the radical of

$$X = \alpha + \beta x + \gamma x^2 = de + (d+e)v + v^2,$$

where $\quad \alpha = de, \qquad \beta = d + e, \qquad \gamma = 1.$

Using Byerly's tables of integrals we find that

$$\frac{z}{C} = \underset{(147)}{\int \frac{\sqrt{X}}{x}\, dx} = \sqrt{X} + \frac{d+e}{2} \underset{(120)}{\int \frac{dx}{\sqrt{X}}} + de \underset{(142)}{\int \frac{dx}{x\sqrt{X}}}$$

where the numbers refer to the formulae used. In the use of (142) the product de or $(c-a)\,(c-b)$ must be positive; this is true, as a and b both lie on the same side of a and the differences are either both positive or both negative. Hence we have

$$\frac{z}{C} = \sqrt{X} + \frac{d+e}{2} \log\left(\sqrt{X} + x + \frac{d+e}{2}\right)$$

$$- \sqrt{de} \log\left(\frac{\sqrt{X} + \sqrt{de}}{v} + \frac{d+e}{2}\frac{1}{\sqrt{de}}\right) + \Gamma$$

$$= \sqrt{(t-a)(t-b)} + \frac{2c-(a+b)}{2} \log\left(\sqrt{(t-b)(t-a)} + t - c + \frac{2c-(a+b)}{2}\right)$$

$$- \sqrt{(c-a)(c-b)} \log\left(\frac{\sqrt{(t-a)(t-b)} + \sqrt{(c-a)(c-b)}}{t-c} + \frac{2c-(a+b)}{2\sqrt{(c-a)(c-b)}}\right) + \Gamma.$$

As a check on this formula, put $a = b$; we obtain

$$\frac{z}{C} = (t-a) + (c-a) \log (t-c) + \text{const.},$$

which is the same as special case (b).

To apply this to our problem, put $a = a$, $b = 0$, $c = 1$; then we have

$$\frac{z}{C} = \sqrt{t\,(t-a)} + \frac{2-a}{2} \log\left(\sqrt{t\,(t-a)} + t - 1 + \frac{2-a}{2}\right)$$

$$- \sqrt{1-a} \log\left(\frac{\sqrt{t\,(t-a)} + \sqrt{1-a}}{t-1} + \frac{2-a}{2}\frac{1}{\sqrt{1-a}}\right) + \Gamma.$$

Using when convenient $m = \sqrt{1-a}$, this equation may be transformed into

$$\frac{z}{C} = \sqrt{t\,(t-a)} + \frac{m^2+1}{2} \log \frac{(\sqrt{t} + \sqrt{t-a})^2}{2}$$

$$- m \log \frac{(\sqrt{m^2 t} + \sqrt{t-a})}{2\,m\,(t-1)} + \Gamma. \quad \text{(c)}$$

Let the origin in the z-plane be chosen at the point B, $t = 0$; now putting $t = 0$ in (c), we obtain

$$\frac{o + o\,i}{C} = \frac{m^2+1}{2}\log\frac{m^2-1}{2} - m\log\frac{m^2-1}{-2\,m} + \Gamma.$$

$$\therefore\quad \Gamma = -\left(\frac{m^2+1}{2}\log\frac{m^2-1}{2} - m\log\frac{m^2-1}{-2\,m}\right).$$

When $t = a$, $\qquad z = o + d\,i.$

$$o + d\,i = C\left(\frac{m^2+1}{2}\log\frac{1-m^2}{2} - m\log\frac{1-m^2}{-2\,\mathrm{m}}\right)$$

$$- C\left(\frac{m^2-1}{2}\log\frac{1-m^2}{-2} - m\log\frac{1-m^2}{2\,m}\right)$$

$$= C\left(\frac{m^2+1}{2}\pi\,i - \frac{2\,m\,\pi\,i}{2}\right) = \frac{C\,\pi\,i}{2}(m-1)^2.$$

$$\therefore\quad d = \frac{C\pi}{2}(m-1)^2.$$

To find h, we notice that where t is very small we may write

$$\frac{d\,z}{C} = \frac{\sqrt{t\,(t-a)}}{t-1}\,dt \quad \text{as} \quad \sqrt{1-a}\,\frac{d\,t}{t-1} + \ldots$$

$$\therefore\quad \frac{z}{C} = m\,[\log\,(t-1) + \text{const.}].$$

Const. $= -\,i\,\pi$, and is so chosen that the imaginary part of z is zero as long as t is less than unity.

$$\therefore\quad z = m\,C\,[\log\,(t-1) - i\,\pi].$$

When $\qquad t = 1 + \epsilon, \qquad z = -\infty - i\,h;$

$$\therefore\quad -\infty - i\,h = C\,m\,[\log|\epsilon| - i\,\pi],$$

or $\qquad h = C\,m\,\pi, \qquad C = \dfrac{h}{m\,\pi}.$

Hence $\qquad \dfrac{d}{h} = \dfrac{(m-1)^2}{2\,m} = K,$ say $\quad m \overset{=}{>} 1.$

From which, given any value of K which determines the configuration of the polygon, we can find m by the formula

$$m = 1 + K \pm \sqrt{K(K+2)},$$

and vice versa.

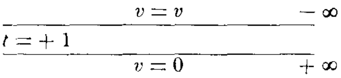
The diagram in the w-plane consists of two parallel straight lines, the angle being zero, and the value of t corresponding being $t = +1$.

Hence

$$\frac{dw}{B} = \frac{dt}{t-1},$$

$$w = \phi + i\psi = B[\log(t-1) + \text{const.}].$$

Const. $= 0$, because $\psi = 0$ as long as $t > 1$. When t passes through the value $t = 1$ and becomes less than unity, ψ increases by $v = B\pi$.



Hence $$B = \frac{v}{\pi}.$$

Now the quantity of electricity on a strip of breadth x and depth unity is by the general equation:

$$q = \frac{1}{4\pi}(\phi_t - \phi_0);$$

and since $$w = \frac{v}{\pi}\log(t-1),$$

$$q = \frac{v}{4\pi^2}[\log(t-1) - \log(-1)]$$

$$= \frac{v}{4\pi^2}\log(1-t), \qquad (c)$$

which gives the amount of electricity on a strip of breadth x, on the under side of the upper plate from the corner B, to a point P at a distance $-x$, from B.

Let us now find a value of $\log(t-1)$ so far from the edge and on the lower surface of the upper plate that t may be considered nearly equal to unity, say $t = 1 - \epsilon$. From (a)

$$\frac{z}{C} = m + \frac{1+m^2}{2}\log\frac{(1+m)^2}{2} - m\log\frac{2m}{t-1}$$

$$- \frac{1+m^2}{2}\log\frac{m^2-1}{2} + m\log\frac{m^2-1}{-2m}$$

$$= m + \frac{1+m^2}{2}\log\frac{(m+1)^2}{m^2-1} - m\log\frac{4m^2}{1-m^2} + m\log(t-1);$$

$$\log(t-1) = \frac{z}{Cm} - 1 - \frac{1+m^2}{2m}\log\frac{m+1}{m-1} + \log\frac{4m^2}{1-m^2};$$

$$\log(1-t) = \frac{-x}{Cm} - 1 - \frac{1+m^2}{2m}\log\frac{m+1}{m-1} + \log\frac{4m^2}{m^2-1}.$$

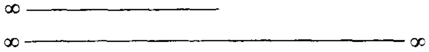
Substituting this value of $\log(1-t)$ in (c) we obtain

$$q = -\frac{v}{4\pi h}\left[x + \frac{h}{\pi}\left(1 + \frac{1+m^2}{2m}\log\frac{m+1}{m-1} - \log\frac{4m^2}{m^2-1}\right)\right] \quad \text{(d)}$$

which is the required expression.

When $a = 0 \quad m = 1$

which gives the case of a semi-infinite plate near an infinite plate

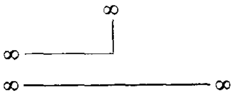


this case (d) reduces to

$$q = \frac{-v}{4\pi h}\left(x + \frac{h}{\pi}\right),$$

which is a correct result.

Where $a = -\infty \quad m = +\sqrt{\infty}$
this should give the expression for the electricity on a square corner near an infinite plate



(d) becomes:

$$q = \frac{-v}{4\pi h}\left[x + \frac{h}{\pi}\left(1 + \frac{\sqrt{\infty}}{2}\log 1 - \log 4\right)\right].$$

We must find the true value of the indefinite term

$$\frac{m^2+1}{2\,m}\log\frac{m+1}{m-1}.$$

Writing it in the form $\frac{0}{0}$ we have

$$\underset{\text{Limit } m \doteq \sqrt{\infty}}{} \frac{\frac{d}{d\,m}\log\frac{m+1}{m-1}}{\frac{d}{d\,m}\frac{2\,m}{m^2+1}} = \frac{(m^2+1)^2}{(m^2-1)^2}_{m \doteq \infty} = +\,1.$$

Hence

$$q = -\frac{v}{4\,\pi\,h}\left(x + \frac{2\,h}{\pi}\,(1 - \log 2)\right)$$

which is the correct result required.

Since for a plate of no thickness the parenthesis of the correction is unity and for infinite thickness is $2\,(1 - \log 2) = .614$.

The correction then always lies between the values: $\frac{h}{\pi}$ and $\frac{h}{\pi}\,.614$ or $h \times .318$ and $h \times .195$.

The amount of electricity on the end $A\,B$ is given by:

$$q = \frac{1}{4\,\pi}\,(\phi_a - \phi_o) = \frac{v}{4\,\pi^2}\,[\log\,(a-1) - \log\,(-1)]$$

$$= \frac{v}{4\,\pi^2}\log\,(1-a) = \frac{v}{4\,\pi^2}\log\,m^2;$$

$$q = \frac{v}{2\,\pi^2}\log\,m.$$

Now the equation $m = 1 + K + \sqrt{K\,(K+2)}$ gives the value of m for any value of $K = \frac{d}{h}$, the ratio of the thickness of the upper plate to its distance from the lower. We must take the positive value of the radical as $m \geqq 1$.

When $d = 0$ or $m = 1$, the amount of electricity on the sharp edge is

$$q = \frac{v}{4\,\pi^2} \log 1 = 0$$

and we find there is none.

To find the quantity of electricity on the upper surface of the upper plate, we find the quantity which lies on the plate from the corner B, at which $t = 0$, to a point P on the upper surface such that the corresponding x is a large multiple of h.

We must, however, agree to keep a small so that we can neglect it in comparison with the value of t we choose. From the general equation (a) we have

$$\frac{z}{C} = t + \frac{m^2+1}{2} \log 2\,|\,t\,| - m \log \frac{(m+1)^2}{2\,m}$$
$$- \frac{m^2+1}{2} \log \frac{m^2-1}{2} + m \log \frac{m^2-1}{-2\,m};$$

$$-\frac{x}{C} = t + \frac{m^2+1}{2} \log |\,t\,| + \frac{m^2+1}{2} \log 4$$
$$- \frac{(m-1)^2}{2} \log (m-1) - \frac{(m+1)^2}{2} \log (m+1)$$

and very approximately since $|\,t\,|$ is large.

$$\therefore\quad 1 - t = 1 + \frac{x}{C} + \frac{m^2+1}{2} \log \left(1 + \frac{x}{C}\right) - \frac{(m-1)^2}{2} \log (m-1)$$
$$- \frac{(m+1)^2}{2} \log (m+1) + \frac{m^2+1}{2} \log 4 \qquad \text{(e)}$$

and $$\frac{1}{C} = \frac{\pi\, m}{h}.$$

The quantity of electricity on the upper plate from $t = 0$ to $t = P$ is

$$q = \frac{v}{4\,\pi^2} [\log (t-1) - i\,\pi] = \frac{v}{4\,\pi^2} \log (1-t).$$

∴ Using the expression (e) we have

$$q = \frac{v}{4\pi^2} \log \left[1 + \frac{\pi x m}{h} + \frac{m^2+1}{2} \log \left(1 + \frac{\pi m x}{h} \right) - \frac{(m-1)^2}{2} \log (m-1) \right.$$

$$\left. - \frac{(m+1)^2}{2} \log (m+1) + \frac{m^2+1}{2} \log 4 \right]. \qquad \text{(g)}$$

The expression for $m = 1$ or $a = 0$ should reduce to the known expression. It becomes

$$q = \frac{v}{4\,\pi^2} \log \left[1 + \frac{\pi\,\kappa}{h} + \log \left(1 + \frac{\pi\,\kappa}{h} \right) \right]$$

which is identical with his result.

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CONTRIBUTIONS FROM THE ZOÖLOGICAL LABORATORY OF THE MUSEUM OF COMPARATIVE ZOOLOGY AT HARVARD COLLEGE, UNDER THE DIRECTION OF E. L. MARK. — No. 149.

METABOLISM AND DIVISION IN PROTOZOA.

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METABOLISM AND DIVISION IN PROTOZOA.

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I. Introduction.

The process of cell-division has been chiefly investigated for the sake of determining the form-changes which characterize it. For this purpose the descriptive-morphological method was found well adapted. The present work deals with a different aspect of the same process and employs a different method. Cell-division, being one of the fundamental powers of organized matter, presents a variety of relations for study. Its morphological aspect has naturally received attention first, but the attempt at further analysis of the results has given rise to a new class of problems. This state of affairs is well summarized by Korschelt und Heider (**:02**, p. 253), who say: "Wir haben bei der Frage nach den letzten Ursachen der im Vorstehenden beschriebenen Erscheinungen immer auf den uns noch unbekannten Chemismus der lebenden Elemente, auf unbekannte feinere Structuren oder auf den noch nicht ergründeten Mechanismus der Bewegungserscheinungen verweisen müssen." It is from the side of the first mentioned of these unknown, but not necessarily incompatible, factors, that the present work has been undertaken. In this work the chemical relations in living organisms, especially with reference to their power to divide, have been studied, and the method has of necessity been experimental.

The work was done under the continuous supervision of Professor E. L. Mark, and I take this opportunity to thank him for his valuable and kindly criticism. I am also under much obligation to him for meeting the numerous and varied material needs of these experiments.

The influence on cell-division of *substances* having a relation to it can be best studied experimentally upon free-living cells. Their media afford the important requirement of the experimental method, namely, the ability of the experimenter to vary the conditions. Since there is an active interchange between medium and cell, even the internal conditions of an organism may thus be varied. However, the distinction between external and internal conditions is at best one of not much logical force. The necessity of performing experiments upon the living object *by means of a liquid medium* made the analysis of physico-chemical relations a primary object. No adequate interpretation of the changes of cells in media is possible without this analysis. Whether it is only a preliminary necessity, or whether, if complete, such analysis would constitute the whole interpretation, must be left for the progress of investigation to determine. In this research the object was to push physico-chemical interpretation as far as the present development of physico-chemical methods permits. This part of the work constitutes one of the two essential modes of explanation here attempted. But it soon became evident that in the present state of knowledge this method of interpretation when applied alone accounts only incompletely for the ascertained facts. Hence the adjustment of the organism to a particular combination of conditions was adopted as a complementary, but in itself also incomplete, principle of explanation. This latter principle is elaborated in the concrete case of Stentor in a subsequent section. It is true that the conception of adjustment is an unanalyzed complex, and we do not know whether it expresses more than, or only as much as, the physico-chemical interpretation could if developed to its limit. At present it is a useful means of description, and the two contrasted modes of interpretation are here regarded as of equal *practical* importance. The media of free-living cells always contain salts in solution. Likewise it is shown by chemical analysis that cells contain salts, although their condition in the living organism is not so precisely known. Salts that are normally present in the cell or its medium I shall for convenience term the *physiological salts.*

It is almost entirely to this class of substances — and, it must be added, to a particular phase only of this extensive subject — that my studies have been directed.

Much work has been done upon the relation of salts to animal cells, the most of which bears only an indirect relation to that which is here attempted. In the experiments of mammalian physiology the principal problem has been to find out the function of the salts in *general metabolism*. The history of this subject is well given by Rywosh (:00). In another class of experiments the aim has been to determine what salts are necessary for the *development* of the animal, or what is the influence of salts upon that process. This is well illustrated by the extensive work of Herbst ('93 to '99) upon marine Invertebrates. The Protozoa, the group dealt with in the present paper, have been made the objects of much chemical experimentation. But the relation between the *process of division and salts physiologically present* in organisms or their media has received but little direct investigation. In Protozoa the division-reaction, if so it may be named, has not been treated heretofore in a *numerical* way as a criterion of the comparative effects of the physiological salts, used either singly or in combination. Of interest in the present work, so far as they bear on division, are the experiments upon artificial fertilization with salts, as done by Loeb ('99), Morgan ('99), and Viguier (:01). These questions suggest themselves: In what relation do the salts *normally* stand to the process of division, and is their action osmotic, or chemical, or both? The analogy between the egg-cell and the protozoan has been frequently stated. The present work was suggested by one feature of the mode of differentiation in the embryo. *Unequal rate of division* is seen to be an important factor in determining the relative positions of parts in the embryo. Our ignorance of the special factors involved here is well expressed by Minot (:03, pp. 38–39). "The reason for the unequal growth is unknown. We have not even an hypothesis to offer as to why one group of cells multiplies or expands faster than another group of apparently similar cells close by in the same germ layer. It is no real explanation to say that it is the result of heredity, for that leaves us as completely in the dark as ever as to the physiological factors at work in the developing individual." This question naturally arises: To what extent, if any, is the rapidity of division influenced by *normal* chemical factors, especially the ever present physiological salts? A protozoan was selected as the object because it seemed to present fewer complications with other possible factors than exist in the case of an association of cells.

Most of the following experiments were made upon *Stentor coeruleus.* This object has special advantages for the particular end in view. The animal was found to have a high degree of sensitiveness to its environment. Its responses differed in kind, and especially in degree, with variations in the reagents applied. Both the raising of cultures and the experiments themselves showed that Stentor differs from some other Protozoa in this respect. This fact permitted a much greater variation of the experimental factors than a more uniform responsiveness would have done. The qualities above described apply in its cell-division and are probably due to the close relation between that process and metabolism. The other interesting reactions of the animal were observed only incidentally.

II. General Methods and Technique.

The methods and technique for experimentation upon Protozoa require much consideration on account of the variety of conditions involved. Usually each case requires special treatment, but in the following experiments certain aspects of the subject were of such constant occurrence that they may be dealt with once for all. Only the essential points will be mentioned.

Since these studies are chiefly concerned with metabolic problems, it was necessary to maintain the animals experimented upon under conditions as nearly normal as possible. Hence the first requirement was some mechanical device for retaining Stentors in a given medium and also for transferring them to it. For the former purpose the hanging-drop method was used at first. Since many animals were required, a large number of drops was necessary. Upon one side of a thoroughly cleaned plate of thin glass (the thin glass used for photographic negatives answers the purpose well) were placed numerous drops, each containing from five to eight Stentors. With a quick motion this plate was inverted over a shallow cylindrical glass vessel, half filled with water, the edge of the vessel having been previously smeared with vaseline. The whole was then placed under a dissecting microscope, a rough map of the field with its contained drops was made, and the number of Stentors in each drop written upon the map. When, after experimentation, the next count was made, the record was continued upon the same map. With a little practice it became easy to regulate approximately by the size of the pipette used the number of Stentors put into a drop. This method was practicable and expeditious for handling large numbers of individuals and afforded excellent opportunity for any desired examination with objectives of low power. It was finally rejected, perhaps unnecessarily, for the method next described in consideration of the fact that conditions in a drop might not be entirely normal for so large an animal as Stentor. For almost all other Protozoa this objection would not hold.

Solid watch-glasses were then adopted; these were polished so as to fit closely when piled on top of one another. They were shallow, about 50 mm. in diameter and 10 mm. deep, and had concave internal surfaces, thus presenting no angles in which the animals might lodge and so escape observation and counting. A watch-glass was only partially filled with the required medium, 2 to 4 cc. being placed in each. No more animals were introduced than could be conveniently counted under a dissecting microscope. When piled up the watch-glasses formed a series of closed chambers with sufficient air space and with a much larger proportion of liquid to each Stentor than in the hanging-drop method. As nearly as could be imitated the conditions of a minute natural pond existed here. This method was used for nearly all experiments.

An important point in any method is the transference of the organisms from their native culture medium to the test-medium in the observation glass. To avoid contamination of the latter, or to reduce it to an infinitesimal amount, the following practice was adopted. A considerable number of Stentors were taken from their usual location upon one side of the culture jar with a medicine dropper and placed in a watch, glass (No. 1). Soon these would gather together in one part of the vessel on account of the presence there of zoögloea, the particular direction of the light, etc. With as small a quantity of liquid as possible, they were then transferred to watch-glass No. 2, which contained a considerable quantity of the medium to be used for the given experiment. In the same way, except that pipettes of smaller bore and a dissecting microscope were now used, they were repeatedly transferred to successive "ponds" of the required medium. From the last watch-glass "pond" a few at a time were transferred with a capillary pipette to the test watch-glass containing the pure medium. This I call the method of multiple transference.

How many such transfers are necessary? Fortunately there is a good test to determine this point. When the electrical conductivity of the medium — using electrodes that can be dipped into the liquid of a watch-glass and then removed — shows no *immediate* change upon the introduction of Stentors, then there is no contamination. In the case of multiple transference from culture liquid to distilled water, it was determined that from two to four transfers were sufficient to prevent any perceptible contamination. The number necessary depends much upon the experimenter's personal method of working. The construction of the pipettes is also an important factor. It appears from the following consideration that the amount of contamination resulting upon the dilution following the use of the pipette must be exceedingly small. Capillary pipettes can be made of such small diameters as to admit but one Stentor at a time, and under the magnifier five to ten animals can be drawn in with a very small quantity of water. This small amount of liquid is furthermore a very small proportion of the 4 to 5 cc. with which it is diluted in the next watch-glass. It follows that successive transfers rapidly multiply the dilution of the original contamination, which at length becomes infinitesimal. This source of matter foreign to the desired medium can be reduced to as low a value as that resulting from the practically unavoidable contact with air and glass. The subject of the solubility of glass will be treated of in the experiments where that factor becomes of importance in drawing conclusions.

The selection of the Stentors for experiment is also a matter of much importance. No other factor that might be overlooked would so quickly bring discrepant results. If the metabolic effects of different reagents are to be compared, it is important that the experiments be conducted, if possible, upon animals in the same condition. When that is not practicable, it is important to have a means of comparison by which different results can be reduced to a common standard. The following experiments offer illustration of both procedures. The experimenter who maintains a continuous supply of material (Stentor) will have on hand cultures in various stages of development. *For comparative experiments the animals should be taken invariably from the same culture.* A good, well-established culture has considerable longevity and makes the above rule practicable. The results are still more favorable when the experimenter has learned to raise healthy cultures by a uniform method. In any case Stentors presenting abnormal or special conditions, such as the undersized, the pale, those that are conjugating or dividing, should be rejected. Material should always be abundant, for if it is, practice soon secures a very uniform selection of normal animals. Among the examples which follow there are some which show the possible reduction of results from different sources to a common metabolic standard.

The question as to the number of organisms that are necessary to a given experiment is one that cannot be answered by a general statement. In experiments upon metabolism where division is the criterion, cumulative experience with the animal's normal condition and its reaction to various substances aids in forming a judgment as to the numbers to be used. Before one knows the normal adjustments of the animal, larger numbers and more repetition may be necessary than later in the course of the work. The present experiments were often so planned either that each involved the one preceding it, or else that the new experiment was based upon the assumption of the validity of those preceding, and so led to discrepancy if the assumption was false. The above discussion applies to a certain class of experiments the validity of which is but little increased by repetition, but such are not numerous in biology.

The animals were segregated into groups, five to ten being placed in a watch-glass and constituting a preparation. This was a convenient number, for it was possible to count them under the dissecting microscope even after multiplication had taken place. The condition after twenty-four to forty-eight hours, usually the latter interval, was regarded as the one most representative of the effects of the medium upon division. Experiment shows that, in general, division keeps pace with metabolism, the latter of course depending upon the supply of food. In experiments with liquid media it is necessary to exclude every form of natural food, if it be desired to study the effect of the pure medium consisting, e. g., of a salt solution. Hence the Stentors in these experiments carried on their metabolic processes at the expense of the reserve nutriment which they contained when taken from their original culture. Consequently the effects upon division have to be observed early in the period of their subjection to a medium practically free from food. Some results observed after long intervals show that Stentor can live a long time without other food than physiological salts, but the process of multiplication always had its maximum in about the initial twenty-four to forty-eight hours. Sometimes it was necessary for a special purpose to take results after still shorter intervals.

The question whether division occurs in cycles has an important bearing upon any method of investigation whose results are based upon counting the number of organisms after a period of division. If there is such periodicity, it would be important to know whether the animals taken for comparative experiments had been taken in the maximum or in the mimimum of the cycle. In the case of Stentor neither direct observation nor the experiments made, furnish evidence of any *inherent* periodicity of division. The present experiments show that, except when some special modification of the medium exists (e. g. presence of potassium chloride in excess), multiplication runs, in the main, parallel to metabolism. Furthermore, while the supply of food in cultures and the constitution of the medium, especially with reference to its salt content, are normal, no maxima and mimima of division occur. This condition never lasts indefinitely, and both observation and chemical examination show that there is a definite course of changes natural to cultures from the period of their beginning with a food supply to that of its exhaustion. In an undisturbed culture the decline of Stentor as well as its initial rapid multiplication is due to metabolic conditions. In the present experiments old cultures were frequently revived, and rapidly multiplying ones were sometimes brought to sudden decline, by an apparently slight change in the nutritive conditions of the medium. The experiments on salt media described in this work show the pronounced sensitiveness of these animals to this factor. The response of Stentor to these conditions points to *metabolic exigency* as the efficient factor. To demonstrate experimentally, i. e. by *more efficient means than observation of cultures*, that periodicity of division occurs in a protozoan, two conditions at least must be fulfilled. First, the experimenter must know the exact composition with reference to salts, food, etc., of the media to which he subjects the animals. Such promiscuous mixtures as hay infusion *of unknown composition* will not suffice. Since frequent chemical analyses are impracticable, it will be necessary to construct by trial artificial media of known composition. The second requirement is that the experimenter should determine the normal adjustments of the animal to the different factors of the proposed media. Without this knowledge it is impossible to ascertain whether observed increase and decrease of multiplication is due to simple metabolic exigency or to an inherent tendency to cyclic division. A beginning towards meeting both requirements above indicated has been made for Stentor by experiments subsequently described.

In the preparation of reagents such precautions were taken as were appropriate to the end in view, and hence varied somewhat in different experiments. In general this part of the work was made to rest as much as possible upon objective demonstration. Reagents were either tested, even though their labels might be those of reliable manufacturers, or they were prepared personally. The measures used for volumetric purposes were calibrated by means of weighed quantities of liquid, using Sutton's ('96, p. 25) table for air weighing with brass weights.

Standard solutions were carefully made, and consistency among dilute solutions, chlorides, etc., maintained. Dilutions with water from an originally strong solution were made by weight with reference to the temperature standard. Perhaps distilled water was the reagent demanding most attention. In testing waters of all sorts, as well as culture liquids and reagents in special cases, the method of electrical conductivity was applied. For this purpose the apparatus devised by Nernst (**'94**) and modified by Maltby (**'95**) for measuring conductivity was found most useful. For its construction and use the original descriptions must be consulted.

For biological purposes a great number of test-cells of various constructions was found necessary. Variation in the size and distance apart of the electrodes was indispensable in some critical experiments where the use of platinized electrodes was not allowable. On this account and owing to the good qualities of the apparatus itself for the avoidance of polarization, it was never found necessary to use platinized electrodes in the test-cells. For convenience in the subsequent description of experiments I shall adopt the same notation for the measuring tubes of the instruments as are given to them by Maltby (**'94**, Fig. 4, p. 142). The tubes most referred to there are on the right-hand side, the larger one being marked W'' and the smaller W.

III. CONDITIONS OF GROWTH IN STENTOR, AND THE MANAGEMENT OF CULTURES.

For my study it was soon found desirable to maintain a continuous supply of vigorous material. *Later* it became evident that it was still more necessary to know the origin of the Stentors used. These requirements led to the artificial culture of Infusoria in the laboratory, instead of the continual acquisition of new material from ponds in the vicinity. Artificial cultures incidentally exhibit some otherwise unnoticed aspects of the metabolism of Stentor, and are a valuable means of confirming by mass-culture the results of special experiments. Such cultures furnish to observation, and especially to chemical tests, much indication of the continuous change in constitution taking place in a medium and of the phases of these changes to which the different kinds of Infusoria are respectively adjusted. But the practice of making artificial mass-cultures had the disadvantage of consuming much time. Numerous empirical trials had to be made before sufficient insight was obtained into the conditions to which Stentors adjust themselves. This animal was observed to differ from some other Infusoria (e.g., Paramaecium) in having a more circumscribed range of favorable conditions. The salt media devised as the result of successive experiments, and described in the sections treating of adjustment, furnished the most successful starting points for mass-culture.

The original cultures were obtained by collecting a variety of solid material from the edge of ponds in the vicinity of Cambridge. The collection must not be too rich in fermentable matter. If this precaution is not observed, such Stentors as are brought in with it will be killed off by an excess of the initial fermentation characteristic of every culture. Brown leaves and dead reeds were found most useful. Both in artificial and in natural cultures *decaying cellulose proved to be the best source of the food supply for Stentor.* It is of course only the source, not the food itself. An excess of fleshy green matter must be avoided, but some thread algae may be included. No more pond water need be taken than is necessary to cover the collection. It is best to transfer the solid matter to a jar while the latter is in the water. It was customary to set mass-cultures, whether natural or artificial, in cylindrical glass jars of about 4000 cc. capacity. The object in using such large vessels was to obtain cultures whose longevity would be great in proportion to the food supply. To each jar was transferred enough of the collected matter to occupy about one tenth of its volume. The jar was then filled with tap water and kept at room temperature, or if convenient a little warmer. Jars should never be allowed to cool to low temperature, should be covered with a pane of glass to prevent evaporation, and should stand in diffuse daylight. Provided Stentors are originally present, some of these cultures will be successful, and after one or two weeks will show a growth of Stentors localized on the side of the jar away from the light. If too much fermentable material was present, it will take longer for the culture to develop, because Stentors will not appear in abundance until after the initial fermentation has ceased. In some cases a culture develops after weeks of standing. The above procedure, although frequently successful, carried with it no certainty of result and hence pointed to the necessity of a further study of conditions. Cultures were set with an abundant supply of oxygen-producing water plants, in addition to the leaves and reeds above mentioned. The result was signal failure. This fact, coupled with the observation that many successful cultures contained no evident abundance of algae, shows that the experimenter need take no special precautions to supply oxygen to a Stentor culture. The decaying algae frequently noticed in good cultures probably acted as a source of food.

Observations were made to determine directly the immediate food of Stentor. The animals were compressed under a thin cover glass by gradually withdrawing the liquid medium with filter paper. Their movements were thus stopped and they were pressed flat. Finally the contents of the cell escaped. Unidentified unicellular vegetable matter of low organization, Euglenae, and Arcellae were the most frequent kinds of food present. These organisms were abundantly consumed. Occasionally a Stentor was observed in the act of ingesting a Paramaecium. Although Stentor is omniverous, this fact was of not much avail in raising a culture. Very often it was observed that an *aborting* culture with enough Stentors still present to make a successful start contained an abundance of the direct foods upon which the animals could easily subsist. This observation was repeated so often that it was quite evident that some other important condition than a supply of food had not been fulfilled.

Gradually it became clear that *no other single condition is of such determining influence for free-living cells as the salt content of their liquid medium.* The special experiments, subsequently described, made during the period of these culture experiments, furnished direct evidence for the above proposition. Hence attention was turned to the determination of a favorable salt medium for Stentor. The failures and the partial successes that attended this effort will not all be recorded here. They were instructive in showing that the determination of an animal's adjustments to substances brings to light a metabolic status not so well exhibited by any other method.

The most successful salt medium for raising cultures was obtained by the experiments described in the section on Single and Combined Salts (p. 497). Special tests upon the most successful medium are there described. It consisted of the following combination of salts dissolved in water:

$CaCl_2$	.00055 m.
$NaNO_3$	.00015 m.
$MgSO_4$	.00015 m.
K_2HPO_4	.00015 m.

The sum total of its equivalent concentration is 100 molecular parts in 100,000. The most important constituents are $CaCl_2$ and K_2HPO_4, and the proportions, especially high for $CaCl_2$, are important. As subsequent experiments show, Stentor can live a long time in a favorable salt solution, but multiplication soon ceases. To maintain this, a food supply must be added to the salt solution, and this requirement has proved to be a difficulty. For the addition of any food that has been found available utterly changes the salt content both qualitatively and in its proportions. The effect of this upon the animals has been partially overcome by doubling or even trebling the concentration of the above formula so that, if added to distilled water, it would have represented 200 to 300 molecular parts in 100,000. The *excess* of constituents known to be favorable was relied upon to nullify, in part at least, the disturbance of balance due to the addition of food. The same observation applies to the use of tap-water, which was customary for mass cultures.

Having added to about 3500 cc. of tap-water, contained in a 4000 cc. jar, sufficient of the above salts to make the required concentration when the jar is full, the next step was the addition of some dry leaves or dead reeds or both. The smaller the amount the more quickly will the culture develop, but when the supply is too scanty the culture cannot become long-lived. Sometimes it is desirable to heat the leaves and reeds to boiling and then to add the cooled mixture to the salt solution. This causes the solid material to sink immediately, thus facilitating observation at the upper part of the jar, and it also hastens the chemical changes that must precede the rapid growth of Stentor. The final step is to "seed" this culture with a mixture of all sorts of Infusoria and other living material from thriving cultures, including, of course, a large proportion of Stentor. If good cultures are on hand, it is advisable to stir up one of them and transfer about 500 cc. of its liquid to the new culture. If the new culture was successful, it showed the previously described localized growth of Stentors at some time within a week after it had been started. Among the various foods tried was a decoction of barley made by boiling a few grams for about one-half hour. The liquid, containing many particles in suspension, was poured off from the barley grains. When this was very largely diluted with tap-water and seeded, many Infusoria, especially Vorticellae, grew well, but no abundant culture of Stentor could be so raised. The results were somewhat better when the extract of barley was added to the salt medium mentioned above. Tests made independently of the cultures showed that the Stentors did not use as food the particles obtained by boiling the grain. But no other medium equalled this extract in the rapidity of the divisions at first produced. The Stentors became smaller without subsequent growth. Evidently the effect produced was stimulation, not feeding.

The medium probably had a favorable salt constitution, derived from the grain, and it had comparatively little power of fermentation. The classical hay infusion was also tried. While Paramaecia and many other Infusoria grew abundantly in cultures set with hay, Stentor uniformly failed to produce successful cultures. When a hay infusion became very old, so that it was reduced to the condition of a decaying cellulose medium, it frequently produced Stentors. Likewise when the amount of hay put into the above salt solution was very sparing a transient culture might result. A series of chemical determinations was made upon hay infusion and upon other media for comparison. The results showed that hay furnished so large an amount of fermentable matter as to produce too much acidity in a medium intended for Stentor. This fact also explains the observations upon the age of a hay infusion referred to above, and the amount of hay that may permit a growth of Stentors.

Some operations will now be described that are useful for improving poor cultures or for reviving old ones. Some account will also be given of chemical estimations by the volumetric method that were made in order to compare cultures in different stages. The latter observations were found to furnish a good index of the conditions prevailing in the culture liquid. Only the estimation of acidity, the most useful of these determinations, will be described. Operating upon 5 cc. of the culture liquid I determined the amount of 0.01 normal acid (hydrochloric) or alkali (sodic hydrate or calcic hydrate) required to produce reaction with methyl orange and phenolthalein respectively. The results were compared and other estimations also were made. By making a series of observations upon a single culture from the time of setting it, some idea was gained of its progressive chemical changes. Experience showed that the titration with phenolthalein was the most instructive and the only one necessary. This indicator has the advantage for our purpose of being useful in the titration of carbonic acid and organic acids. These come especially into play in fermenting cultures. In these estimations it was important not to deprive the culture of much liquid, as refilling it disturbed the naturally prevailing conditions. Yet repeated tests were required, and so for this reason the small quantity of 5 cc. was invariably adopted for a test. In the titration of carbonic acid with 0.01 normal alkali (usually sodic hydrate) much difficulty was at first experienced owing to the absorption of this gas from the atmosphere. Hence constant results could not be obtained in successive estimations of the same liquid. The following procedure was devised to overcome the difficulty. The liquid (5 cc.) to be titrated was transferred with a pipette from the culture jar to a short thick-walled test-tube about one cm. in diameter. A few cubic centimeters of kerosene were then poured in to cover the liquid and so prevent absorption of CO_2 by the alkali introduced for titration. Phenolthalein was added. A glass rod was also kept in the tube for stirring. The titration was performed over a white tile. If necessary, the burette could be made to deliver its reagent under the layer of kerosene, by means of tubing of rubber and glass. But without this additional precaution consistent results were easily obtained and the acidity of different cultures or of the same culture from day to day could be compared. Of course carbonic acid was not *the only* acid which was estimated in the above process. The end reaction took place when the first trace of normal carbonate or of excess of sodic hydroxide was present.

By means of this process the following observations were made. If separate hay and leaf cultures be set with a salt solution as previously described, the acidity of 5 cc. at the beginning may be zero or perhaps equivalent to 0.1 cc. of 0.01 normal sodic hydroxide. After one day an acidity of several tenths will have been reached by both cultures, but the hay culture has the greater acidity. After two to three days the acidity of 5 cc. of the hay cultures was usually equivalent to approximately 0.7 cc. of 0.01 normal sodic hydroxide. After the third day there was no important increase of acidity unless too much fermentable matter had been added. Observation of the Stentors originally placed in these cultures for seed showed that when the acidity exceeded an equivalency of 1 cc. of 0.01 normal sodic hydroxide they became pale, rested upon the *bottom* of the jar, and diminished in numbers. From the bottom of the hay culture their disintegrating forms could be taken and examined. A further examination of successful and unsuccessful cultures showed that the former were either neutral or had an acidity of a few tenths only, whereas the latter were nearly always characterized by a considerable degree of acidity. The fact that heating any of the culture liquids up to the boiling point greatly diminishes their acidity, often indeed producing a condition that gives an akaline reaction with phenolthalein, points to the presence of CO_2, produced by fermentation. As is well known, organic acids also are developed in this process, but in smaller proportions. Soon after the condition of maximum acidity is reached, a covering of Stentors appears over the surface of the culture. In a successful culture the increase in Stentors, Paramaecia, and some other Protozoa may have already reached a great abundance before this condition is attained. Multiplication of Stentors was never observed in an actively fermenting liquid. Observation both of natural and experimental cultures shows that *Stentor is an animal that has adjusted itself to such conditions as prevail in a late stage of a fermenting liquid.* To raise Stentors in a salt solution, it is necessary to reduce to the minimum the intensity of the fermentation inherent in the food supply. This fermentation process is, however, indispensable, for it is the source, direct or indirect, of all the nutriment of Infusoria. Success sometimes attended the introduction of Stentors into a liquid that had completed its active fermentation.

An excellent method of removing the excess of CO_2 and noxious gases from a culture, whether new or old, is to transmit a current of air through it for some time. By reason of the comparatively high partial pressures of the gases in contact with the air, either at the surface or deeper, the whole liquid loses much of these undesirable constituents. This removal of gases is probably of as much importance to the Infusoria as the accompanying increase in oxygen. Unless the odor and other conditions of a culture indicated the need of such aeration, it was found that a growing culture could be ruined by this process. In all cases an estimation of acidity should be made. For some time the practice was resorted to of precipitating the CO_2 as calcium carbonate by the addition of calcic hydroxide. Neutrality or a slight excess of calcic hydroxide are not unfavorable to Stentor. The former condition cannot, of course, be maintained in a culture liquid. This process is inferior to aeration and was abandoned in favor of the latter.

If a culture is old and declining, it is sometimes easier to revive it than to start a new one. This is probably due to the fact that the liquid of a former thriving culture is a favorable salt medium for Stentor. Aeration together with the addition of some brown leaves or dried reeds frequently restored a culture that had run its course. If the liquid was not already laden with salts, it was often found advantageous to add 100 to 200 molecular parts in 100,000 of the usual salts. Such addition of salts was extensively practised, but to what limit it could be carried successfully was never determined. By the use of any or all of the preceding methods, when the conditions suggested them, the same cultures were maintained for months without emptying the contents of the jar.

IV. ACCELERATION OF DIVISION.

My earlier direct experiments upon the rate of cell-division were suggested by the well-known examples of artificial stimulation produced by the treatment of unfertilized eggs, for a short time only, with chemicals. I refer to the experiments of Tichomiroff ('86), O. und R. Hertwig ('87), Morgan ('99), Loeb ('99), Winkler (:00), and others. Among the substances so used occur the physiological salts, — the class in which we are especially interested, — as well as acids, alkalis, extracts of physiologically active substances, and alkaloids. A possible analogy suggested itself between the artificially stimulated egg, beginning and continuing its cleavages after an initial impulse, and a possible acceleration in the rate of division in Stentor after similar treatment. I set the question, "Can a temporary impulse due to a substance accelerate the *rate* of division in Stentor or Paramaecium?" The first two of the following experiments were made with this question in view and gave an affirmative answer. It seemed reasonable that the same effects might be kept up by the continuous application of the same substance, and such proved to be the case. I therefore describe these experiments, among others made with a different aim, because they are interesting from another point of view. The comparison of the reaction to a temporarily applied substance with one continuously applied will make clearer the mode of action in the former case. The experiments whose main result was the acceleration of division in Stentor and Paramaecium were as follows.

The Specific Action of Potassic Chloride on Stentor.

Expt. No. 6, Oct. 21, 1901.

For the purpose of applying a liquid medium for a limited time, and to enable me both to apply and to withdraw the medium rapidly, I made use of the apparatus I have elsewhere (Peters, :01) described as a U-cell. This was, essentially, two slides, between which there was placed, in U-form, a properly selected piece of yarn. This cell was placed in an inclined position, the open end of the U uppermost, in a vessel containing enough liquid to permit the immersion of the lower part or the whole of the cell when desirable. By this arrangement liquid can be passed through the wall of the cell without the production of excessive downward pressure upon the contained organisms. The cell was filled with 0.05 m. potassic chloride by means of a capillary pipette, and into it were introduced a considerable number of Stentors, all from the same culture. To this solution the organisms were subjected for ten minutes. During the next ten minutes tap-water was passed through the cell to remove the potassic chloride, and the Stentors were then transferred to a glass vessel containing about 18 cc. of their original culture medium free from other Stentors. In this medium they were mounted in hanging drops, as described in this paper under General Methods and Technique (p. 444). The results are shown in the following table. The time stated is reckoned from the beginning of the experiment. The Mean Result expresses the number to which *on the average* one Stentor increased or diminished.

Time	0 da.	1 da.	3 da.
No. of Stentors	157	202	210
No. of Divisions		?	64
Mortality		?	3
Mean Result		1.29	1.39

A control experiment was made with a similar U-cell in which the Stentors were treated merely with tap-water for twenty minutes, a period of time equivalent to that of the test experiment. The animals were then transferred, as before, to a portion of their native culture medium, free from other Stentors, and mounted in hanging drops. These preparations were set away simultaneously with the test preparations and were kept under the same room conditions. The only essential difference of which I am aware between test and control experiments was the temporary treatment of the former with 0.05 m. potassic chloride. The control gave the following results:

Time	0 da.	1 da.	3 da.
No. of Stentors	188	216	233
No. of Divisions			48
Mortality			4
Mean Result		1.15	1.24

Expt. No. 7, Oct. 24, 1901.

Test experiment similar to Experiment No. 6. Culture medium made to contain 0.05 m. potassic chloride applied to Stentors for ten minutes.

Time	0 da.	1 da.	2 da.
No. of Stentors	212	233	231
No. of Divisions			23
Mortality			3
Mean Result		1.10—	1.09

The control made as described under Experiment No. 6, yielded the following results:

Time	0 da.	1 da.	2 da.
No. of Stentors	139	140	142
No. of Divisions			8
Mortality			6
Mean Result		1.01—	1.02—

Expt. No. 9, Oct. 24, 1901,

was similar to Experiments 6 and 7, except that 0.016 m. potassic chloride was used in the test experiment and that the count was taken after sixteen hours. Test and control contained 142 and 170 Stentors respectively and gave average increases of 1.09 and 1.05 respectively.

I now abandoned the method of temporary application of the reagent, for reasons that will be stated in the subsequent discussion. Hence in all the subsequent tables of this section the given concentration of the reagent was *continuously* applied, except when the time is expressly stated. I also desired to compare the reaction to potassic chloride with that to sodic chloride.

Expt. No. 11, Nov. 1, 1901.

The method of hanging drops was used as previously described. I here adopted the plan, later abandoned, of making both test and control experiments in a medium consisting of diluted and more or less fermented hay infusion. Both media when completed contained the same proportion of strong hay infusion, the dilution being made with tap water. They differed in the fact that one portion of test medium contained potassic chloride added in the proportion of 0.01 m., while the other contained sodic chloride added in the proportion of 0.01 m. The results of three sets of simultaneous preparations were as follows:

	0.01 m. KCl.		0.01 m. NaCl.		Control.	
Time	0 hr.	24 hr.	0 hr.	24 hr.	0 hr.	24 hr.
No. of Stentors . .	247	350	175	209	118	157
No. of Divisions .		106		43		46
Mortality		7		1		2
Mean Result . . .		1.42		1.19		1.33

The Division-Reaction of Stentor to Potassic Chloride.

I shall now describe the phenomenon of division in a potassic chloride medium, which occurred abundantly in this and the following experiments. I observed in the hanging drops of the potassic chloride medium numerous Stentors of sizes ranging from that of a normal individual to that of one so very much smaller than normal, that it was difficult to decide whether to count each as a single individual. The count given above under 0.01 m. potassic chloride does not include all the individuals that might have been counted, but only the larger ones. In all the applications of chemicals to Protozoa during the course of the present investigation, I have never observed this potassic chloride reaction to occur in any Protozoan except Stentor, nor with any other substance than potassic chloride; neither does it occur in Stentor under normal conditions. When the test medium for drop-preparations of Stentor was made to contain, besides its normal salts, from .01 m. to 0.0167 m. potassic chloride, and the results were compared with the control preparations lacking the potassic chloride, the effect of the reagent was unmistakable. With the greater of the two proportions above mentioned, the effect began to appear sooner than with the less. The formation of new Stentors was in a portion of the cases apparently normal, as judged by the normal appearance of the process and by the size of the new Stentors; but in other cases one or the other of the following abnormal methods prevailed. In normal division the attachment of the anterior of the two new organisms to the posterior one is at some point of the *periphery* of the frontal area of the latter. In one of the abnormal methods there resulted a dumb-bell shaped figure in which the attachment was at the *middle* of this area. In these cases there was a gradual constriction extending with equal rapidity from all sides. A wrinkled appearance of the pellicula was usually evident at the narrowest part of the constriction. In the other abnormal method the process had exactly the appearance of budding, a globular protrusion being gradually constricted off at a point not far behind the adoral band. Many of the new Stentors originating by either method, including both large and small sizes, were of globular form. In view of the possibility that these might be simply portions of extruded cytoplasm, I observed them further. Upon closer examination I rarely failed to find in them one or more of the beads of the nucleus. An adoral band of cilia was not present at first, but grew while the individual was taking on the typical conical form of a normal Stentor; this was true of the dwarfs as well as of those of normal size. To test the vitality of these small Stentors, individuals were removed from the drops in which they occurred, examined for the presence of a nucleus, and then placed in a separate preparation in a medium of hay infusion. This test showed a considerable degree of mortality in such dwarf Stentors, but also yielded numerous apparently perfect, though dwarf, animals. Owing to lack of a proper food-containing medium their further history was not pursued to determine whether they would grow to full size. I regard their growth to full size as highly probable. The above account has been confined as much as possible to a description of observations, and a discussion of the results will be reserved until the close of the section.

Experience having taught that Stentors are very sensitive and responsive animals, I decided to make an experiment by which to determine whether the numerous Stentors produced in the manner last described were really due to the influence of potassic chloride upon division. The possibility of this suggested itself, and indeed the following experiment was made, before the above-described detailed examination of the process was undertaken.

Expt. No. 13, Nov. 4, 1901.

Preparations of Stentor by the drop method in a 0.0167 m. potassic chloride medium and the control preparations made in hay infusion were strictly parallel in origin of animals used, in time, in dilution of reagent, and in room conditions. They were all handled with great caution to avoid violence, especially in mixing fluids. The following results were obtained:

	0.0167 m. KCl.		Control.	
Time	0 hr.	6 hr.	0 hr.	6 hr.
No. of Stentors	298	331	192	193
No. of Divisions		33		6
Mortality		0		5
Mean Result		1.11		1.00

As in Experiment No. 11, the number recorded in the potassic chloride preparations did not include all that might have been counted, some having been omitted because of their small size. In the control experiment no such difficulty arose, as the offspring were all of nearly normal size.

A comparison of the division-reaction of different Protozoa subjected to the same essential conditions is of interest. I therefore give, briefly, the results of some experiments made with potassic chloride and sodic chloride upon *Paramaecia*. The conditions were parallel with those employed in the experiments on Stentor.

Expt. No. 20, Nov. 20, 1901.

Hanging-drop preparations of Paramaecia in hay infusion were treated for one minute with a 0.167 m. sodic chloride solution. It was previously determined that a fraction of a minute longer than this was sufficiently injurious to stop locomotion.

	0.167 m. NaCl, 1 min.		Control.	
Time	0 hr.	24 hr.	0 hr.	24 hr.
No. of Paramaecia . . .	39	135	55	205
No. of Divisions		96		150
Mortality		0		0
Mean Result		3.46		3.73

On account of their motion the animals were too numerous to be counted alive, and they were therefore killed before counting.

Expt. No. 21, Nov. 21, 1901.

Purpose and method the same as in the preceding experiment. Paramaecia.

	0.167 m. NaCl, 1 min.		Control.	
Time	0 hr.	30 hr.	0 hr.	30 hr.
No. of Paramaecia . . .	37	49	40	64
No. of Divisions		13		24
Mortality		1		0
Mean Result		1.32		1.60

Expt. No. 10, Oct. 30, 1901.

Hanging-drop preparations 0.01 m. potassic chloride continuously applied in hay infusion. Paramaecia.

	0.01 m. KCl.		Control.	
Time	0 da.	1 da.	0 da.	1 da.
No. of Paramaecia	99	103	87	91
No. of Divisions		4		4
Mortality		0		0
Mean Result		1.04		1.04

Expt. No. 19, Nov. 18, 1901.

Hanging-drop preparations of Paramaecia in hay infusion containing 0.0167 m. potassic chloride.

	0.0167 m. KCl.		Control.	
Time	0 hr.	16 hr.	0 hr.	16 hr.
No. of Paramaecia	59	40	47	45
No. of Divisions		0		0
Mortality		19		2
Mean Result		0.67		0.95

Expt. No. 29, Dec. 7, 1901.

Hanging-drop preparations of Paramaecia in 0.02 m., and 0.01 m., potassic chloride medium, also containing 20 per cent of strong, fermented hay infusion. Medium of control experiment consisted of similar 20 per cent hay infusion only.

	0.02 m. KCl.		0.01 m KCl.		Control.	
Time	0 hr.	42 hr.	0 hr.	24 hr.	0 hr.	42 hr.
No. of Paramaecia	24	23	24	75	24	372
No. of Divisions		2		51		348
Mortality		3		0		0
Mean Result		0.96		3.12		15.50

A number of experiments were made to ascertain the effect of *chloroform* upon the rate of division in Paramaecia. The general result of this was to accelerate division when the reagent was present in the proportion of 5 per cent of a saturated aqueous solution. A concentration of 10 per cent had a retarding influence. I shall give but one experiment.

Expt. No. 22, Nov. 22, 1901.

A sufficient volume of a saturated solution of chloroform in tap-water was added to fermented hay infusion (bacterial food) to make the mixture contain 5 per cent of the strong hay infusion. The control consisted of this diluted 5 per cent hay infusion only. All necessary dilutions were made with tap-water.

	5 % Aq. Chloroform.		0.0167 m. KCl.		Control.	
Time	0 hr.	30 hr.	0 hr.	30 hr.	0 hr.	30 hr.
No. of Paramaecia	53	405	59	226	53	310
No. of divisions		352		166		257
Mortality		0		0		0
Mean result		7.64		3.81		5.85

Summary of Observations.

STENTOR.

				Mean Results.	
No. of Expt.	KCl m.	NaCl m.	Chloroform.	Test.	Control.
6	0.05			1.39	1.24
7	0.05			1.09	1.02
9	0.0167			1.09	1.05
11	0.01			1.42	1.33
13	0.0167			1.11	1.00
11		0.0167		1.19	1.33
		PARAMAECIUM.			
20		0.0167		3.46	3.73
21		0.0167		1.32	1.60
10	0.01			1.04	1.04
19	0.0167			0.67	0.95
29	0.02			0.96	15.50
29	0.02			3.12	15.50
22			0.05 Aq.	7.64	5.85
22	0.0167			3.81	5.85

The general results of the experiments described in this section are as follows:

1. Division in Stentor can be accelerated in rate and modified in character by the presence of an excess of potassic chloride in an otherwise normal medium.
2. Division in Paramaecia can be accelerated by the presence of a certain proportion of chloroform in an otherwise normal medium.
3. Stentor and Paramaecia differ markedly in their division-reaction when subjected to the same substances under similar conditions. In other words, the two organisms are *adjusted* to different conditions.
4. Practically the division-effects which are obtainable by the temporary application of a given concentration of a substance may be attained by the continuous application of a more dilute solution of the same substance.

General Discussion.

My subsequent experiments have given me an insight into the significance of those just described which is different from what the preceding description would seem to imply.

The work described in the following sections shows that it is very difficult to determine the degree of activity of particular factors, even when such factors have been purposely reduced to the smallest number that the vital conditions of the animal permit. In the preceding experiments we have an unknown mixture of both salts and food, dead as well as living, with an excess of one salt. In view of the control experiment, differing only in the absence of the excess contained in the test, any difference in result can be ascribed undoubtedly to that excess. But in view of the further fact, developed in subsequent experiments, that the simultaneous presence of several substances modifies their individual activity, it is difficult to see how any satisfactory insight into the meaning of the above processes can be obtained from the use of mixtures of large and unknown complexity. This consideration shows that there is a limit to the value of the method previously employed, and also suggests a better plan of experimentation.

With the above reservations, I attribute to potassic chloride a degree of specific action not very commonly observed among salts, but probably of more frequent occurrence than our present means of observation indicate. The acceleration due to chloroform is not at all different from that common to nearly all poisons used in appropriately low concentrations. In both the above cases there is a reaction-chain having a stimulating substance at one end of the series and the division-reaction at the other. Except by mere speculation, I am unable to interpolate other elements into the chain.

The action of a temporarily applied substance seems to me to depend much on the greater or less permeability of the cells to the substances applied. Either enough of the permeating substance was retained by the organism to keep up the effect, or the modification at first produced by it was strong enough to affect the activities of the organism after the removal of the reagent. To the subject of permeability in general I shall frequently return in later sections.

V. Observations on Single Salts and Water.

I next planned a series of experiments in which the object was to compare the effects of a number of physiological salts upon the division-reaction when applied singly and in the higher ranges of concentration. The number of salts so experimented with was for several reasons small. First, the time required for this and subsequent work was much more than at first had seemed probable. The precautions taken in making the preliminary preparation, much of which is described in the section on General Methods and Technique (p. 444), frequently consumed the major part of the time and labor expended. Secondly, the problems suggested by the results with these substances could not be solved by multiplication of similar experiments with other substances. A new line of experimentation was required, and one suggested itself.

Mass-cultures were carried on simultaneously with the special experiments here described, and in most cases furnished the animals for the latter. From these cultures it became more and more evident that different species of Protozoa present peculiar and characteristic adjustments. The observations that gradually impressed this fact upon me are described, but not in their full detail, in the part devoted to the Conditions of Growth in Stentor and the Management of Cultures. This important fact of *specific adjustment* suggests that cumulative experience with a single organism is the necessary preliminary basis for any reliable interpretation of comparative experiments with differently adjusted Protozoa. Accordingly I confined my subsequent experiments almost solely to *Stentor coeruleus*. I regret that limited time and more pressing questions prevented the extension of these experiments to other Protozoa. However, in compensation for this deficiency I have obtained greater certainty for the conclusions drawn, even upon problems of a general nature, by confining my attention to Stentor.

I selected potassic chloride, sodic chloride, calcic chloride, and magnesic chloride as being four fairly representative physiological salts for animal cells in general, as is shown both by ash-content and by feeding experiments (Forster, '73, and others). Since they represent important chemical classes of substances, it was thought that they might facilitate a possible chemical interpretation of physiological action. Furthermore, chlorides alone were selected in order to reduce the number of factors involved and to provide a common factor that would make comparison more practicable. The selection of the chloride, rather than some other radical, though a matter of judgment, was to a certain extent a matter of hazard. It proved to be a fortunate venture. Tests made at various times between the different experiments hereafter described convinced me that the chloride is better adapted to the end in view than any other inorganic radical of the bases used. The chlorine ion is, upon the whole, less injurious and exhibits fewer abrupt gradations in physiological action from salt to salt among the salts here used. This point is always capable of a satisfactory test by selecting different bases with the same acid radical, and *vice versa*. Another organism than Stentor might of course show a different adjustment. The most difficult preliminary problem in the plan of experimentation was the determination of a common physico-chemical condition for comparison. If solutions of the salts here used be so made as to contain *equal molecular concentrations*, they will still differ in *osmotic concentration* (the latter term used as by Hamburger, :**02**, p.14). Both of these factors are physically active in any given solution, and biological experiment has shown that they are also distinguishable in their physiological action. To facilitate inference from effect to cause, it seemed advisable to eliminate if possible one of the two factors. There is, in general, a closer correspondence between the intensity of physiological action and the osmotic concentration of a solution than there is between physiological effect and the molecular concentration producing it. Hence the desired elimination of a factor was most easily accomplished by experimenting, in the beginning at least, with solutions of *equal* osmotic concentrations. The molecular concentrations of course *varied*, and differences (or equalities) in the effects obtained could be interpreted accordingly, other conditions being alike. Curves (p. 473), plotted from the mean results so obtained, represent directly the variation in effects which was produced by variation in the molecular concentration of each salt.

Five series of experiments were made, each series representing the simultaneous application of the above selected salts. These series are designated both in the tables which follow and in the curves on p. 473 by the Arabic numerals 1 to 5. Excepting the curves for series 5, the Roman numerals I, II, III, IV, signify potassic chloride, sodic chloride, calcic chloride, magnesic chloride, respectively. The effects of different salts at nearly equal osmotic concentrations would be represented by an additional set of curves (not shown) connecting points numbered with the same Arabic numerals (excepting the 5's, which represent different molecular concentrations of calcic chloride). Between the points designated by the same Arabic numerals, e. g., all the 1's, the secondary curves should be so drawn as to connect these points of equal osmotic pressure in the order of their highest mean results. This order illustrates the variation in mean results when osmotic pressures are equal, better than connection in the order of concentration would. However, either method of connection shows the great lack of parallelism between the curves and the axis of abscissas, and this is the point of importance.

To make such solutions, I took seven tenths as much of a molecular weight of calcic chloride or magnesic chloride as of potassic chloride or sodic chloride. The limits of error on the biological side are so wide that I abandoned my original intention of making the solutions used strictly isotonic, as measured by the dissociation coefficient, i.

Closely fitting solid watch-glasses only partially filled with the liquid medium were used. The test media consisted simply of distilled water containing, accurately, the specified concentration of the salts. The control medium, when used, was a filtered culture liquid from the native culture of the Stentors employed. In presenting the results of experiments the origin of the Stentors is always stated by giving the number of the mass-culture. Nearly all of the animals used in the following experiments came from the same or a few continued cultures. The results are therefore all the more comparable. Customarily I placed ten Stentors in each dish, and prepared either five or ten dishes (in all fifty or a hundred Stentors) of each medium to be tested. All the preparations described under the serial number of a single experiment were made simultaneously and carried on under strictly identical surroundings. Further details of the above procedure have been given in the section on General Methods and Technique (p. 444).

The mean result states the number of individuals to which, on the average, one Stentor increased or diminished in the given time. My general experience in the use of solutions has taught that for my purposes the number of molecular parts in 100,000 is an appropriate measure of their concentration from a physiological point of view.

Series 1. Expt. No. 45. April 8, 1902.

(All Stentors originated from culture No. 20318.)

.01000 m. KCl.

Time	0 hr.	29 hr.
No. of Stentors	100	6?
No. of Divisions	0	0
Mortality		94
Mean Result		0.06

.01000 m. NaCl.

Time	0 hr.	29 hr.
No. of Stentors	100	0
No. of Divisions		0
Mortality		100
Mean Result		0

.00700 m. $CaCl_2$.

Time	0 hr.	29 hr.	4 da.	9 da.
No. of Stentors	100	102	103	105
No. of Divisions		2	3	5
Mortality		0	0	0
Mean Result		1.02		

.00700 m. $MgCl_2$.

Time	0 hr.	29 hr.
No. of Stentors	100	0
No. of Divisions		0
Mortality		100
Mean Result		0

Control.

(Culture medium No. 20318.)

Time	0 hr.	29 hr.
No. of Stentors	100	104
No. of Divisions		44
Mortality		0
Mean Result		1.04

Series 2. Expt. No. 46, Apr. 10, 1902.

(All Stentors originated from culture No. 20318.)

.00500 m. KCl.

Time	0 hr.	28 hr.	47 hr.	5 da.
No. of Stentors	100	88	84	61
No. of Divisions		0	0	
Mortality		12	16	
Mean Result			0.84	

.00500 m. NaCl.

Time	0 hr.	27 hr.	47 hr.
No. of Stentors	100	20	0
No. of Divisions		0	0
Mortality		80	100
Mean Result			0

.00350 m. $CaCl_2$.

Time	0 hr.	29 hr.	46 hr.	5 da.
No. of Stentors	100	91	88	91
No. of Divisions		0	0	3
Mortality		9	12	
Mean Result			0.88	

.00350 m. $MgCl_2$.

Time	0 hr.	27 hr.
No. of Stentors	100	0
No. of Divisions		0
Mortality		100
Mean Result		0

Series 3. Expt. No. 50, Apr. 14, 1902.

(All Stentors originated from culture No. 20318.)

.00300 m. KCl.

Time	0 hr.	24 hr.	48 hr.
No. of Stentors	100	90	91
No. of Divisions	10	0	1
Mortality		10	10
Mean Result			0.91

.00300 m. NaCl.

Time	0 hr.	24 hr.	48 hr.
No. of Stentors	100	80	85
No. of Divisions		0	5
Mortality		20	20
Mean Result		0.85	0.85

.00210 m. $CaCl_2$.

Time	0 hr.	24 hr.	48 hr.
No. of Stentors	100	99	101
No. of Divisions		0	2
Mortality		1	1
Mean Result			1.01

.00210 m. $MgCl_2$.

Time	0 hr.	24 hr.
No. of Stentors	100	2
No. of Divisions		0
Mortality		98
Mean Result		0.02

Series 4. Expt. No. 48, Apr. 12, 1902.

(All Stentors originated from culture No. 20318.)

.00100 m. KCl.

Time	0 hr.	43 hr.	3 da.
No. of Stentors	100	105	109
No. of Divisions		5	9
Mortality		0	0
Mean Result		1.05	

.00100 m. NaCl.

Time	0 hr.	43 hr.	3 da.
No. of Stentors	100	106	117
No. of Divisions		6	17
Mortality		0	0
Mean Result		1.06	

.00070 m. $CaCl_2$.

Time	0 hr.	42 hr.	3 da.
No. of Stentors	100	99	105
No. of Divisions		0	6
Mortality		1	1
Mean Result		0.99	

.00070 m. $MgCl_2$.

Time	0 hr.	42 hr.	3 da.
No. of Stentors	100	80	77
No. of Divisions		0	0
Mortality		20	23
Mean Result		0.80	

It is evident from the preceding experiments that for the three salts, potassic chloride, sodic chloride, magnesic chloride, I have reached the *destructive maximum* of concentration. The following experiments were made to determine the same point for calcic chloride:

Series 5. Expt. No. 57, Apr. 17, 1902.

(All Stentors originated from culture No. 20318.)

0.05000 m. $CaCl_2$.

Time	0 hr.	22 hr.
No. of Stentors	100	0
No. of Divisions		0
Mortality		100
Mean Result		0

0.04000 m. $CaCl_2$.

Time	0 hr.	22 hr.
No. of Stentors	100	0
No. of Divisions		0
Mortality		100
Mean Result		0

0.03000 m. $CaCl_2$.

Time	0 hr.	22 hr.
No. of Stentors	100	0
No. of Divisions		0
Mortality		100
Mean Result		0

0.02000 m. $CaCl_2$.

Time	0 hr.	22 hr.	49 hr.
No. of Stentors	100	16	11
No. of Divisions		0	0
Mortality		84	89
Mean Result		0.16	0.11

.01000 m. $CaCl_2$.

Time	0 hr.	22 hr.	49 hr.
No. of Stentors	100	51	49
No. of Divisions		0	0
Mortality		49	51
Mean Result		0.51	0.49

.00700 m. $CaCl_2$.

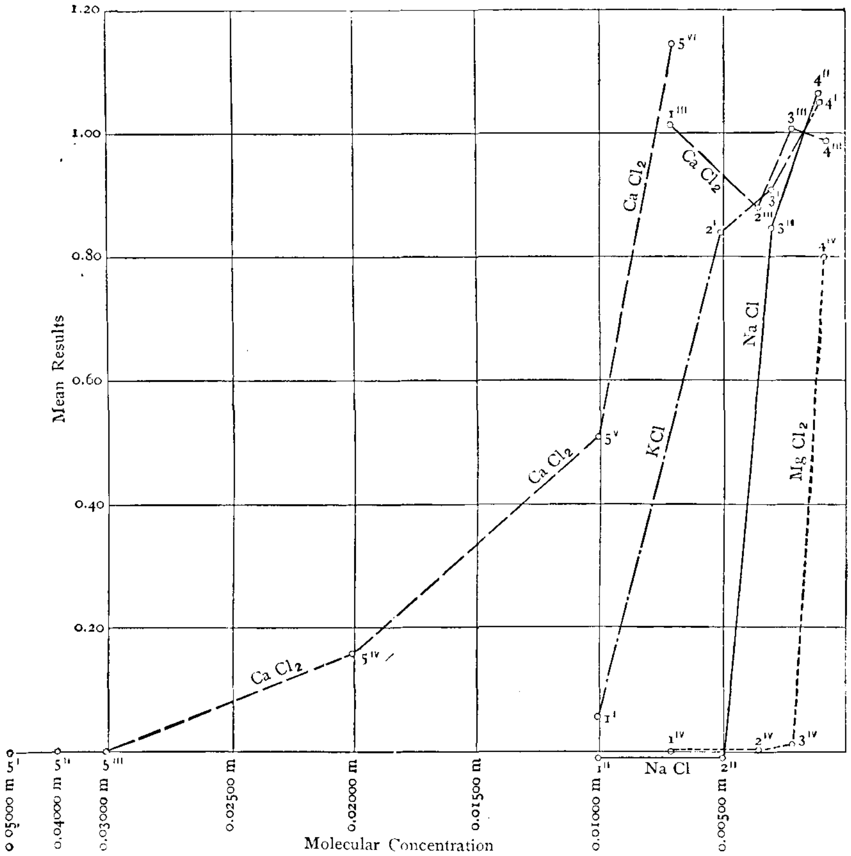
Time	0 hr.	22 hr.	49 hr.
No. of Stentors	100	115	109
No. of Divisions		15	15
Mortality			6
Mean Result		1.15	1.09

Control.

(Culture medium No. 20318.)

Time	0 hr.	26 hr.
No. of Stentors	100	127
No. of Divisions		27
Mortality		0
Mean Result		1.27

The results of the experiments thus far described and plotted in the accompanying curves (p. 473) permit some inference as to the relative magnitude of the *factor of molecular concentration*, and to this subject I shall recur in detail later. To get an independent estimate of the *osmotic factor*, I determined to make some experiments in which this should be the principal factor involved. Some trials were made with cane sugar, but the samples used were probably contaminated and not sufficiently recrystallized. However, the most serious objection to this substance, for my purpose, was its strong tendency to acid fermentation. The presence of acid occurs so soon after the solution is made as to become an important factor in results taken after twenty-four to forty-eight hours or more. After several trials the use of cane sugar was abandoned. Fortunately milk sugar proved to be practically innocuous to Stentor, and it has the advantage over cane sugar that its fermentation is very slow. Tolerably pure samples were purchased and freed from dust and other contaminations by repeated crystallization from hot solution. Parallel with the application of milk sugar I applied to Stentor of the same origin a concentration of 0.00700 m. sodic chloride, sufficient, according to previous experience, to produce a marked effect. I selected sodic chloride for comparison because of its want of any marked specific effect upon division (cf. potassic chloride). The osmotic concentration of a 0.00700 m. sodic chloride solution is about equivalent to that of a 0.01250 m. milk-sugar solution, and this latter is the lowest concentration of milk sugar used in this experiment. For comparison with the salts, the application of a substance (milk sugar) found to have practically no other



For explanation of curves, see pp. 480–492; for discussion, see § 6, pp. 480–492.

influence upon Stentor than that of osmotic pressure is well adapted to show the relative magnitudes of the osmotic and the specific factors.

Expt. No. 55, May 14, 1902.

(All Stentors originate from culture No. 20502.)

0.01250 m. Milk Sugar. Nearly isotonic w'th 0.00750 m. NaCl.

Time	0 hr.	10 hr.	48 hr.
No. of Stentors . . .	50	54	53
No. of Divisions . . .		4	4
Mortality			1
Mean Result			1.06

0.01500 m. Milk Sugar. Nearly isotonic with 0.00900 m. NaCl.

Time	0 hr.	16 hr.	48 hr.
No. of Stentors	50	55	56
No. of Divisions		5	6
Mortality			0
Mean Result			1.12

0.01750 m. Milk Sugar. Nearly isotonic with 0.01050 m. NaCl.

Time	0 hr.	16 hr.	48 hr.	5 da.
No. of Stentors	50	50	54	46
No. of Divisions			4	
Mortality			0	
Mean Result			1.08	

0.02000 m. Milk Sugar. Nearly isotonic with 0.01200 m. NaCl.

Time	0 hr.	16 hr.	48 hr.
No. of Stentors	50	56	59
No. of Divisions		6	9
Mortality		0	0
Mean Result			1.18

0.02500 m. Milk Sugar. Nearly isotonic with 0.01500 m. NaCl.

Time	0 hr.	16 hr.	48 hr.
No. of Stentors	50	51	56
No. of Divisions		1	6
Mortality		0	0
Mean Result			1.12

0.03000 m. Milk Sugar. Nearly isotonic with 0.01800 m. NaCl.

Time	0 hr.	16 hr.	48 hr.	5 da.
No. of Stentors	50	53	56	57
No. of Divisions		3	6	7
Mortality		0	0	0
Mean Result			1.12	

0.00700 m. NaCl. Nearly isotonic with 0.01250 m. Milk Sugar.

Time	0 hr.	3.5 hr.	30 hr.
No. of Stentors	50	5	0
No. of Divisions		0	0
Mortality		45+	50
Mean Result			0

To facilitate comparison this experiment is summarized in the following table:

Milk Sugar.	Isotonicity.	0 hr.	16 hr.	48 hr.	Mean Results.
.01250 m.	.00750 m. NaCl	50	54	53	1.06
.01500 m.	.00900 m. NaCl	50	55	56	1.12
.01750 m.	.01050 m. NaCl	50	50	64	1.08
.02000 m.	.01200 m. NaCl	50	56	59	1.18
.02500 m.	.01500 m. NaCl	50	51	56	1.12
.03000 m.	.01800 m. NaCl	50	53	56	1.12
NaCl.					
.00700 m.	.01250 m. $C_{12}H_{24}O_{12}$	50	5	0	0

These data on milk sugar do not seem to me to be sufficient to yield any satisfactory conception of *how* the salts act upon the organism. I have worked out only the higher range of the concentration curve, whereas from a physiological point of view it is a lower range of concentrations that represents normal conditions and is therefore of especial interest. From the logical standpoint the investigation might have begun with the action of pure water. Perhaps by this method additional factors could have been introduced into the physico-chemical reaction chain sooner than by beginning with higher concentrations. A few tests of the reaction of Stentor to ordinary distilled water and to various samples of supposedly pure water were suggestive of a new and interesting phase of the general problem treated of in this research. It was evident upon preliminary analysis that the physico-chemical phase would have to be worked out carefully before the biological aspect could be approached. As a measure of the purity of water I adopted its electrical conductivity. This is altogether the most delicate and practicable of the known methods for the end in view. For the measurement of conductivity I adopted the Nernst apparatus (Nernst, **'94**, and Maltby, **'95**). For details of the construction and use of the apparatus, and for the preparation of water for these tests, I refer to the original articles and to the part of this paper dealing with General Methods and Technique (p. 444).

I determined that the average conductivity of the ordinary distilled water which I used was about $\kappa = 6.8 \times 10^{-6}$, and I also prepared a quantity of water whose conductivity was $\kappa = 0.9$ to 1.0×10^{-6}. An estimation of the degree of purity these numbers represent and of the probable nature and importance of the contaminations present are stated or referred to in Kohlrausch u. Holborn (**'98**, pp. 111 ff). Suffice it to say here that the purest water I used as a reagent for Stentor

($\kappa = 1 \times 10^{-6}$, or $1- \times 10^{-6}$) was very nearly as pure as one can produce when working in contact with air. This statement assumes of course that the containing vessel in which the experiment is performed is not a source of contamination. Wherever this possibility was a factor of importance I have taken pains to prove the absence of contamination by the objective method of a conductivity determination. In some cases, depending on the end in view, it suffices to know the degree of contamination or its rate, or the fact that the contamination is so small in a given time as to defy detection by even so delicate a method as that here used.

What, then, are the phenomena of the reaction of Stentor to pure water? I will first describe this experiment as it presents itself to simple direct observation. The distilled water used, if freshly prepared, should be aerated by repeated pouring from one vessel into another in a thin stream of some height. I applied both the ordinary distilled water which I was in the habit of using and such as had a conductivity of $\kappa = 1- \times 10^{-6}$. Great care is necessary to avoid the introduction of contamination with the organisms, as it is of course impossible to handle them except as they are contained in liquid. The method of multiple transference, described on pp. 445–446, meets the requirements and permits an objective (conductivity) test of its efficiency in any given case. Quadruple transfers were made in this test. The solubility of the glass used proved upon examination to be a negligible quantity in this experiment. The animals show at first no noticeable signs of discomfort. If undisturbed they gradually come to rest in an expanded condition, as they do in any harmless medium. But after about one and a half to two hours in the water of $\kappa = 7 \times 10^{-6}$, or after about three fourths of an hour to an hour in the water of $\kappa = 1- \times 10^{-6}$, the first evidence of disintegration appears in the form of irregularities of the surface. The margin of the optical section of the Stentor looks roughened instead of smooth, as it normally is. Within a short time afterwards there remains only a mass of disintegrating material. The two samples of water yield the same qualitative phenomena, but the onset of disintegration occurs sooner in the pure water. The length of time that the animals resist pure water depends of course upon their physiological condition and is therefore variable. A sample of water causing disintegration within a few minutes (complete destruction within fifteen minutes, as I once observed) justifies the suspicion of its containing solid or gaseous impurities that are foreign to properly distilled water. Experience with personally prepared and tested waters will amply prove this statement. The treatment of Stentor with more or less pure distilled water was frequently employed in the course of this investigation, and always with essentially the result above described.

While it is possible to make an inference from the experiments with salts and with water, as to the method of destruction by distilled water, much more satisfactory objective evidence is furnished by the next experiment. This consisted essentially in the measurement of the changes in electrical conductivity exhibited by very pure water in which Stentors were contained. It is true that my original object in making this experiment was to obtain a quantitative expression for the respiratory activity of these small organisms. The measurement of CO_2 excretion seemed *a priori* feasible. The result in the sense in which the question was put was negative, but the facts developed are of material importance to a general explanation of the action of water and salts as observed in my experiments. To the subject of respiration I shall return in the section on permeability.

For measuring the conductivity of the medium I employed the Nernst apparatus previously alluded to. Important sources of error that threaten to vitiate the results of so delicate an experiment need to be carefully eliminated. One of these is the change in conductivity due to change of temperature, and another is the introduction of contamination into the medium from the solution of portions of the apparatus itself. As is well known, increase in temperature increases the absolute conductivity, κ, of liquids, and *vice versa*. A favoring circumstance in the present experiment is that the temperature coefficient of pure water is comparatively small, but constant values are not known (Kohlrausch u. Holborn, **'98**, p. 115), and if any temperature change occurs during the experiment its corresponding change in conductivity must be determined for the case in hand. This latter I have done when necessary.

The portion of the apparatus requiring the most scrupulous control of conditions is the test-cell in which the medium with the Stentors is placed. I have found by experience that good chemical glassware, such as is used for beakers, frequently furnishes, especially after usage or preliminary treatment (Kohlrausch u. Holborn, **'98**, p. 113 ff), a glass vessel of such resistance to solution as to give no electrically measurable quantity of solute within the length of time occupied by my experiment. The thimble-sized beaker here used, capable of containing about 5 cc., was of this nature. Because of the high resistance of the medium (pure water) I was able to use *unplatinized* electrodes without experiencing any difficulty from polarization in obtaining a good minimum in the telephone. Aside from this, freedom from polarization is one of the important advantages of Nernst's apparatus. The circumstance of being able to use unplatinized electrodes relieved me of what would otherwise have been an unavoidable source of contamination. The electrodes were fixed at adjustable distances apart upon a frame of hard rubber, which, when set over the thimble beaker, suspended the electrodes in the contained water. Between them, but a little to one side, was suspended a mercurial thermometer with bulb immersed. This arrangement of test-cell is not recommended as the best that could be constructed to serve the required purpose. It is described here principally to give information of the conditions under which the experiment was performed. It results from the construction that glass (of beaker and thermometer), platinum, and air are the only objects in contact with the contained liquid. If necessary the bulb of the thermometer and the inside of the beaker could be coated with paraffine. To test the efficiency of the conditions, I made the following control experiment. The test-cell was set up with a content of 3 cc. of very pure water. Practically simultaneous readings of time, temperature, and the length (L) of the measuring column of resistance-liquid were made and recorded. The test-cell was connected in parallel with the measuring tube; it must be remembered that in this mode of connection the length (L) increases with the conductivity of the liquid in the test-cell and *vice versa*. The numerical value of the conductivity is not important to my argument. With the mirror scale and the magnifier with cross-threads that were in use when this experiment was made it was possible to read one twentieth of a scale-division, one tenth of a scale-division being a change in length of significance, the scale-division being 1.2 mm. The position of the electrode was read directly. The dial attachment (Nernst, '**94**, and Maltby, '**95**) for very much smaller readings was not used in this experiment.

Under the control conditions above described the following record was secured.

Time,	8.30	8.37	8.45	8.55	9.05	9.15
Temp.,	21°.00	20°.85	20°.75	20°.70	20°.70	20°.75
L.,	10.5	10.5	10.5	10.5	10.5	10.5

Time,	9.25	9.30	Variation = +60 minutes.
Temp.,	20°.80	20°.90	Variation = –0°.1 C.
L.,	10.5	10.5	Variation = 0 divisions.

The result shows satisfactorily that the apparatus itself (that is, the test-cell) is not a source of contamination to the contained medium, when in use for a period of one hour.

I now placed in the same test-cell 3 cc. of very pure water containing ten Stentors introduced by the process of multiple transference, and made a series of observations corresponding to the above with the following result :

Time,	4.07	4.15	4.20	4.27	4.31	4.38	4.47
Temp.,	23°.30	23°.30	23°.35	23°.35	23°.30	23°.15	23°.00
L.,	10.6	10.6	10.6	10.6+	10.6+	10.65 (.7?)	10.65

Time,	4.54	5.07	Variations = +60 minutes.
Temp.,	23°.00	23°.00	Variations = −0°.30 C.
L.,	10.7	10.7	Variations = 0.1 divisions.

Since there was a decrease in temperature, a correction of *L* for this would result in a larger value than 0.1 scale-division. Although it is not important to my purpose to establish more than an appreciable value for *L*, a rough trial was made immediately with the same materials, but with a temperature change of 2° C. This was equivalent to a change of 0.5 scale-division. This correction applied would give a value to *L* of about 0.17 scale-division.

I will record here one more similar experiment. The same thimble-beaker with paraffined thermometer bulb and 3 cc. of pure water only, gave the following readings for the control experiment:

Time,	3.02	3.10	3.17	3.25	3.32	Variations = +30 min.
Temp.,	21°.75	21°.65	21°.50	21°.50	21°.50	Variations = −0°.25 C.
L.,	10.5	10.5	10.5	10.5	10.5	Variations = 0 div.

With ten Stentors added I obtained the following results:

Time,	3.35	3.50	3.56	4.05	4.18	4.28
Temp.,	21°.50	21°.50	21°.50	21°.50	21°.50	21°.50
L.,	10.5	10.5	10.5	10.5	10.5	10.6?

Time,	4.35	Variations = +60 minutes.
Temp.,	21°.50	Variations = ±0° C.
L.,	10.6	Variations = +0.1 divisions.

Experiments like the above two were often performed, but not always with the care here exercised. *The results uniformly showed an increase in the conductivity of the water containing the Stentors.* I have however learned to regard the above values of *L* as not being proportioned to the number of Stentors employed.

One further feature of these observations I desire to emphasize here. The increase in value of L above observed did not continue indefinitely, but *was only initial, at least in the above intensity.* To the meaning of this fact I shall recur (p. 505).

Sufficient data have now been secured at different points on the concentration curve to furnish at least the basis for a general interpretation, and I therefore defer the description of further experiments upon combined salts until after the following general discussion.

VI. Discussion of Single Salts and Water.

I desire to interpret the preceding experiments from two initially different points of view. From one standpoint I shall attempt a physico-chemical explanation as far as the facts warrant; from the other I shall seek to explain the same and additional facts as the expression of specific adjustments in the metabolism of Stentor. The two views are certainly complementary to each other. Whether or not they can be merged into one, I prefer to leave to the reader's opinion upon a fundamental question in biology.

The results of the two experiments last described, together with others that are similar, show that when Stentors are transferred from their ordinary mass-culture medium to very pure distilled water, an increase in the conductivity of the water takes place during at least the initial hour or two. What is the source of this variation? The control shows that it cannot be from any part of the test-cell. It might be urged that the observed increased conductivity was due to the unavoidable introduction of a small amount of the culture medium with the Stentors, notwithstanding the multiple transference of the animals; for the concentration in the immediate vicinity of a solid object, whether animate or inanimate, would be much greater than in the surrounding medium, generally, and furthermore the ciliate covering of Stentor would certainly increase considerably the danger of the retention by the animal of more highly concentrated solution immediately around it at each transfer. But that this cannot be the true explanation of increased conductivity is evident from the fact that this increased conductivity does not manifest itself promptly, but only after the lapse of some time. These facts leave us room for no other explanation as to the cause of increased conductivity than the metabolism of the organisms themselves. Reflection upon the change in the conditions presented by the Stentors when transferred shows, furthermore, that this conclusion is entirely reasonable.

There is abundant evidence that the culture media contain salts. The later, artificial cultures that furnished material for the present work were raised in favorable salt media, as described in other sections. That the earlier, natural cultures brought into the laboratory from the ponds also contained salts in considerable quantities is scarcely open to doubt. Decaying organic matter and earth in contact with water are abundant sources of salts in solution. Furthermore, evaporation of a few cubic centimeters of culture medium always left a perceptible residue. Conductivity measurements of the natural media, determined by comparison with solutions of calcic chloride, usually gave a value equivalent to that of a 0.00100 m. to 0.00200 m. calcic chloride solution (see p. 508). From whatever culture the organisms for this experiment came, it is certain that their original medium contained a greater concentration of salts than the very pure distilled water to which they were transferred.

The presence of salts in the medium does not necessarily imply that the cell living in it contains the same salts. The discovery of the relation between the salts in the medium and those contained in the free-living cell itself, being the object of the present investigation, this relation is not to be assumed. For example, in Chaetomorpha and Spirogyra, according to Jansen (cited by Schäfer, **'98**, p. 277), we have illustrations of a great difference between the concentration of the salt within the cells and that in the surrounding media. The two plants have about the same internal osmotic pressure, although the former is a marine, the latter a fresh-water form, Spirogyra thus having a greater and Chaetomorpha a less pressure than their respective media. The permeability of the organism, which probably governs such conditions, will receive some attention for Stentor in a later section. Here will be made only the very probable assumption that the Stentor cell does contain a certain quantity of salts. That Stentor contains physiological salts in certain proportions may be inferred from the results of chemical analysis in the case of all organisms obtainable in sufficient quantities to permit such analysis, and from the results of all experiments on nutrition made with reference to salts (e. g., Forster, **'73**, Herbst, **'92–98**, and others). The experiments upon adjustment given in a later section of this paper also lend support to the proposition that Stentor contains some of all the salts found necessary for a medium in which its metabolic processes may normally take place. What these salts are individually in the case of Stentor is not of importance at this stage of the discussion; but I may point out that the salts within the cell probably originate from the medium, either directly, by osmotic absorption and by ingestion, or indirectly, being contained in other organisms used as food, which organisms originally obtained their salts from the medium. It is assumed here that although some of the salts contained in the cell may possibly be in a state of chemical combination with proteid, yet a considerable proportion of them exist in a condition to produce osmotic phenomena. They are either free and in simple solution (in vacuoles, Bütschli), or at most in such unstable combination, physical or chemical, as to be easily broken down into more numerous portions, thereby becoming osmotically more active.

When, therefore, Stentors are transferred from a normal medium to very pure distilled water, there results the osmotic relation of a salt pressure within the Stentor cell that is higher than that existing in the distilled water externally adjacent to it. What process ensues under these conditions depends, at least in large part, upon the degree of permeability of the protoplasm, including the walls of the protoplasmic alveoli (Bütschli) as well as the cell-walls. If the cell-wall of Stentor had been wholly impermeable to salts, much water would have been absorbed (assuming of course free permeability for water), no increase of salts would have taken place in the distilled water, and the animals would have died of swelling; but neither of these phenomena was exhibited. *On the contrary, the evidence that there is an increased salt-content in the distilled water demonstrates the greater or less permeability of Stentor to physiological salts* when the animal is surrounded by distilled water. But I will note in passing that I do not conceive the permeability of Stentor when in very pure distilled water to be the same as when the cell is surrounded by the normal conditions of partial pressures of salts. I have been led to this opinion by the results with milk sugar, as I shall presently explain. We are now able to assign *loss of salts as the chief factor in the destruction produced by distilled water*, possibly brought about, as implied above, by an abnormal condition of the cell-wall induced by contact with water free from salts. The more permeable the cell-wall, the more rapid would be the loss of salts, and the less, consequently, would be the absorption of water. Careful observation failed to show any evidence of swelling due to an increased water-content, which, however, probably took place to some extent. Evidently the loss of a portion of its salts has much more serious consequences for the animal and its metabolism than some increase in its water-content. I must here emphasize, for the purpose of a subsequent comparison (p. 500), the fact that *all* the permeating salts share in this process of withdrawal.

The phenomenon of shrivelling, observable when molecular or hypermolecular concentrations of either physiological salts, or of practically indifferent substances (sugars, e. g.), are applied to Stentor, permits us to make at least the following inference. Shrivelling is due to loss of water from the cell to the solution outside it, and this movement of water proves that *the Stentor under these osmotic conditions is permeable to water*, a fact which it might perhaps have been permissible to assume rather than demonstrate. But whether loss of those salts which have unbalanced partial pressures was also a factor in the process of destruction by concentrated solutions, is a more difficult question to decide from the evidence. The conditions here are so closely related to those in the milk-sugar experiments that I will consider them in connection with the latter.

We have now considered the conditions that prevail at opposite ends of the concentration curve, and shall next proceed from both regions towards the isotonic or normal region. That portion of the curve which extends from distilled water up to those osmotic pressures which are approximately isotonic with the natural culture media from which the animals for experiment originate, I shall term the *hypisotonic* range. The conditions here require separate description, which I will therefore defer till I have given (in the next section) the experiments on the lower concentrations of single and combined salts.

There will remain, then, the question of the effect and the mode of action of those concentrations which lie in the *hyperisotonic* range (above about 200 molecular parts of calcic chloride in 100,000) of the curve. A comparison of the mean results presented on the one hand by the curves (p. 400) for the four salts, and on the other hand by the milk-sugar experiments, shows that we have to distinguish between two well-marked classes of effects. *The application of milk sugar in hyperisotonic concentration was practically harmless*, whereas like concentrations of the four salts were destructive. Mean results of unity, or something above unity in the former case, and of zero in the latter, show the strong contrast between the two kinds of bodies. The curves (p. 473), taken in connection with the sodic chloride control, give sufficient indication of the general reaction of Stentor to these concentrations to show what would have been the effects obtained from a simultaneous trial of each of the four salts in parallel with each test with milk sugar. Mean results of zero would have been their uniform outcome.

In what way shall we picture to ourselves the processes that bring about these contrasted effects? If Stentors be removed from their native mass-culture liquid to a solution of a single salt of higher concentration, what processes follow?

Of physico-chemical phenomena we have, assuming the permeability of the cell to water, the loss of water from the cell to the solution outside of it, because its osmotic concentration within the cell is in excess of that outside. This process will take place both in the application of the four salts and of milk sugar, but with a very probable difference in degree, depending upon a difference in permeability of the cell to the two kinds of substances. Milk sugar belongs to a class of substances, the sugars in general, found to have very low, if any, power of permeation. Upon the other hand the physiological salts, and especially ionic constituents of them, have been found by Hamburger and others to have more or less power to permeate cells. We now have a better knowledge of the permeability of the mammalian blood-corpuscle than of any other free animal cell. Although the subject of its permeability with reference to particular substances, especially electro-negative radicals (ions) of salts, has been much in dispute, certain results are now established with considerable certainty. These are well summarized in tabular form by Hamburger (:02, p. 260). As the results — thus far much too meagre in proportion to the importance of the subject — upon different cells accumulate, it becomes evident that among different kinds of cells much similarity exists in regard to the permeating power of such classes of substances as sugars, alkali salts, and possibly alkali earths. This makes the conclusions above stated regarding the permeating power of milk sugar and the four salts, as well as the quantitative difference between the two, in the case of Stentor extremely probable. The experimental evidence which I have thus far worked out, as described in the section on Permeability (p. 503), is very limited, but such as it is it coincides with the results found in other cells.

Returning from this necessary digression to the case of Stentor in hyperisotonic media, we would have a higher osmotic concentration of milk sugar and of single salts, respectively, outside the cells than inside them. Granted non-permeability in the former case, and a certain degree of permeability in the latter, more water would be abstracted from the cells by milk sugar then by the salts. For the inward movement of the salts or their ions would tend to establish as high osmotic concentration of these salts within the cell as existed outside of it. This process of equilibration would be absent from the cells in milk sugar. We may now answer with much probability of correctness the following important question. In the total destructive effect produced by the application of hyperisotonic concentrations of these four salts respectively, is the abstraction of water, i. e., the simple osmotic factor, of determining influence? It is not. For while the abstraction of water was greater in milk sugar than in the salts, the mean results show that milk sugar was, in comparison with the salts, practically harmless. It is true that abstraction of water by sufficiently high concentration of sugar, or of inactive salt, if such there be, may be at least a prominent factor in the destruction of the cell. But at present we are describing lower, though still hyperisotonic, concentrations, where there is the possibility that both osmosis and a metabolic activity of the same substance are present. The demonstration of these two modes of action, and especially their separate estimation, is the first important step toward a physico-chemical analysis of the action of physiological salts upon cells. *We have now carried the analysis far enough to be able to exclude abstraction of water as the determining factor in the destruction produced by these salts at the higher ends of the curves.* Where, then, shall we look for the efficient factor?

I now return to the process of equilibration which takes place when the cell is more or less permeable to the salt. With the outward movement of water there is also, because of unequal salt pressure already described, a concurrent inward movement of the *single* salt applied (or of its ions). There results an introduction into the cell of an excess of that salt in proportion to other salts present in normal amount. We have to deal, then, with the physiological effect of an excess of one salt within the cell, a normal proportion of which would not interfere, in the case of physiological salts, with its usual metabolism. *The ruin of the cells in the given hyperisotonic solutions was due to an excessive proportion of a single salt.*

In the next section I shall present experimental evidence that there is an increased division-reaction with Stentor under the influence of several salts together, as compared with that under the action of one, and especially of the adjustment of Stentor to *limited proportions* of different salts.

From the experiments with distilled water it might be suggested here that an objective determination should be made — either by electrical conductivity or by quantitative chemical estimation — to prove that water has increased or that salt has proportionately diminished in the external medium. This was not done because the above described conditions of concentration are made sufficiently certain by the ascertained osmotic concentration of the native mass-culture medium of the Stentors and the purposely prepared higher concentration of the reagents used. Granted a certain degree of permeability, of which the experiments themselves furnish evidence, the above described processes tending to equilibration of pressure follow as a physical necessity. Furthermore there would be required a correction (as explained in the section on Permeability) for change of volume of the Stentors used, which was, from the nature of the case, not required in the experiment with distilled water.

That the destructive action of the excess of a single salt was not osmotic in its nature, is evident from the absence of injury under the influence of milk sugar in a concentration exerting equal or greater osmotic pressure. Mechanical pressure and movement of water, characteristic osmotic processes, resulted in no harm to the cells. Simple osmotic action, that is *redistribution* of water and salts, as it would occur in a physical experiment, is not sufficient to account for the result, even though the curves may be subdivided into osmotic regions. In fact, the experiments upon Stentor, taken as a whole, permit the conclusion that osmosis, though a factor necessarily present, is secondary in importance to the *specific* activity of physiological salts. The modification of membranes included in that class of phenomena denominated by Ostwald "mechanical affinity" (Schäfer, '**98**, p. 275), and the distinctively chemical processes included in metabolism, exert a controlling influence superior to that of simple osmotic distribution of salts and water. An important problem suggests itself here, namely, the determination of the specific activities of the salts with reference both to permeability and to their individual metabolic functions.

It might be supposed that the physiological salts, not being active substances chemically, owe most of their physiological effects to their osmotic action. It is highly probable that the maintenance of a certain mechanical pressure in the medium surrounding a cell is at least *one* of the important functions of physiological salts. But the magnitude of this factor as compared to the chemical, metabolic activity of these salts is strikingly shown by the secondary curves on page 473. As explained on page 466, these are to be so drawn as to connect those concentrations of the different salts which represent approximately equal osmotic pressures. If the salts had produced mean results in proportion to their osmotic pressures these secondary curves would have been parallel to the axis of abscissas. But at equal pressures the mean results have such values as to produce curves, making nearly the largest angle possible with the axis of abscissas. These facts, as represented by the curves, justify the conclusion that *the physiological salts have a distinct chemical, that is, specific activity in addition to their osmotic function.*

We have now attempted a separate estimation of the relative shares, for Stentor, of the osmotic factor and of the qualitative chemical factors, both of which are exerted by the same substance at a given concentration. Because the latter factor depends upon the nature of the substance whose native properties come to act physiologically *when that substance forms an essential, that is, a constitutive proportion of the living organism*, and because I shall need a well-defined term to express this relation, I shall denominate it the factor of *constitutive proportion* or, here, simply the constitutive factor. The idea of proportion will have importance in the next section.

Substances which act by virtue of being in the organism form a general series of some range with respect to the constitutive and osmotic factors. If the substance is actively toxic the osmotic factor becomes practically *nil*, the constitutive factor accounting for nearly the whole result. A modification of the permeability of membranes through alteration of constitution, which may possibly be important in toxic action, I reckon of course with the constitutive factor. At the opposite end of the general series there is a class of substances, here called the physiological salts, which were once considered to be practically inert (ash) constituents of the organism. But investigations beginning with Liebig have shown the necessity of a different interpretation. Their importance as osmotically active substances is now demonstrated. But their constitutive, metabolic activity, although evidently present, is practically unknown, so far as regards its mode of action. To the question, what is the chemical activity in the case of each salt, physiology is at present able to give but very uncertain answer. Suggestions as to what these activities may consist in are to be found in Rywosh (:00).

We are now prepared to consider some peculiar conditions in our experiments, made evident by analysis. When the animals were contained in a medium of distilled water, there existed within each the several partial pressures of their physiological salts. On the outside of the organism respective partial pressures to balance the former were absent. Salts were withdrawn and the animals died in consequence. Upon the other hand, when Stentors were placed in a hyperisotonic medium of pure milk sugar exactly the same unbalanced partial pressures existed within the cell, yet they did not die for the want of salts. In fact the animals suffered no appreciable harm from the milk sugar medium. The same unbalanced partial pressures existed when a single hyperisotonic salt was applied, excepting for the single salt in question. Death resulted *mainly* from a penetrating excess of the salt applied. First, how shall we explain the contrasting effects above referred to between milk sugar and distilled water? Evidently some other condition than simply the unbalanced partial pressures of the salts comes into play. It is well known that in experiments on osmosis a membrane may during the course of the experiment alter its constitution in consequence of having, for example, a greater affinity for the solvent than for the solute. Thus a copper ferrocyanide membrane (Schäfer, '98, Vol. I, p. 275) absorbs water. Some such process as this I conceive to take place with Stentor in distilled water. Absorption of water by the cell-wall or the protoplasmic meshwork might very probably result in a permeability for the contained salts much greater in degree than its normal permeability. Furthermore we do not know what important changes in metabolic processes the presence of more water than normal might entail. The processes of molecular condensation, perhaps indicated by greater or less relative dryness of protoplasm, give indications of being of fundamental importance in the anabolism and katabolism of living organic matter. Whatever the process may be, the presence of an extremely high concentration of water is its efficient condition, and it is this same condition that is absent in the hyperisotonic application of milk sugar and of salts also. If, then, the protoplasmic *membrane* swelled, and so increased its permeability to an abnormal degree in distilled water, such swelling would be absent in hyperisotonic media of milk sugar and physiological salts. The permeability of the cell could be less than normal, especially toward the *outward* movement of its physiologically necessary salts. This condition by no means necessitates the preclusion of the *entrance* of a single salt existing externally under high pressure. Some membranes, for example the shell membrane of the hen's egg and the skin of the frog (Schäfer, '98, Vol. I, p. 280), are known to be permeable to a particular substance in one direction only. The outward movement of salts with unbalanced partial pressures into distilled water *is due to the alteration of the permeability of the membrane by contact with water free from salts.* Milk sugar, on the contrary, does not alter the normal permeability, which presumably does not permit the escape of salts physiologically necessary to the life of the cell. It seems quite probable that abnormal alteration of the important membrane that mediates nearly all interchange between the surrounding liquid and the cell contents, is a frequent, perhaps constant, factor in the action of salts and other agents upon free-living cells. The *physiological condition* of the membrane may embrace such a variety of possibilities that physico-chemical analysis cannot proceed far without the determination of this factor. This is a difficult undertaking, that must be made in each particular case.

The above explanation may be considered only as a possibility, and as showing that there is no necessary inconsistency in the previous explanation of the results with distilled water on the one hand, and on the other, single salts at higher concentration and milk sugar. These considerations further emphasize the importance of permeability in some fundamental physiological problems.

We pass finally to the middle, or approximately isotonic, region of the curves. As these concentrations are approached from either direction it is observable that the values of the mean results tend upward toward unity, or even to a point above that. It is true that the curves given lack a series of values between those afforded by distilled water and by the isotonic region. Data in this hypisotonic range are given in the next following section. Yet it is plain that the addition of even a *single* physiological salt to distilled water produces a rise in the curve that may cause it to reach unity in approximately the isotonic region, as is here actually shown, or possibly even sooner. The corresponding conditions have already been considered to some extent in describing the osmotic relations in normal or native culture media. Upon the previously applied principles of permeability, partial pressures, and the physiological conditions of the cell-wall in the particular experiment, it is not difficult to construct a picture of what probably occurs. It is not necessary to restate these details, but I desire to add the following considerations. The metabolism of the cell will cause the partial pressures of the physiological salts inside the cell to undergo continual and probably unequal fluctuations for the different salts. In other words, anabolic and katabolic changes will result in a continually varying number of salt molecules, and a consequent equilibration of partial pressures, which is, in all probability, unequal for the different individual salts. A slight change in the composition of the external medium would also entail a rearrangement of osmotic pressures. Complete osmotic equilibrium, that is, complete isotonicity, probably never exists at any time during the life of the animal. These continual changes are part of the normal, vital metabolism of the cell. They show the deep significance to the organism of what seems at first to be merely the fulfilment of the necessities of physical law.

The conditions cannot, of course, be entirely normal as long as we apply externally only a single salt. But with even a single salt the approach towards normal conditions becomes evident by a rise in the curve under the following conditions. As the concentration increases from the hypisotonic towards the isotonic region, or as the concentration diminishes from the hyperisotonic towards the isotonic region, the two limbs of the curve show by their rise an approach to normal conditions. In other words, *all concentration curves of a single physiological salt have the characteristic median elevation with depression towards either extreme.*

We are compelled to admit that the physico-chemical phenomena involved in the interaction of cells and media, while yielding some results in the attempt at analysis, are still very obscure. However, if we begin analysis with the larger and more evident complexes of phenomena (in Roux's sense), we can treat them from another standpoint, that of adjustment. But the description of the phenomena can never be complete without an account of the physico-chemical processes, which are, to say the least, one form of the expression of the life of the organism.

The Adjustments of Stentor.

As my experiments in the treatment of Stentor with salts increased in number, certain apparent inconsistencies, as well as certain constant relations, attracted my attention. There is insufficient detail in the data thus far obtained to explain these conditions by physico-chemical methods. This fact led me, not to the abandonment of physico-chemical explanation, but to consider the phenomena from another point of view also. This has to do with the adjustment of the animal to different conditions.

Before drawing conclusions upon adjustment from the preceding data and the curves partially representing them, it is important to estimate their range of applicability. Are the mean results above recorded, fixed values? I do not maintain that in their absolute, numerical value they are even approximately invariable. Occasionally repetitions with solutions of previously used concentrations gave results surprisingly close to the first ones; but in other cases there was much deviation. Further observation showed this *variation in results corresponded to the conditions of the cultures* from which the animals originated. As I was engaged throughout this research in the management of cultures as my source of material, I had favorable opportunities to become familiar with newly established, with flourishing, and with declining cultures. Experiment convinced me of the great importance of knowing the origin of the Stentors used. Results vary in their numerical value with the vitality, i. e. the physiological condition, of the animals. A Stentor culture as it passes through its successive stages offers excellent examples of this variation. In general I selected as the source of my material only healthy cultures. Perhaps the best criterion of this condition was the longevity of the culture. It is to be especially observed that the curves of the four salts experimented with give results obtained with animals mostly from one culture, No. 20318, and that these experiments were made within a short period of time. These Stentors originated from a good culture under conditions as nearly representing a physiological norm as can well be obtained. The particular results are therefore thoroughly comparable among themselves. But comparison within the limits of what is practically a single experiment does not show the extent of their applicability. When a certain relation between the effects of given substances has been satisfactorily established within the limits of an experiment, that relation can be transferred logically to other experiments and will be found to be applicable; but it may be pitched upon a relatively higher or lower scale *according to the metabolic condition of the animals.* In other words, nutritional variations within certain limits do not nullify fundamental adjustments of the organism, as one might be led to suppose by a comparison of numerical values obtained at different times. Protozoa are not more capricious in reaction than other groups, their reputation for this resting upon a failure to investigate conditions. There exists for each species a normal physiological condition which the experimenter must learn to recognize. There also exist within the cell, as proved by characteristic reactions, normal substance-relations that are identifiable in series of experiments even when these are conducted at different pitches of metabolic activity. For the detection of these, absolute numerical values are no guide. But the relation of values to each other, and a common standard of comparison, enable us to compare experiments differing widely in numerical results. The possibility of doing this is an important consideration when experiments necessarily extend over a long period of time. Concrete illustrations of this method will be given in this and the following sections.

We may now put the question, What adjustments, that is, what correlations of conditions, in Stentor are shown by the preceding data? Let us begin with the curves (p. 473). An unmistakable feature is their serial arrangement. This is shown by the fact that the rise in the value of the mean result above the zero point occurs serially in the order of diminishing concentration as follows: calcic chloride, potassic chloride, sodic chloride, magnesic chloride. Practically the same order holds for the opposite ends of the curves (or the curves produced), for the points at which the mean results reach the value of unity, as is graphically represented by their intersections with the axis of abscissas. More or less parallelism is evident in the curves, and no intersection with each other occurs on the paths of the curves between the zero points and the axis of abscissas. Quite other conditions than the above, carrying with them a different significance, are conceivable. The concentration at which a substance *begins* to be favorable for the physiological processes of the animal is shown by the point at which the upward curvature occurs. Since these upward curvatures, and even the paths of the curves as a whole, stand in the serial order above mentioned, *Stentor must be best adjusted to calcic chloride, next to potassic chloride, then to sodic chloride, and least to magnesic chloride.* It is not maintained that in another series of experiments the curvature would occur at the same absolute points of concentration, or that the mean results would reach unity at the same points; but it *is* maintained that, if comparative experiments with corresponding concentrations of two or more of the four salts be made upon Stentors having the same physiological condition, although the results obtained might be numerically different, they would continue to stand in the same *relative* order, the results most favorable to the animal being those for calcic chloride. In other words the curves are valid, as I have previously explained, only *relatively*, not absolutely. Further evidence that calcic chloride favors the life processes of Stentors more than the other salts do can be found in the next following section.

In the case of milk sugar we have a different form of adjustment. The physiological salts being natural constituents of the organisms, adjustment to them may be distinguished from that to milk sugar, which is probably an artificial, though certainly an *indifferent*, substance for Stentor.

The significance of adjustments for the internal organization of the cell will be discussed in the following section.

VII. Observations on Single and Combined Salts.

The experiments of the present section were made with a twofold aim. First, it was desired to complete the physico-chemical description of the action of salts in the hypisotonic range of concentration, the aim being here theoretical; secondly, to determine the *normal adjustments* of Stentor to physiological salts. The purpose was to find such salts and such combined *proportions* of these as seemed most favorable to the life-processes of the animal. From this would result the determination of, first, the *qualitative range of adjustment*, that is, the number and kinds of substances to which the animal reacts normally; secondly, the *quantitative range of concentration* that was normal for the animal in the case of each salt. The two series of facts constitute the ground for the theory of *constitutive proportion* referred to in the preceding section. A complete knowledge of an animal's relation to substances, even if only to a class, for example to physiological salts, would require more experiments than I have been able to make. But I have extended my studies far enough to illustrate this method of attacking some of the problems of the internal chemical conditions of a free-living cell.

Polished watch-glasses, closed by piling them on top of one another, were used to contain the salt media and the Stentors. It was customary to make the test for each substance with fifty animals, ten each in five different watch-glasses. In the following records, unless a statement to the contrary is made, there were ten animals to a watch-glass. All the Stentors for a given experiment came from the same culture and they were therefore subject to like conditions. But differently numbered experiments frequently included animals of different origin. Their comparability has been previously discussed. The number of Stentors surviving after the expiration of stated intervals of time are written in their respective columns.

Expt. No. 44, April 2, 1902.

The difficulty in finding a uniformly successful method of raising Stentors in mass-cultures finally led me to give special attention to the salt-content of the media so used. Many such cultures were carried on parallel to the following experiments and with media whose composition was based upon them. From among the salt solutions in common use for the water-culture of plants, I selected Pfeffer's medium (Pfeffer :00, p. 240), made as follows. Of calcic nitrate 4 grm.; of potassic nitrate, magnesic sulphate, acid phosphate of potassium, each 1 grm.; of potassic chloride 0.5 grm.; all dissolved in 7 litres of distilled water. A preliminary trial showed that much dilution was necessary for use with Stentor. The above medium was diluted to seven times its original volume. It is not necessary for my purpose to determine what may be the chemical condition of the above salts after their interaction in dilute solution. Stentors placed in this medium gave the following results:

ONE SEVENTH PFEFFER'S MEDIUM.

Time	0 hr.	15 hr.	46 hr.	10 da.	15 da.
No. of Stentors .	50	55	96	99	76

The medium contained no food in the ordinary sense of the term and the numerical results are another illustration of the propriety, under this circumstance, of taking results for comparative purposes after an approximately forty-eight-hour interval.

Evidently this is a medium to which Stentors are well adjusted. But the addition of salt-containing food, for example hay or leaf extract, gave a mass-culture that was a failure. Possibly such an addition brought about a disturbance of the salt balance, the existence of which will be established by experiments described further on in this section. Inspection of the radicals represented by the salts of this medium shows a duplication between different salts that will render it unnecessarily difficult to determine their separate shares of activity by the method of excluding one salt after another. A more favorable opportunity to do this would be afforded by a salt medium in which duplication of radicals had been avoided, provided the physiological necessities of the animal should be compatible with it. For several reasons it seemed to me much better to begin with single salts and proceed step by step to the more complex combinations. Hence each experiment of the following series is based upon the more or less tentative conclusions drawn from those which precede it. General and conclusive results can be shown only from the series as a whole.

Expt., Jan. 8, 1903.

The Stentors used in this experiment were lighter in color than hardy Stentors, and were somewhat undersized. Of the chemical conditions in the combination $Na_2HPO_4 + CaCl_2$, it is for our purpose necessary to know only that all the ions, or radicals, used are represented in the solution. $CaHPO_4$ is soluble in dilute solution and still more so in the presence of some other salts, for example NaCl (Dammer, '94, Band II. Theil 2, p. 320).

Media.	0 hr.	1 da.	2 da.	4 da.
.00050 m. Na_2HPO_4	50	50	50	50
.00050 m. $CaCl_2$	60	62	64	64
.00025 m. Na_2HPO_4 and .00025 m. $CaCl_2$	50	52	48	45
.00025 m. Ca2OH	50	30	33	31
.00050 m. Ca2OH	50	0		
.00050 m. HCl	50	0		

This experiment permits the following *tentative* conclusions: (1) Stentor is well adjusted to Na_2HPO_4 and to $CaCl_2$. (2) The low results of the combination $Na_2HPO_4 + CaCl_2$ were due to a sudden loss in one set of ten out of five such sets (50 Stentors). We may therefore infer from the uniformity of the other four sets, that Stentor is also well adjusted to this combination. Subsequent experiments uphold this view. (3) Stentor is not at all adjusted to excess of acid or alkali.

Expt., Jan. 9, 1903.

Stentors good, but slightly pale. Chemical conditions of solution like those of di-sodic phosphate + calcic chloride solution.

Medium.	0 hr.	1 da.	3 da.	4 da.	8 da.
00050 m. $CaCl_2$ and .00050 m. K_2HPO_4 . .	50	57	57	59	59

Thus far the result indicates that Stentor is equally well adjusted to di-potassic phosphate and to di-sodic phosphate, when either is in combination with calcic chloride.

Expt., Jan. 14, 1903.

The Stentors used in this experiment were pale and not well fed.

Media.	0 da.	1 da.
.00100 m. $CaCl_2$	50	47
.00100 m. KCl	50	7
.00100 m. KNO_3	50	1
.00100 m. $MgSO_4$	50	10
.00100 m. K_2HPO_4	50	10

Of this list of physiological salts used *singly* and in *hypisotonic* concentration calcic chloride shows a mean result near unity and stands in marked contrast to the low values for the other salts. This effect is in harmony with the curves of the preceding section, where the same comparative results appear. It suggests the use of calcic chloride as a basis of comparison between different experiments when it occurs as a common factor in each. It gives a measure of the metabolic pitch characterizing the experiment.

Expt., Jan. 16, 1903.

Stentors are pale.

Media.	0 hr.	1 da.	7 da.
.00100 m. $CaCl_2$	50	30	
.00100 m. K_2HPO_4	50	0	
.00050 m. $CaCl_2$ and .00050 m. K_2HPO_4	50	48	34

Using calcic chloride as a standard, we should say that the Stentors here used present a low metabolic pitch. In view of this the higher mean result, and especially the longevity shown by the combination, strongly indicate that *Stentor is better adjusted to a combination of physiological salts than to any single salt of that combination*, provided that the *constitutive proportions* of the salts be observed. This is an important proposition for my subsequent discussion.

Expts., Jan. 17 and 22, 1903.

Media.	0 hr.	1–2 da.
JANUARY 17.		
.00100 m. $CaCl_2$	50	29
.00080 m. $CaCl_2$ and .00020 m. $MgSO_4$	50	32
JANUARY 22.		
.00100 m. $CaCl_2$	50	33
.00075 m. $CaCl_2$ and .00025 m. $MgSO_4$	50	35
.00050 m. $CaCl_2$ and .00025 m. K_2HPO_4 and .00025 m. $MgSO_4$	50	60

The animals at the end of this last experiment were healthier in color than at the end of the following experiment:

.00075 m. $CaCl_2$ and .00025 m. K_2HPO_4	50	54

I have ventured to record together the experiments of January 17 and 22 on the basis of their nearly equal results with calcic chloride.

The combination of the largest number of physiological salts used has given the best result, and there is additional evidence of the *good adjustment of Stentor to calcic chloride and di-potassic phosphate as is shown by the following experiments:*

Expt., Jan. 28, 1903.

Media.	0 hr.	1 da.	3 da.
.00055 m. $CaCl_2$ and .00015 m. $MgSO_4$ and .00015 m. $NaNO_3$ and .00015 m. K_2HPO_4	50	77	85
.00055 m. $CaCl_2$ and .00045 m. K_2HPO_4	50	76	83

This experiment merely adds evidence for the good adjustment to a combination. To test further the proposition regarding the comparative action of salts singly and in combination, I selected the above mixture of four salts for study in the next experiment. The high results obtained indicate that it represents constitutive proportions and is therefore adapted to the proposed test.

Expt., Feb. 2, 1903.

Media.	0 hr.	1 da.
.00055 m. $CaCl_2$ and .00015 m. $MgSO_4$ and .00015 m. $NaNO_3$ and .00015 m. K_2HPO_4	50	56
.00055 m. $CaCl_2$	50	26
.00015 m. $MgSO_4$	50	27
.00015 m. $NaNO_3$	50	4
.00045 m. K_2HPO_4	50	3
.00015 m. $MgSO_4$ and .00015 m. $NaNO_3$	50	37

The results of the experiment show (1) the excellent adjustment of Stentor to the four-salt mixture; (2) the superiority of this combination to any of its constituents used singly, it being double that of the metabolic pitch indicated by calcic chloride.

From the preceding experiments taken together inference of considerable reliability can be drawn as to the relative importance, or share of activity, of each of the constituents. The result of making partial combinations like the mixture of magnesic sulphate with sodic nitrate, as shown above, gives information on this point, and this method is further applied in the following final experiment.

Expt., Feb. 6, 1903.

Media.	0 hr.	3.5 hr.	2 da.
Distilled Water	50	0 (all had disintegrated.)	
.00055 m. $CaCl_2$ and .00015 m. $NaNO_3$ and .00015 m. $MgSO_4$ and .00015 m. K_2HPO_4	50		65 (+16 dead).
.00055 m. $CaCl_2$ and .00015 m. $NaNO_3$	50		13
.00055 m. $CaCl_2$ and .00015 m. $MgSO_4$	50		33
.00055 m. $CaCl_2$ and .00015 m. K_2HPO_4	50		46 (+10 dead).

The results show, as before, the excellent adjustment of Stentor to the four-salt medium. They further show that the adjustment to two-salt media is best with calcic chloride + di-potassic phosphate; but it is also evident that the other constituents aided in producing the still better result of the four-salt medium.

I have used the above four-salt medium successfully for the establishment of mass-cultures. In such cases, owing to the addition of dry leaves or reeds for food, I have doubled or trebled the proportions in one hundred thousand, in order to overbalance the unknown salts introduced by the food material.

This four-salt medium represents at least one set of combinations to which Stentor is well adjusted, but it may not be the only useful combination of this description. However, numerous trials in making departures from the qualitative or quantitative constitution of this medium indicated, by their frequent failure, *that both the qualitative and quantitative range of substances to which Stentor is adjusted, is comparatively limited.* It would be of interest to have corresponding media well adjusted to other Protozoa tried with Stentor for the purpose of comparing the results. The empirical determination of a successful combination for Stentor we may now dismiss, and proceed to the consideration of some general aspects of the employment of single and of combined salts.

VIII. General Discussion of Single and Combined Salts.

The preceding experiments taken as a whole show that a medium consisting of several salts is more favorable to the physiological processes of the animal than one containing only a single salt. The physico-chemical phenomena underlying this fact seem to me to be the following. The experiments being made at hypisotonic concentrations, there were lower partial pressures outside the cell for all its contained salts except, probably, the one present in the medium. For this one salt there was a lower partial pressure within the cell as soon as the concentration applied outside exceeded the normally low concentration within. The low ash-content thus far obtained in all analyses of protoplasm shows that the normal concentration of individual salts within the cell is low. It is probably so low that most of the hypisotonic concentrations used in the preceding experiments exceeded the physiological limit for a single salt. When the contrary condition obtains, we approach, more and more as the concentration lowers, the conditions shown by cells in distilled water. In distilled water, as previously described, loss of salts brought about by constitutional alteration of the cell-wall explains the result. But under the conditions more probably existing in the above experiments we should have, in consequence of differences in partial pressures, two opposite movements of salts. Assuming permeability, the one salt applied would penetrate the cell and the other physiological salts would pass into the external medium. We are unable to determine how much of the effect to assign to each of the two processes, but they both contribute to the disturbance of the normal proportions of physiological salts. *They inhibit the division-reaction in Stentors by altering the constitutive proportion of its salts.* When the number of salts applied externally increases, the tendency to disproportion diminishes in consequence of fewer unbalanced pressures, and the results, as shown by division, are better. This is especially true if the normal relative proportions of the salts applied be preserved, as the four-salt medium probably does approximately. Changes in water-content within the limits here employed are not of essential or even perceptible influence upon metabolism. That under the prevailing conditions this factor is not important for our reckoning appears from a consideration of its ineffective magnitude at the extremes of the concentration curve. Stentors in distilled water did not suffer from excessive inward osmosis of water, nor did Stentors near the opposite extreme in milk sugar of hyperisotonic concentration suffer from excessive abstraction of water. If the movement of water at either extreme did not occur in sufficient intensity to prove harmful physiologically, such effect may of course be excluded from the intermediate ranges.

We may now attempt to describe in summary the physico-chemical processes for all concentrations of physiological salts. The form of the general curve obtained from the results expressing division-reactions shows a range of elevation more or less broad for medium concentrations, with depressions on either side which reach zero at extremely high or extremely low concentrations. At extremely high osmotic concentration of an indifferent substance (milk sugar) the destructive process is abstraction of water. At extremely low concentration (distilled water) the efficient factor in the harmful result is abstraction of physiological salts. For intermediate concentrations the osmotic introduction or abstraction of water, as the case may be, is a factor necessarily present, but of insufficient intensity to produce a disturbing physiological effect. Nor does simple osmotic redistribution of salts, that is, physical alteration of pressures of osmotically active particles, account for the result. At equal osmotic pressures, as curves previously given show, effects differ according to the individual salt. In the intermediate range of the concentration curve alteration of *constitutive proportion* is the factor of primary efficiency. It is worth while to notice that, owing to unbalanced partial pressures, the phenomena in hypisotonic and hyperisotonic concentrations are essentially similar processes. These consist in equilibration of pressures leading to the same result, — the alteration of the normal constitutive proportions of the physiological salts. Taking into account, then, the whole range of the general curve, there may be distinguished three classes of efficient factors. *At one extreme there is* (1) *abstraction of water ; at the other,* (2) *abstraction of all permeating salts, probably in the relative proportion in which they are present in the organism ; and in the intermediate range,* (3) *alteration of constitutive proportion.* Finally we must recognize in *the permeability of the cell-membrane* or the protoplasmic alveoli a fundamentally important, but thus far often undetermined, factor. Without the determination of this factor physico-chemical explanation cannot proceed far.

These considerations lead to the following conception of the functions of the salts normally occurring in cells. The activity of the organism, in so far as it is dependent upon the physiological salts, depends, first of all, upon the qualitative nature of the constituent salts, that is, the function of these salts is primarily *constitutive*. In the activity of the organism they do service by virtue of their native qualities. If the organism had a different normal composition it would no longer exhibit the same reaction (physiological, chemical, physical) by which it now preserves its identity and individuality.

Secondly, the reactions of the organism depend, so far as they depend at all upon the physiological salts, upon the relative *proportions* of these to one another. Their function is quantitative as well as constitutive. A change in these proportions outside the normal limits of variation, would result, if successful, in loss of identity, if unsuccessful, in disease or death.

This theory by no means asserts that the salt which we know as a compound must exist as such in the organism. But the modified qualities, or energies of the substances contribute to and determine the activities of the organism, by virtue of their constitutive and quantitative relations to it.

From one point of view, at least, the organism is an organization of various *substances*, and the word organization means neither more nor less than correlation. If we choose to take the most advanced physico-chemical point of view, we may describe an organism as a specific correlation of energies.

The principles of constitutive and quantitative functions are nearly axiomatic. With less certainty, but yet with a large degree of probability, we can make a third proposition, which is derived from a consideration of the facts of adjustment. What indication do these give of the internal constitution of the cell? *The external medium to which the cell is normally adjusted represents more or less closely the internal qualitative and quantitative constitution of the cell.* This statement does not necessarily apply to the external medium of the whole plant or animal body, but it does apply to the liquid media in which the majority of cells pass their entire existence. Further, the proposition is not intended to exclude the possible presence in the medium of substances to which the cell is approximately indifferent.

The evidence for the above view of adjustment is mostly indirect. Where direct chemical analysis can give information of the internal constitution of any particular protoplasm, the method of determining adjustments would not be resorted to for that end. But where analysis cannot be applied, or where the results of analysis give too little indication of the original active form of combination of the radicals found by that process, the method of adjustments has considerable value. It is not of value because of its accuracy, for it is only a method of approximation, but it affords a wider physiological knowledge of substance-relations than can be obtained by analysis alone.

From the nature of the case we can adduce evidence of only a general and probable character for the view of adjustment above outlined. In our experiments we found Stentor well adjusted to certain calcium compounds and to certain phosphates. Other salts normally found in various kinds of protoplasm, if added, improved the degree of adjustment. It is probable that, had the experiments been longer continued the need of these latter salts, though present in small proportion, would have become more evident. All the salts used in the experiments upon adjustment are found in the list of necessary salts as determined by various feeding experiments, and also in the list of general protoplasmic constituents as determined by chemical analysis. Further general evidence comes from the consideration that the liquid medium is the sole source of income and the sole recipient of the outgo of the metabolic substances of the cell. Under the circumstances a general correspondence between medium and cell constitution could scarcely fail to exist, especially in the case of the physiological salts. That the metabolic activity of the organism is a metamorphosing process is not inconsistent with the above view.

In the case of Stentor an interesting ecological problem suggests itself in this connection. What relation is there between the adjustments which the animal has shown in the laboratory and the conditions in regard to substances which prevail in the natural homes of Stentor in the field? I have no data to present upon this subject, although it would be practicable to obtain such in this vicinity.

I now turn to a brief consideration of the question, What relation does the process of division in Stentor bear to the physiological salts? The substances experimented with fall into two classes. In the one stands solely potassic chloride. Of all the substances tried, it alone seemed to have the power of specifically disturbing the process of division. In what manner this was effected we are unable to say. All the other substances stand in another class, whose activity in division I attribute primarily and principally to their share in metabolism. The manner of destruction of the cell (disintegration, for example) indicates a fundamental metabolic disturbance. Moreover, such divisions as occur seem to be normal. Culture experiments also show that when food is abundant, that is when metabolic conditions are good, division is increased, and *vice versa*, without anything peculiar appearing in the division process itself. The absence of any perceptible modification of the normal process of division, and, under given conditions, a fairly constant deviation of *rate* of division from the normal, are evidence of a primarily metabolic influence of these salts. However, it is true that calcic chloride and sodic chloride, for example, applied under the same conditions affect the rate of division in different degrees. Numerical differences that are fairly constant for a given metabolic condition of the animals result from the application, for example, of calcic chloride and of sodic chloride. From this fact it appears that these differences result from specific qualities of the salts. It is possible, and indeed probable, that these peculiarities extend even to those metabolisms which underlie the process of division.

IX. The Permeability of Stentor.

A few experiments were made upon permeability in Stentor. In one series the change in conductivity occurring in a salt medium containing the animals was measured. In the other series a volumetric determination of the chemical change resulting under similar conditions was made. I have records of eight experiments upon conductivity and of four chemical estimations. As all the experiments of a series showed the same essential results only a few examples will be described.

Expt., Oct. 14, 1902.

The test-cell used for the measurement of conductivity consisted of a beaker, 2 cm. in diameter and 8 cm. high, a thermometer with bulb immersed, and unplatinized electrodes 25 mm. square standing at a distance of 10 mm. apart in the fluid of the beaker. The latter contained in both test and control experiments 5 cc. of a salt medium, the composition of which will be given presently. Stentors were placed in the liquid in the test experiment only. The test-cell was connected in parallel with the measuring tube W″. The first column of the following record shows the number of the observation; the third gives the temperature read to within 0.05° C; the fourth gives the position of the electrode in the measuring tube W'' as read directly against a millimeter scale placed behind it. The fifth column shows the position of the electrode as determined by means of the disk placed upon W''. The latter reading is accurate to within 0.02 mm. The physical relations of the apparatus are such that an increased reading in the position of the electrode is due to increased conductivity through the test-cell, and *vice versa*, when the latter is connected in *parallel* with the measuring tube. The absolute values of the conductivity would not add to the significance of the experiment and are not determined. Without the presence of Stentor in the test-cell the following record was obtained.

Observ.	Time.	Temp.	Scale.	Disk.
Wp 1	9.47	19.60	547.5	.222
Wp 2	9.50	19.60	547.5	.222
Wp 3	10.20	19.60	547.5	.222
Wp 4	10.33	10.60	547.5	.222
Wp 5	10.36	19.60	547.5	.229
Wp 6	10.47	19.60	547.5	.250
			Difference = +	.028 mm.

This test shows that the conditions in the test-cell are nearly constant for a period of one hour.

Corresponding measurements, made under the same conditions, except that the 5 cc. of the medium contained about fifty Stentors, gave the following record.

Observ.	Time.	Temp.	Scale.	Disk.
Wp 1	7.20	21.00	547.0	.125
Wp 2	7.26	21.00	547.0	.159
Wp 3	7.40	21.00	547.2	.222
Wp 4	7.49	21.00	547.3	.298
Wp 5	8.00	21.00	547.3	.375
Wp 6	8.06	21.00	547.3	.417
Wp 7	8.11	21.00	547.35	.437
Wp 8	8.21	21.00	547.4	.468
	1 hr.	0.00		.343 mm.

After the experiment fifty-two live Stentors in apparently good condition were recovered. Only one dead animal was observed.

The results show that in a period of one hour the conductivity of 5 cc. of this salt medium containing about fifty Stentors had increased by an amount far above the limit of error of the measurement.

The medium here used consisted of the following salts dissolved in ordinary distilled water.

Na_2HPO_4	.00030 m.
KNO_3	.00010 m.
Fe_2Cl_6	trace

In this case the concentration would of course be much higher. Determinations, as made in the next experiment, of the concentration of mass-cultures gave a range equivalent to that of a 0.00100 m. to 0.00200 m. calcic chloride solution. Without doubt Stentors taken from any of my mass-cultures and placed in the above test-medium made with distilled water would suffer a change from higher osmotic concentration to lower, that is, this medium presents for them hypisotonic conditions. Within the cell there exist unbalanced partial pressures of such physiological salts or ions as are not represented by the medium. In the latter di-sodic phosphate is *probably* present in a concentration that exceeds the amount of the same salt normally contained within the cell. In consequence of the unbalanced partial pressures on both sides there ensues a double movement of salts or ions, if the membrane is permeable to them. Since the medium is hypisotonic to the cell, there is also an inward movement of water tending to produce an increase of volume in the cells. The total result of all these processes is to increase the conductivity of the medium. *This result signifies an increased concentration of salts.* It does not follow directly that the membrane is permeable to the salts in question. The case is different from that of pure distilled water, where absorption of water from the medium by the cell could not in itself increase the conductivity of the medium. No possible loss from pure water could affect the conductivity of the remainder, whereas any outward movement whatever of salts would result in increased conductivity. But if in the present experiment, conductivity having increased, it could be shown that the cells exhibited no change in volume, then permeability would be demonstrated. Or if from the increase in concentration of the medium there were subtracted a correction, expressing the concentration-equivalent due to loss of water from the medium to the cell, a remainder, if any, would measure the permeability. Such a correction, determined by the haematokrit method, has been applied to chemical estimations in the case of blood corpuscles. I have not attempted this method with Stentor, and to find a concentration-equivalent in terms of conductivity may not be feasible. Hence the above experiment is inconclusive. It is not probable, however, that the inward movement of water alone accounts for all the increase in conductivity. The probabilities are in favor of the permeability of the cell, for it is not likely to possess a membrane permeable to water alone.

Upon the whole, the method by measuring conductivity is too general to yield specific information upon permeability, because conductivity is influenced by too many factors, and is affected by various salts. Moreover, as Hamburger (:02, pp. 255, 259) has pointed out, under certain conditions some exchange of ions between medium and cell could take place without change in the conductivity of the former.

An experiment similar to the above was attempted upon Paramaecia, but these animals disintegrated under the process. Furthermore, two experiments were made upon *Spirostomum ambiguum.* In the first experiment the animals were placed in the same hypisotonic medium of di-sodic phosphate + potassic nitrate as that which was used for Stentor. The result showed a similar increase in conductivity. In the second experiment the test-medium consisted of native culture liquid that had been filtered through a washed filter and had then been well shaken for the purpose of aeration. This was practically an isotonic medium. *In a period of* 1.7 *hours no increase in conductivity was detected.* This result has two important bearings. It indicates the correctness of the view above expressed that increased conductivity in the hypisotonic medium was due to osmotic redistribution of salts and water. It furthermore indicates that the respiratory activity of the animal, though constant, is not of such quantitative or possibly qualitative nature as to increase the conductivity of the medium within a short period of time. The motive for making this experiment was to test the possibility that respiration by the cell as a whole, or excretion by the contractile vacuole, might result in the accumulation in the medium of measurable ionized products. The quantity of these should increase continuously and uniformly, and its determination by the measurement of conductivity would have been a valuable method for the investigation of metabolism in Protozoa. Various determinations upon isotonic media containing Stentors for longer periods than above described were made, but the results were inconclusive.

Expt., Dec. 13, 1902.

The object of this experiment was to determine the permeability of Stentor to chlorine. The method consisted in the estimation of the change in the content of chlorine and in the degree of alkalinity of a medium containing Stentors under isotonic conditions. This method is due to Hamburger, and was used successfully by him for the demonstration of permeability in blood corpuscles.

In the selection of a medium two considerations were important. First, the medium must be favorable to the physiological processes of the animal. Previous experiments indicate that of all one-salt media calcic chloride would best meet the requirement. Secondly, the solution of calcic chloride must be as nearly as possible isotonic with the native culture medium from which the animals originate. How to obtain the osmotic value of culture liquids was a question of some difficulty. From conductivity measurements it was possible to infer that their salt-content was so low as to render most methods uncertain in result. After some consideration the following approximate, but for our purpose sufficiently accurate, method was adopted. It was determined what concentration of calcic chloride had the same conductivity as the culture liquid, and the former was then regarded as the osmotic equivalent of the culture medium. When the culture was started it had the following composition, the salt being dissolved in tap-water.

Na_2HPO_4	.00050 m.
KNO_3	.00005 m.
KCl	.00005 m.
$FeCl_3$ calculated	.000005 m.

To this had been added some hay, some leaves, and some aqueous extract of barley. The effect of these additions is to make the medium consist of a *miscellaneous mixture of physiological salts.* Exactly which of these are present and in what proportions is of course unknown. The method by measuring the depression of the freezing point is especially applicable in such cases, but here this value would have been too small to be reliable. As is well known, the coefficient of dissociation, i, of a salt solution in relation to its equivalent conductivity, Δ, is expressed by the equation $i = 1 + (k - 1)\,\alpha$ and $\alpha = \frac{\Delta_v}{l_K + l_A}$. In these formulae i is the coefficient of dissociation, k is the number of ions into which the salt splits, α is the coefficient of activity of Arrhenius, Δ_v is the equivalent conductivity at the concentration v, l_K and l_A are the velocities respectively of the kation K and anion A. But α may be found with sufficient accuracy for our purpose by taking the ratio of Δ at the given concentration, represented by Δ_v, to Δ at or nearly at infinite dilution, represented by Δ_∞, that is $\alpha = \frac{\Delta_v}{\Delta_\infty}$. For a number of physiological salts the ratio $\frac{\Delta_v}{\Delta_\infty}$, taken from a table of conductivities to the nearest integer for $\Delta_v = .002$ n, is as follows: $CaCl_2 = 110/115$, $Na_2HPO_4 = 100/126$, $NaH_2PO_4 = 84/126$, $KNO_3 = 122/125$, $NaNO_3 = 101/104$, $KCl = 127/130$, $NaCl = 107/110$, $Na_2SO_4 = 105/110$, $K_2SO_4 = 126/133$. This ratio has the approximate value of 1. For univalent salts, the number of ions, κ, into which a molecule of salts dissociates $= 2$; for bivalent salts $= 3$, for the above phosphates $= 4$. Hence $i = 1 + (k - 1)\ \alpha = 2, 3, 4$. In other words, the osmotic concentrations of those salts at .002 n. is equivalent to 2, 3, 4 times that of an undissociated salt. Hence if it be desired to take any one of the physiological salts to represent the osmotic value of a mixture of all of them, one whose value for $i = 3$ is probably the best choice. Partly for this reason and partly for the sake of good physiological adjustment I selected calcic chloride. Its degree of dissociation is near the average of that of all the salts in question, provided none of them is present in excessively large proportion. A concentration of calcic chloride having the same conductivity as that of the culture medium of the Stentors of this experiment may hence be regarded as *approximately* isotonic with the medium. Conductivity determinations were made with the test-cell previously described connected in parallel with the measuring tube W'', upon which the dial was used. The conductivity of the culture liquid from which the Stentors were to be taken was then measured, that is, the position of the electrodes in W'', as indicated by the pointer on the dial, was noted. It was not necessary to know the absolute value. Successive equal quantities of calcic chloride solutions of different concentrations were then introduced into the test-cell and the position of the electrode noted as before. After two calcic chloride solutions had been measured in this way the concentration for the third trial was found by interpolation from the first two. It developed incidentally that under the conditions of the apparatus then prevailing the sensitiveness of the instrument was = .0001 n. calcic chloride solution. *It was found that a* .0022 *n., that is a* .0011 *m. calcic chloride solution had the same conductivity as the culture medium.* Of course the same result could be obtained with fewer measurements and more calculation, but since conductivity measurements are made with certainty and rapidity, this method seemed the better.

Several hundred Stentors were introduced into 200 cc. of .0011 m. calcic chloride solution. The liquid was contained in a narrow graduate and the animals settled upon the bottom. A glass siphon of narrow bore was placed in the graduate for the purpose of withdrawing portions of the liquid. Its inner end was bent upward, so that the process would not disturb the Stentors lying below. Its outer end was closed by a piece of rubber tubing with a clamp placed upon it. Portions were withdrawn for chemical examination according to the following schedule:

Dec. 13.	1.49 P. M. —	Placed Stentor in $CaCl_2$ solution.				
	1.53 P. M. —	Withdrew	about	35 cc.	Portion	No. 1.
	2.18 P. M. —	"	"	35	"	" 2.
	2.50 P. M. —	"	"	35	"	" 3.
Dec. 15.	11.00 A. M. —	"	"	45	"	" 4.

The several portions withdrawn were immediately estimated for chlorine by argentic nitrate with neutral potassic chromate for indicator.

The amount of acid or alkali necessary to produce reaction with methyl orange and phenolthalein was also found. The estimations with argentic nitrate gave the following result:

CHLORINE.

Portion.	Time (approximate).	
1	5 min.	5 cc. required 10.95 cc. of .01 n. $AgNO_3$.
2	30 min.	5 cc. required 10.55 cc. of .01 n. $AgNO_3$.
3	1 hr.	5 cc. required 10.60 cc. of .01 n. $AgNO_3$.
4	2 da.	5 cc. required 11.80 cc. of .01 n. $AgNO_3$.

These determinations were made by means of a 20 cc. burette graduated to .05 cc. The sensitiveness and accuracy of the process exceeded this amount of the argentic nitrate solution. The estimations for acidity and alkalinity were as follows:

Portion.	Time.	
1	5 min.	5 cc. required .15 cc. of .01 n. HCl, methyl orange. 5 cc. required .05 cc. of .01 n. NaOH, phenolthalein.
2	30 min.	5 cc. required .15 cc. of .01 n. HCl, methyl orange. 5 cc. required .05 cc. of .01 n. NaOH, phenolthalein.
3	1 hr.	5 cc. required .15 cc. of .01 n. HCl, methyl orange.
4	2 da.	5 cc. required .18 cc. of .01 n. HCl, methyl orange.

The result shows that within a period of one hour immediately following the transfer of Stentors *the concentration of chlorine in the medium decreases.* If the Stentors did not change their volume, the result would demonstrate the penetration of chlorine into the cell. In that case, conditions being isotonic, some other ion would pass outward into the medium. In experiments on blood corpuscles Hamburger (:02) found that one CO_3 ion could, under nearly isotonic conditions, change position with two chlorine ions, the corpuscles being permeable. The evidence of this was the increased alkalinity of the medium, due to the substitution in it of alkali carbonate for alkali chloride. In the present experiment the tests for alkalinity have fallen out negatively. An insoluble carbonate of calcium may nevertheless have been formed. Perhaps also an uncertain result was to be expected unless the Stentors were previously treated with a stronger carbon dioxide solution than their native culture medium (Hamburger, :02, p. 258). Upon the other hand if the Stentors changed volume, — as they would probably do even in isotonic media (Hamburger, :02, p. 259), — a correction would have to be applied to the concentration of chlorine as found by chemical estimation. If the Stentors swell by absorption of water, the concentration of chlorine in the medium would be increased thereby. It was, however, diminished. If change of volume is to account for this, we must assume a shrinkage in the Stentors. Since no haematokrit determination was made, we have no means of estimating this factor accurately. It remains to be found out by experiment whether this operation is practicable with these animals. But in the approximately isotonic solution of calcic chloride used it is improbable that an important amount of swelling or shrinkage occurred. The various experiments upon Stentor recorded in the course of this research indicate that the animal does not readily react by a change in volume to even considerable variations of osmotic pressure of the physiological salts. This implies a good degree of permeability. The decrease in chlorine found in the present experiment may, therefore, be attributed *mostly* to the inward permeation of that ion, but this is largely a matter of opinion. The experiments upon permeability had not reached a decisive point when this work was closed, but I shall continue work on this subject, and hope to publish results soon.

X. THE EXTRUSION OF COLOR IN STENTOR.

This phenomenon, though obscure, and bearing no known direct relation to any of the preceding work, will nevertheless be described briefly. In the application of reagents to Stentor it was observed that in some media a mass of blue-green matter was extruded by the cell. This occurred instantly upon transference to the new medium, or, in some cases, after the lapse of a minute. The animal swims away leaving the extruded mass or cloud behind. Occasionally the extruded mass was observed to adhere to, and to be trailed along with, the moving animal.

No evident injury resulted to the Stentor either immediately or remotely. The animal later divided in media that had produced this effect. However, those media which were found most favorable to the organism did not produce extrusion, although these media contained a proportion of certain substances which alone would have produced it. The effect seemed to be inhibited by other substances present, which were never observed to produce extrusion. Observation showed that the rejected matter might originate from any part of the surface of the Stentor, its boundary frequently forming a line parallel to the outline of the animal. The growth of this envelope in thickness could easily be observed. The extruded matter varied somewhat in consistency and depth of color. At times it was scarcely visible, and seemed to disappear in the surrounding water. In other cases it was a heavy deep-green mass. It always showed a more or less gelatinous consistency, so that it could be shaken as a whole by jarring the watch-glass containing it. Its nature and mode of production made it a difficult object to examine. I was unable to discover any structure in it, and I think it was a homogeneous mass.

The close analogy of this phenomenon with extrusion of coloring matter from blood corpuscles was evident. It led to the suggestion that there might be here a relation to osmotic conditions similar to that shown by the process of "laking" in blood. Had this been true, the same use as in the latter case might have been made of Stentor cells to arrive at an approximate estimation of the osmotic concentration of their medium. However, the results of tests which entirely disprove this view of the phenomenon are given in the following table. The first column gives the number of the experiment, the second indicates the substance, and the third gives the extremes of concentration in molecular parts at which and between which extrusion was observed, — E signifying that extrusion occurred, 0 that none was observed. A parenthesis encloses the nearest concentration at which non-extrusion was observed. The original record contains observations at many intermediate concentrations, but these are not given in the table.

No.	Substance.		Range molecular parts.
1.	KCl	E	.155–.001 (.0001)
2.	NaCl	E	.1–.001 (.0001)
3.	KNO_3	E	.06–.0006 (.0001 ?)
4.	Na_2SO_4	E	.2–.001 (.0001)
5.	$(NH_4)_2SO_4$	E	.2–.001 (.0001)
6.	Na_2HPO_4	E	

No.	Substance.		Range molecular parts.
7.	NaOH	E	.022–.0001 (.00005)
8.	HCl	E	.001–.0004 (.0001)
9.	Lactose	E	.2–.04 (.02)
10.	Chloroform	E	Sat. aq. sol.–.1 sat.
11.	$CaCl_2$	0	.193–.0001
12.	Ca2OH	0	.0144–.00014
13.	$CaSO_4$	0	Sat. aq. solution
14.	$MgSO_4$	0	.014

The extrusions with sodic sulphate are peculiarly dense (heavy), deep in color, and of strong consistency. That with chloroform has much the same character, and well illustrates the formation of an envelope of this matter around the cell. Most of the salts and the acids give lighter-colored and less consistent extrusions.

The addition of calcic chloride to sodic sulphate solutions raises the concentration of the latter required to produce extrusion, and in sufficient quantity prevents this effect of sodic sulphate altogether.

This result demonstrates clearly enough that this is not an osmotic phenomenon, like the laking of blood corpuscles with some salts. Even in the latter case certain reagents, for example ammonic chloride (Hamburger, :**02**, p. 169), cause extrusion in any concentration. For Stentor the substances tested form a series, at one end of which practically any concentration results in extrusion, at the other no concentration is able to produce this result. The series expresses the animal's adjustment to the different substances in regard to this phenomenon. The extruded substance is of such nature that simple osmotic change does not cause its rejection from the cell. The conditions point to a chemical process. It is well known that different classes of proteids exhibit characteristic solubilities or insolubilities toward different salt solutions. By means of these they are separable from one another. In view of these facts the suggestion is ventured that the extruded coloring matter is a proteid separated from the living protoplasm by some salts, and not at all by others.

XI. GENERAL SUMMARY.

1. In the growth and normal action of free-living cells, the salt content of the liquid medium in which they live is a most important factor.

2. In cultures of Infusoria the different stages in the development of a culture have each their characteristic animals. The fermentation of food material in the early stages of a culture prevents the growth of

Stentor at that time, owing to the acidity of the medium. When the initial fermentation has ceased the acidity diminishes and Stentors multiply.

3. Division in Stentor can be both accelerated and modified in character by the presence of an excess of potassic chloride in an otherwise normal medium. In Paramaecium an acceleration which is normal in character occurs under the influence of a small proportion of chloroform in an otherwise natural medium.

4. The effects on the division of Stentor produced by the chlorides of calcium, potassium, and magnesium are represented by the curves given on p. 473. Each curve taken by itself shows the effect of the factor of concentration. Secondary curves connecting experiments at the same osmotic concentration (see explanation of curves, p. 466) make an angle with the axis of abscissas; that is, they show that the salts have specific effects in addition to their osmotic activities. Milk sugar at concentrations equal to or greater than those used with the above four salts is harmless for Stentor. This fact shows that abstraction of water was not a prominent factor in the action of the four salts. Excessive proportion of a single salt was the destructive factor. Measurement of conductivity shows that distilled water kills Stentors by abstraction of *salts*. At extremely high concentrations sugar, as well as salts, kills cells, partly at least by abstraction of *water*.

In regions of the concentration curve extending between these extremes the effects of different concentrations do not differ sharply, as shown, for example, by the curve for calcic chloride. Hence the theoretical concentration curve for a single salt has a more or less broad elevation in its middle, or isotonic, region and descending limbs towards both hyperisotonic and hypisotonic extremes.

5. The permeability of the cell membrane is altered by distilled water, probably being increased by it. In milk sugar it probably remains normal. The effect of salts upon the permeability of the cell is a highly important factor in their action on free-living cells.

6. The curves on page 473 show different degrees of adjustment of Stentor to the chlorides of calcium, potassium, sodium, and magnesium in the order named.

7. A combination of *several* physiological salts results in a more favorable adjustment than a medium containing only *one*.

8. Stentor is well adjusted to a medium composed of calcic chloride + sodic nitrate + magnesic sulphate + di-potassic phosphate in given proportions.

9. The method employed for determining the adjustments of an organism gives an approximate physiological indication of the internal chemical organization of the cell.

10. The relation of physiological salts to the cell depends upon the "constitutive proportions" of the salts in the cell. The effects of these salts upon division are in general produced through their influence on metabolism, but with specific differences of intensity in each case. Potassic chloride, used alone, is the only one that has been observed to exert a qualitative effect upon division.

11. The cells of Stentor are probably permeable to the chlorine ions.

12. The extrusion of coloring matter by Stentor is not directly due to osmotic action.

13. The precipitation of the coloring matter by neutral salts suggests that it may be a proteid.

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SPECTRA FROM THE WEHNELT INTERRUPTER.

By Harry W. Morse.

With Three Plates.

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By Harry W. Morse.

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Nearly every one who has written on the subject of the electrolytic interrupter of Wehnelt has noticed and described the brilliant light which is produced about the "active" electrode when the interrupter is in action. Wehnelt himself [*] examined this light, and states that he found in its spectrum lines of hydrogen, the D-lines of sodium, and, when the active electrode was made negative, many other bright lines. Voller and Walter [†] made a more complete qualitative study of the spectra, and found that apparently any metal, when used as active electrode in the Wehnelt arrangement, gave its characteristic spectrum. They speak also of the fact that it was necessary to frequently change the electrolyte, as the metal which had been used in preceding experiments as active electrode contaminated succeeding spectra, and showed its own lines together with those of the metal under examination. These same facts have been brought out more fully, though still in a very crude and qualitative way, by Hoppe [‡] and Werner von Bolton [§], both of the latter suggesting the use of the phenomenon for producing colored light and spectra for demonstration. Simon [‖] states that in the form of interrupter which he used, in which the break takes place, not at a metal point, but in the electrolyte at a narrow opening in a dividing diaphragm between two large electrodes, the same light-phenomena are produced, but no further data on the spectra are given. Hale [¶] examined the spectrum of an iron point in the Wehnelt interrupter in connection with other spectra produced by the arc under liquids. Konen,[**] in a paper on the same subject, also speaks of these spectra, but appears to consider the light too feeble to permit of photography of the spectrum.

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[*] W. A., **68**, 233 (1899).

[†] W. A., **68**, 539 (1899).

[‡] Electrotech. Ztsch., **21**, 507 (1900).

[§] Ztsch. f. Electrochem., **9**, 913 (Nov. 1903).

[‖] W. A., **68**, 860 (1890).

[¶] Astrophys. J., **15**, 131 (1902).

[**] Drudes An., **9**, 742 (1902).

The present paper contains the first part of a research on the spectra produced by an arrangement similar to the Wehnelt interrupter, with tables of the wave-length and approximate intensities of lines of the following metals, together with comparison tables and plates of lines produced in the arc and condensed spark: —

Lithium. Carbon point in solution of lithium chloride.
Sodium. Carbon point in solution of sodium chloride.
Potassium. Carbon point in solution of potassium chloride.
Magnesium. Wire in hydrochloric acid.
Calcium. Platinum point in solution of calcium chloride.
Strontium. Platinum point in solution of strontium chloride.
Barium. Platinum point in solution of barium chloride.
Aluminium. Wire in hydrochloric acid.
Silver. Wire in hydrochloric acid.
Zinc. Wire in hydrochloric acid.
Mercury. Platinum point in solution of mercuric nitrate.
Tin. Wire in hydrochloric acid
Lead. Wire in hydrochloric acid.

It is the intention to present in this paper the more general study of the phenomena, and those spectra have therefore been selected from the plates at hand which are simplest and show marked points of interest.

At first sight of the phenomenon one is reminded of the production of spectra by allowing the spark to pass from a platinum point *to* a solution of a metallic salt, and the spectra do show marked similarity to those produced in this way. The researches on the mechanism of the interrupter have shown, however, that the breaking of the circuit is caused by the formation of a layer of vapor about the active point and that the water of the solution is in large measure dissociated by the high temperature reached.* There seemed therefore the possibility that we might possess in an arrangement of this kind another step in our scale of spectra and possibly a temperature midway between that of flame and arc, or arc and spark.

The importance of a more thorough study of spectra produced under water and in gases and vapors under pressure, and the fact that among von Bolton's drawings of spectra of the Wehnelt interrupter there are several which are *banded*, gave interest to a more exact study of the phenomena.

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* Vid. Voller and Walter, l. c.

In order to reach some degree of accuracy in the comparison of wave-lengths the spectra were photographed with a Rowland concave grating of 163 cm. radius, of about 2500 lines to the cm. This grating has a ruled surface of 8×14.5 cm. and gives a very bright spectrum indeed. The dispersion is of course small, the length of the first spectrum from λ 3200 to λ 6000 being only about 5 cm. This grating was kindly loaned us by Professor Langley and has proven indispensable for the work.

The instrumental arrangements were simple. The Wehnelt cell was made of a beaker, the large electrode of platinum, lead, or aluminium, and the point was a wire of the metal under investigation, or where the metal was in solution as a salt, of platinum or carbon. In this latter case the platinum lines or the lines of impurities in the carbons used often appeared, and they were used as standards with which to compare the spectrum sought. The lines of the metal forming the large plate also appeared after a prolonged exposure. The image of the light about the point was focussed on the slit by means of a condensing lens of aperture sufficient to fill the whole of the large grating with light.

The exposures required were long, as the light is at best weak compared with that of the spark or arc or even that of a Geissler tube. For a slit opening of 0.10 mm. the exposure in the first spectrum was from one to two hours. In the third spectrum, where photographs were taken for the more accurate comparison of wave-lengths, the exposure reached six hours, the intensity of the light varying greatly with different metals.

The spectra were photographed on orthochromatic plates, without color screen, and the range of most of the photographs is from λ 3200 to λ 6000. In some cases the photograph extends much farther into the red, the line of lithium at λ 6709 being, for example, clearly visible. The plates were developed with Amidol, to which only a very small amount of sodium sulphite and a little potassium bromide were added. This developer acts slowly, but permits of prolonged development without the production of chemical fog. It is to be highly recommended for work of this kind, where under-exposure is the rule and every possible detail must be obtained from the plate.

During the exposure the light was observed frequently with a direct vision spectroscope, and the various parts of the glowing gaseous envelope about the point were examined in the hope of finding differences in the spectrum at various points. Such differences were not found, the spectrum being apparently the same in all parts of the envelope and remaining remarkably constant throughout the exposure.

As the electrolyte became hot the intensity of the light became less, and for the sake of economy in time the solution was usually changed every ten or fifteen minutes. The points of most of the metals used also required frequent replacing, and this was done by feeding the wire down through a simple clamp as fast as it was eaten away.

The first series of observations showed that the spectrum is exactly the same, whether the metal in question is used as active electrode or is present as a salt in solution ; further, whether the metallic point is anode or cathode, though a great difference in the intensity of the light exists in the two cases. The metallic point as anode gives only a feeble zone of light. As cathode several distinct stages of the condition about the point are to be distinguished, one of these being accompanied by a rapid "zerstäubung" of the metal. This condition is unfavorable for the production of a bright luminescence, but goes over with increase of current density into the more favorable condition.

It was found that the spectrum is exactly the same whether produced by direct or alternating current. With direct current the electrolyte heated faster and the point was more rapidly eaten away. The greater part of the work was therefore carried out with the commercial alternating current, which is of 110 volts and 60 cycles. The current through the cell averaged 2.5 amperes, and it was found best to keep the resistance of the electrolyte at a point which would give about this current. The large plate had a contact surface of about 10 sq. cm. and the wires used were of diameter 0.3 to 1.5 mm.

No direct experiment on the effect of inductance in the circuit was made, but the cell was used alternately without inductance and as interrupter with a large induction coil for the production of the spark spectra, without any change visible in the small direct vision spectroscope.

It was expected that many gaseous lines would appear, and they were looked for, but the only one found under the conditions of the experiment was the red hydrogen line at λ 6562.* This is visible clearly in many cases; in others it is either very faint or entirely invisible.

In the following tables the wave-lengths of the principal lines are given, with their intensities in the spark, arc, and Wehnelt spectra on a scale of 1 to 100; 1 is the intensity of a barely measurable line, and 100 that of very strong lines, like the principal lines of the alkali- and alkaline earth-metals. It is of course clear that such intensity measurements with the eye, on plates where the exposures are by no means comparable, are only rough approximations. The attempt has therefore been made to confine any conclusions drawn from such comparisons to cases so obvious and striking that there could be no possible chance of error introduced by difference in exposure or width of slit.

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* Werner von Bolton used the hydrogen lines of the Wehnelt as standards for comparison, and shows them in his drawings of spectra. Certainly none appear on any of the author's plates.

Careful comparison of the spectra produced by a platinum or carbon point in solutions of various salts of the same metal shows that there are no differences whatever corresponding to different salts. This has been proven for zinc by photographing the spectrum of a platinum point in zinc sulphate, nitrate, chloride, bromide, and iodide, and also that of a zinc point in sulphuric, nitric, and hydrochloric acids. These spectra are all identical within the limits of the method. The same has been shown for aluminium for an equally extended series of salts and acids, and in the cases of other metals for a less number of combinations. The tables give the data for a stated combination in each case, but apply equally well to any other for the same metal or for any salt of the same metal.

The present paper is concerned only with the photographs of the Wehnelt in the first spectrum, where comparison can be made with the full spectrum of spark and arc without disturbance from overlapping. The dispersion is only sufficient to give an accuracy of about 1 Ångstrom unit in the comparison of wave-lengths.

LITHIUM. [Plate 2, Fig. 7.]

Wave-length.	Spark.	Arc.	Wehnelt.	Wave-length.	Spark.	Arc.	Wehnelt.
3233.0	6	10	2	4273.5	5	30	5
3795.0	. .	5	. .	4602.5	100	100	100
3915.0	. .	8	. .	4972.0	40	50	15
3986.0	. .	8	. .	6104.0	10	20	30
4132.5	15	100	10	6708.0	. .	2 rev.	5

Spark. Lithium chloride on carbon electrodes.

Arc. Lithium chloride on carbon electrodes.

Wehnelt. Carbon point in solution of lithium chloride. (The impurities in the carbon are iron, calcium, and aluminium.)

It seems probable that the exposure of the Wehnelt is at least as great in proportion as the others, since the line at λ 6708 comes out clearly and lines of shorter wave-length than λ 3233 are visible.

The resemblance to the spark spectrum is striking, but there are considerable differences in intensities, λ 4972 being much weaker and λ 6104 much stronger. Many of the strong *arc* lines are either absent or greatly reduced in intensity. It is of interest to note that the lines which retain a part of their intensity belong to the principal and first subseries of Kayser and Runge, while those which lose all or a great part of their intensity from arc to Wehnelt belong to the second subseries. The lines λ 4972.0, λ 4273.5, λ 3986.0 are of this latter class.

SODIUM.

Wave-length.	Spark.	Arc.	Wehnelt.	Wave-length.	Spark.	Arc.	Wehnelt.
3303.0	15	40	10	5688.0	15	40	30
4983.0 d	. .	10	. .	5890.0	50	50 rev.	50
5155.0 d	. .	4	. .	5896.0	40	40 rev.	50
5682.5	15	15	10				

Spark. Sodium chloride on carbon electrodes.

Arc. Metallic sodium on carbon electrodes.

Wehnelt. Carbon point in solution of sodium hydroxide. (Impurities same as in lithium.) (Plate 1, Fig. 2.)

A rather close agreement with the spark spectrum is to be seen, with a noticeable difference in the relative intensities of the lines at λ 5682 and λ 5688. The lines which are present in the arc spectrum and do not appear in the Wehnelt spectrum belong in this case also to the second subseries, but it is quite probable that the exposure was insufficient to bring out these relatively weaker lines. λ 5896 is at least as strong as λ 5890. Taking into account the rapid decrease in the sensitiveness of the plates in this region, it must in reality be stronger.

POTASSIUM.

Wave-length.	Spark.	Arc.	Wehnelt.	Wave-length.	Spark.	Arc.	Wehnelt.
3217.5	. .	3	. .	5340.0	2	20	6
3447.0	. .	40	20	5343.5	1	. .	. .
4044.5	30	100	100	5359.5	. .	15	6
4047.0	. .	. .	. .	5783.0	3	20	6
5099.5	. .	4	1	5802.0	3	20	10
5112.5	. .	3	1	5812.5	. .	3	1
5323.5	2	10	3	5832.0	2	10	6

Spark. Potassium chloride on carbon electrodes.
Arc. Metallic potassium on carbon electrodes.
Wehnelt. Carbon point in solution of potassium carbonate. (Impurities in carbons same as in lithium.) (Plate 1, Fig. 3.)

The spectrum resembles the arc more closely than the spark. The difficulty of obtaining the spark spectrum unobstructed by air lines and the lack of a resolution of the doublets make comparisons in the case of little value.

MAGNESIUM.

In the table the following points are of especial interest:

1. The triplet λ 3720, λ 3724, λ 3730. These lines do not appear in the spark or arc spectrum under ordinary circumstances, but are strong in the spectrum of magnesium in the oxyhydrogen flame, and have always been considered lines belonging to a low temperature. Liveing and Dewar* observed that when the arc passed between electrodes of metallic magnesium these lines were visible, provided the atmosphere about the arc was one which could provide oxygen. They come out clearly in air, oxygen, and carbon dioxide, but do not appear in hydrogen, nitrogen, cyanogen, chlorine, or ammonia.

2. The presence of the arc line at λ 4352 and the strong spark line at λ 4481. One or the other of these is usually very faint under ordinary conditions. In the arc between magnesium electrodes both lines are strong, λ 4481 sometimes quite as strong as in the spark.* Hart-

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* Proc. Roy. Soc., **32**, 189 (1881); and ibid, **44**, 241 (1888).

MAGNESIUM. [Plate 2, Fig. 8.]

Wave-l'gth.	Spark.	Arc.	Flame.	Wehnelt.	Wave-l'gth.	Spark.	Arc.	Flame.	Wehnelt.
3330.0	7	8	. .	7	4966.5	. .	. .	. .	4
3332.5	10	10	. .	10	4974.5	. .	. .	. .	b
3337.0	12	15	. .	12	4984.5	. .	. .	. .	b
3341.0	. .	. .	. .	2	4995.0	. .	. .	. .	b
3437.5	15	. .	. .	. .	5006.0	. .	. .	. .	b
3720.0	. .	. .	strong.	10	5020.0	. .	. .	. .	b
3724.0	. .	. .	strong.	15	5028.3	. .	. .	. .	b
3730.0	. .	. .	strong.	20	5036.0	. .	. .	. .	b
3829.5	20	10	strong.	20	5063.0	. .	. .	. .	b
3832.5	30	15	strong.	30	5072.0	. .	. .	. .	b
3838.5	40	20	strong.	40	5079.5	. .	. .	. .	b
3855.0	. .	. .	. .	1	5086.5	. .	. .	. .	b
4352.0	3	12	. .	5	5168.0	. .	20	. .	20
4443.5	. .	. .	. .	b	5172.5	5	30	. .	30
4449.0	. .	. .	. .	b	5183.0	5	50	. .	50
4481.0	30	. .	. .	30	5527.5	4	. .	. .	3
4570.0	. .	1	strong.	. .	5529.0	. .	20	. .	. .
4704.0	4	15	. .	6	5711.0	. .	3	. .	. .
4960.5	. .	. .	. .	10	5880.0	. .	. .	. .	b

Spark. Magnesium terminals.
Arc. Metallic magnesium on carbon electrodes.
Flame. Liveing and Dewar's measurements.
Wehnelt. Magnesium wire in hydrochloric acid.

mann and Eberhard* have shown that in the spectrum of the arc between magnesium terminals under water, spark lines become strong, λ 4481 showing clearly. Hartmann† has also shown that there is reason to doubt that λ 4481 corresponds to a higher temperature than λ 4352. The spectrum of the arc between magnesium terminals in an atmosphere of hydrogen is nearly identical with that produced under water.[*]

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* Astrophys. J., **17**, 229 (1903).

† Ibid., **17**, 270 (1903).

3. The absence of certain strong lines in the Wehnelt spectrum. Among these the arc line λ 5529, the flame line λ 4570, the spark line λ 3437.5. The presence of other lines usually appearing, and supposed to correspond to the same temperature, is also of importance.

4. The presence of the bands usually ascribed to the oxide. In the band with head at λ 5006 seven flutings are visible. These are also visible in the arc spectrum, but are much fainter. This band was described by Liveing and Dewar[†] and resolved into fine lines by Crew and Basquin.[‡] The first-named authors obtained the band only in oxygen, air, or carbon dioxide, and the evidence seems to show conclusively that it is due to the oxide. It seems clear from Liveing and Dewar's work that the presence of this band does not necessarily indicate a lower temperature than that corresponding to the other lines of the spectrum of magnesium. This band also appears when the spark is allowed to pass from a platinum point to a solution of a magnesium salt.[§]

5. The *appearance* of certain lines. The line at λ 4481 is perfectly sharp, as it is when inductance is introduced into the circuit of the spark in air.[‖] The triplet λ 3829.5, λ 3832.5, λ 3838.5 is very intense, and the lines are broadened and diffuse in contrast to their appearance in air. Wilsing[¶] finds that when the arc passes between magnesium terminals under water, this triplet is displaced, and broadens out into diffuse absorption bands.

CALCIUM.

The spectrum is very like that of calcium in the spark, with many differences of intensity, and absence of some fairly strong lines. The spectrum from λ 5500 to λ 5900 is of interest. There is evident diffuseness in the lines in this region, in marked contrast to the sharp lines of the spark and arc, but whether this is due to a real difference in the spectra or a possible lack of adjustment of the apparatus a more extended investigation must decide. So far as the plates obtained show, there are bands with heads corresponding to sharp lines in the arc and spark spectra.

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[*] Astrophys. J., **17**, 229 (1903).

[†] L. c.

[‡] Astrophys. J., **2**, 101 (1895).

[§] Vid. Lecoq de Boisbaudran, Spectres Lumineux.

[‖] Vid. Huggins, Astrophys. J., **17** 145 (1903).

[¶] Astrophys. J., **10**, 113 (1899).

CALCIUM.

Wave-length.	Spark.	Arc.	Wehnelt.	Wave-length.	Spark.	Arc.	Wehnelt.
3170.5	20	. .	. .	4878.5	5	8	. .
3275.0	. .	8	. .	5041.5	3	4	. .
3631.0	5	30	2	5189.0	3	4	. .
3644.5	10	20	2	5262.5	3	4	3
3706.5	50	10	20	5265.7	2	6	2
3737.5	50	25	30	5270.5	5	10	. .
3933.5	100	100	60	5349.5	3	5	. .
3968.5	60	70	40	5472.0	. .	2	b ?
4227.0	40	100	30	5509.0	2	3	b ?
4283.0	6	10	5	5537.0	. .	. .	b ?
4289.5	4	10	5	5582.0	3	6	b ?
4299.0	3	5	3	5589.0	10	10	15
4302.5	5	8	8	5594.0	4	6	b ?
4308.0	3	8	3	5598.5	4	4	b ?
4319.0	7	10	3	5601.5	3	3	b ?
4425.5	7	15	3	5603.0	3	2	b ?
4435.5	10	20	8	5857.5	. .	5	4
4455.0	15	30	15				

Spark. Calcium chloride on carbon electrodes.
Arc. Calcium chloride on carbon electrodes.
Wehnelt. Platinum point in calcium chloride solution.

STRONTIUM.

The spectrum is in general similar to that of the spark, the order of intensities agreeing well in the majority of instances. A few of the stronger spark lines, with intensities sufficient to make them visible if they were present in normal intensity, are not to be seen. Among these the group λ 5225–λ 5257. The "oxide" bands are bright.

The same question arises here as in the case of calcium. In the

STRONTIUM.

Wave-length.	Spark.	Arc.	Wehnelt.	Wave-length.	Spark.	Arc.	Wehnelt.
3381.0	30	8	5	4869.0	4	. .	. .
3465.0	40	10	20	4874.0 d	5	5	5
3475.0	10	8	3	4893.0	2	. .	. .
3706.0	. .	10	3	4962.0	7	8	3
4030.0	. .	8	. .	4968.0	2	4	. .
4078.0	100	50	50	5221.5	2	. .	. .
4162.0	15	5	6	5224.0	2	. .	. .
4215.5	100	40	40	5238.5	2	5	. .
4303.0	. .	2	. .	5256.5	3	5	. .
4306.0	40	4	20	5456.5	3	. .	. .
4319.0	. .	8	. .	5480.5	8	. .	. .
4438.0	. .	30	. .	5485.0	1	. .	b ?
4531.5	. .	5	. .	5504.0	6	. .	b ?
4607.5	25	50	30	5520.5	4	. .	. .
4722.5	5	10	5	5534.0	20	. .	b ?
4742.0	5	10	3	5540.0	3	. .	. .
4784.0	3	8	2	5587.0	. .	. .	b ?
4812.0	7	10	6	5850.0	4	. .	. .
4832.0	6	10	3	5890.0	. .	. .	b ?
4855.0	2	. .	. .	5895.0	. .	. .	b ?

Spark. Strontium chloride on carbon electrodes.
Arc. Strontium chloride on carbon electrodes.
Wehnelt. Platinum point in solution of strontium chloride. (Plate 1, Fig. 5.)

Wehnelt spectrum of strontium there are diffuse bands in the yellow and yellow-green with maxima which seem to correspond closely with sharp lines in the spark spectrum. Whether or not greater dispersion will show these to be really bands with heads in the same places as are occupied by sharp lines in the other spectra the plates at hand give insufficient evidence. A more exact study of these bands and the similar ones in the calcium spectrum offers great interest.

BARIUM.

Wave-length.	Spark.	Arc.	Wehnelt.	Wave-length.	Spark.	Arc.	Wehnelt.
3377.0	. .	8	. .	4674.0	2	5	2
3420.5	. .	10	6	4691.0	1	5	. .
3501.0	5	15	. .	4727.0	1	10	. .
3525.0	. .	6	. .	4900.0	30	. .	30
3545.0	. .	6	. .	4903.0	. .	10	. .
3892.5	50	8	50	4934.0	80	30	100
3910.0	. .	8	2	5160.0	. .	10	. .
3993.5	5	15	3	5424.0	. .	40	5
4131.0	40	40	60	5519.0	5	30	6
4166.0	15	10	15	5534.0	20	80	100
4283.5	. .	20	3	5620.0	. .	10	. .
4351.0	. .	20	2	5680.0	. .	8	. .
4403.0	3	10	3	5778.0	. .	50	5
4432.0	3	8	5	5800.5	. .	10	. .
4506.0	2	4	4	5826.0	. .	10	2
4525.0	30	30	30	5854.0	8	10	8
4554.0	100	100	100	6142.0	. .	2	5
4580.0	. .	5	4				

Spark. Barium chloride on carbon electrodes.
Arc. Barium chloride on carbon electrodes.
Wehnelt. Platinum point in solution of barium chloride. (Plate 1, Fig. 4.)

The above comparison table for barium does not include the bands in the green which are strong in the spectrum of the Wehnelt. The following table gives the approximate position of these bands.

BANDS IN THE WEHNELT SPECTRUM OF BARIUM.

Wave-lengths.
4570
to
4575.
4598
to
4605.
5134.0 centre of band.
5167.0 centre of band.
5242.0 head of fluting (violet edge).

Wave-lengths.
5272.0 (red edge of same fluting).
5325.0 head of fluting (red edge).
5446.0
to
5462.0
5477.0 centre of band.
5497.0
to
5519.0 (line. Intensity 6).

The spectrum produced about a platinum point in a solution of barium chloride is in many points similar to the spark spectrum. Many arc lines, absent from, or extremely weak in the spark spectrum, make their appearance. λ 3910.0, λ 4283.0, λ 4351.0, λ 4580.0, λ 5425.0, λ 5778.0, are examples. At the same time many strong arc lines are absent in the Wehnelt spectrum, which has the appearance of a rather incomplete composite of arc and spark spectrum so far as the *line* spectra are concerned. The brightness of the "oxide" bands in the green is remarkable. Altogether the spectrum of the Wehnelt is much like that produced by sparking from a platinum point to a solution of a barium salt. (Vid. Lecoq, l. c.)

ALUMINIUM.

The spectrum of an aluminium point as active electrode in an acid or an alkali or a salt solution, and that from a platinum point in a solution of an aluminium salt, seem to be exactly the same as that obtained by allowing a spark discharge without condenser to pass in air between aluminium terminals. The author has obtained the same banded spectrum in tubes containing oxygen at a pressure of a few millimeters of mercury, and with the same distinctness in tubes containing hydrogen at low pressures, by allowing the spark, condensed or uncondensed, to pass between aluminium terminals. In the case of tubes filled with hydrogen the bands persist for only a comparatively short time, and the spectrum of hydrogen remains. The same spectrum, with the exception of some lines not belonging to the bands, is obtained from aluminium in the arc.

Hasselberg * has measured a great number of lines in these flutings,

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* Kon. Svensk. Akad., 24.

ALUMINIUM. [Plate 2, Fig. 9.]

Wave-length.	Spark.	Arc.	Wehnelt.
3587.0	50	20	25
3602.0	30	4	4
3612.5	15	. .	2
3702.5	2	. .	. .
3714.0	8	. .	2
3944.0	50	60	50
3961.5	100	100	100
4034.3			heads of flutings in Wehnelt spectrum.
4083.7			
4243.2			
4262.5			
4271.8			
4274.0			
4276.6			
4279.3			
4282.7			
4306.8			
4332.0			
4341.7			
4354.3			
4371.7		heads of flutings in arc spectrum.	
4375.6			
4384.6			
4395.6			
4414.8			
4432.5			
4448.0			
4464.0			
4472.0			
4480.0			
4495.3			
4512.2			
4517.7			
4530.0			
4538.7			
4558.9			
4577.6			
4595.0			
4608.4			
4625.9			
4642.3			
4648.5		2d band. Heads.	2d band. Heads.
4662.8			
4671.9			
4695.2			
4717.1			
4736.9			
4755.1			
4782.6			
4803.3			
4817.8			
4842.0		3d band. Heads.	3d band. Heads.
4861.0			
4866.6			
4889.6			
4951.6			
4960.1			
4980.1			
4994.0			
4999.5			
5011.0			
5017.8			
5080.6		4th band. Heads.	4th band. Heads.
5093.3			
5102.7			
5111.4			
5118.1			
5124.0			
5142.6			
5148.0			
5161.0			
5176.0			
5190.0			
5262.0			
5207.6			
5222.4		bands.	bands.
5328.5			
5337.0			
5465.0			
5593.0		5	5 lines.
5697.0		3	3
6234.0		. .	3
6244.0		. .	5

and his measurements have been used for comparison, the agreement being as close as could be expected from the small dispersion and poor definition of the plates measured.

These bands have been for many years attributed to the oxide, but there exists a considerable amount of evidence contradictory to this view. Arons [*] found this same banded spectrum in the arc between aluminium points in an atmosphere of nitrogen or hydrogen, and concluded that the banded spectrum corresponds to the metal and not to the oxide. Hemsalech [†] agrees with Arons, and he has shown that by introducing inductance the line spectrum of the spark between aluminium terminals in nitrogen changes into this same banded spectrum. Berndt [‡] concludes from his experiments that the presence of oxygen is necessary for the production of the bands. Lockyer also attributes the bands to the oxide, Wüllner to the metal, Kayser to the oxide, etc.[§] Simple experiments of the author in hydrogen in closed tubes have shown that the band spectrum, which is present in considerable strength when the discharge is first sent through the tube, decreases rapidly and disappears in a short time, to appear again after the tube has been allowed to recover for a time. The simplest apparent explanation is that the oxide coating of the aluminium terminals reacts with the hydrogen under the influence of the spark with the formation of water vapor, probably until equilibrium between metallic aluminium, aluminium oxide, hydrogen and water vapor is reached. On standing the oxide is re-formed and the equilibrium at the lower temperature re-established.

The tendency already spoken of under barium, toward a composite of arc and spark spectra, is very evident in the case of aluminium. Not only are the bands and lines of the arc spectrum present, but also several lines which do not belong to it.[||] Most of the lines of the spark spectrum are present in the Wehnelt, some of them with changed intensities. The general appearance is as though a rather weak spark spectrum had been superimposed over a stronger arc spectrum.

It is the intention of the author to return to this and other band spectra of the Wehnelt in a later paper. The intensity of the light from an aluminium point in hydrochloric acid is sufficient to permit of a photograph with a grating of higher dispersion without an excessively long exposure. The spectrum being free from the overlapping carbon bands of the arc, the long band in the violet, which was not examined by Hasselberg, should be very easy of access.

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[*] Drude's Ann., **1**, 700 (1900).

[†] Ibid., **2**, 331 (1900).

[‡] Ibid **4**, 788 (1901).

[§] Vid. references, Hemsalech, l. c.

[||] Vid. Kayser and Runge, W. A., **48**, 126.

SILVER.

Wave-length.	Spark.	Arc.	Wehnelt.	Wave-length.	Spark.	Arc.	Wehnelt.
3281.0	60	50	30	3982.0	. .	3	. .
3330.0	. .	. .	2	4055.5	1	15	5
3358.9	. .	. .	2	4212.0	1	10	10
3383.0	60	50	50	4228.5	. .	. .	1
3546.5	. .	2	1	4312.0	1	. .	1
3583.0	. .	2	2	4476.0	1	12	8
3639.5	. .	. .	1	4668.5	. .	15	10
3682.0	1	15	1	4875.0	1	. .	1
3781.0	. .	. .	1	5209.0	100	50	40
3811.0	. .	. .	1	5402.0	20	. .	. .
3892.0	4	. .	. .	5465.0	. .	50	100
3907.5	. .	1	1	5471.5	. .	20	. .
3943.0	1	1	. .	5709.0	10	. .	. .
3963.0	. .	. .	2				

Spark. Silver electrodes.
Arc. Metallic silver on carbon electrodes.
Wehnelt. Silver wire in hydrochloric acid.

With silver as active electrode striking differences of intensity from those obtained in the spark and arc are the rule. Only a few lines come out strongly enough to be reproduced, and the spectrum is in general more like that of the arc than that of the spark. This is shown by the lines at λ 4055.5, λ 4212.0, λ 4476.0, λ 4668.0, λ 5465.0. A number of lines which are strong in the arc are not visible, as, e. g., λ 5471.0, and others are much weaker, like λ 3682.0.

Traces of what appears to be an underlying band spectrum are visible, but the maxima are of such small intensity that a very long exposure would be required to bring out the structure clearly. Such maxima as could be distinctly seen have been placed in the Wehnelt column of the table of wave-lengths.

ZINC. [Plate 3, Fig. 10.]

Wave-length.	Spark.	Arc.	Wehnelt.	Wave-length.	Spark.	Arc.	Wehnelt.
3282.5	20	20	10	4680.5	30	40	40
3302.5	30	30	15	4722.5	40	50	50
3345.5	40	50	20	4811.5	50	60	60
3683.5	. .	50	8	4911.5	30	. .	8
4058.0	. .	25	5	4924.0	40	. .	5
4630.0	. .	10	3				

Spark. Zinc electrodes.
Arc. Metallic zinc on carbon electrodes.
Wehnelt. Zinc wire in hydrochloric acid.

Beside the above lines there are present in the spectrum of the Wehnelt the lines of an underlying band spectrum with heads at approximately λ 3848.0, λ 4238.0, λ 4257.0, λ 4299.0, λ 4325.0. The finer detail of these bands, from photographs in the third spectrum, will be given in a later paper. The heads of the flutings of the strongest and longest band have wave-lengths as follows:

4163.2
4170.1
4176.5
4182.4
4187.6
4193.7
4199.6
4203.9
4208.0
4212.9
4216.1
4220.2
4226.7
4232.2
4237.8
4242.4
4247.8
4252.5
4256.9 *Head.*

The spectrum of a zinc point used as active electrode offers many interesting differences from the spark and arc spectra of the metal.

1. The triplets λ 3282.5, λ 3302.5, λ 3345.5, and λ 4680.0, λ 4722.0, λ 4811.0, are present with about the same relative intensities as in spark and arc. The lines λ 3683.0, λ 4059.0, λ 4630.0, which are strong in the arc and absent from the spark, are present in the Wehnelt spectrum with lower intensities, but about the same ratio of intensity as in the arc. The lines λ 4911.0 and λ 4924.0, strong *spark* lines which are absent

from the arc spectrum, are present in the Wehnelt, but with less intensity, and are not decreased proportionately.

2. These two lines, λ 4911.0 and λ 4924.0, belong to a class which will be referred to again under tin and lead, broad and diffuse, and distinctly different from the other lines of the spark in appearance. The position of these lines in Kayser and Runge's series * has not been determined, and the question as to whether they belong to the same class as the tin lines noticed by Crew † is still an open one.

3. The marked underlying band spectrum. This is so faint as to be difficult of reproduction, though a large part of it may be measured, especially the brighter band from λ 4163.0 to λ 4257.0. Here 19 maxima are visible, but it is in many cases difficult to decide whether lines or the heads of flutings are being measured. This band spectrum seems to be characteristic of this method of producing a spectrum, and the author knows of no other notices or measurements of it.

MERCURY. [Plate 3, Fig. 11.]

Wave-length.	Spark.	Arc	Wehnelt.	Wave-length.	Spark.	Arc.	Wehnelt.
3278.0	1	. .	3	4348.0	. .	7	3
3342.0	7	20	. .	4359.0	30	40	40
3381.0	. .	. .	10	4916.5	. .	5	2
3543.5	1	6	. .	5217.0	2	. .	. .
3650.5	50	15	50	5426.0	10	. .	. .
3655.0	5	10	8	5461.0	50	100	50
3663.0	7	15	10	5595.0	3	. .	. .
3681.0	. .	8	. .	5678.0	20	. .	. .
3984.0	40	5	4	5769.0	20	50	20
4047.0	30	20	40	5790.0	20	50	20
4078.0	2	15	6				

Spark. Amalgamated copper terminals.
Arc. Metallic mercury on carbon electrodes.
Wehnelt. Platinum point in solution of mercuric nitrate.

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* Vid. W. A., **43**, 395 (1894).

† Astrophys. J., **12**, 167 (1900).

The Wehnelt seems to lie about half-way between the arc and the spark in relation of intensity of lines. λ 3278.0 is plainly visible, though of lowest order of intensity in the spark and invisible in the arc spectrum. The lines at λ 3650.0, λ 3655.0, and λ 3663.0 have about the same intensities as in the spark. λ 3984.0 is very feeble, of intensity 4 in the table, as against 40 for the next line, λ 4046.0, and 6 for λ 4078.0. The arc line at λ 4916.5 is plainly visible, while the line at λ 3681.0, also belonging to the arc spectrum, is not to be seen. The spark lines at λ 5426.0 and λ 5678.0 are not present. There is a strong line at λ 3381.0 in the spectrum of the Wehnelt which has not been placed satisfactorily to the credit of any impurity. No traces of a banded spectrum are to be seen.

TIN.

The points of especial interest in the table are: —

1. Marked differences in intensities in spark, arc, and Wehnelt spectra. λ 3283.5 is much weaker in the last than in the spark, and λ 3330.0, λ 3655.5, λ 3801.0, λ 4525.5, λ 5632.0 are much stronger. The spectrum of the Wehnelt is in many parts more like that of the arc.

2. The presence of an underlying band spectrum, containing many flutings, and not present either in the spark or the arc spectrum of the metal. The wave-lengths of these maxima have been placed in the table without certainty as to whether the maximum in question is a sharp line or the head of a fluting. So far as the author knows this spectrum of tin has not been observed under any other circumstances.

3. The appearance of certain lines. Of striking interest in this connection is the pair of lines at λ 5562.0 and λ 5588.0. These lines are strong in the spark, absent from the arc, and of small intensity in the Wehnelt spectrum as compared with the spark. They are also the most striking lines in the spark spectrum on account of their marked breadth and diffuseness. The lines at λ 3283.5 and λ 3352.0 have the same peculiarities in a much less degree, and they also follow the same course in their varying intensities in the three spectra.

A similar change in the latter pair of lines has been observed in the arc between tin electrodes in an atmosphere of nitrogen, these lines being much weaker than in the spark in air.* In ammonia gas and hydrogen they are greatly *enhanced.* The possibility of the formation of metal-hydrogen compounds in the Wehnelt and of a similar influence

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* Vid. Porter, Astrophys. J., **15**, 274 (1902).

TIN. [Plate 3, Fig. 12.]

Wave-length.	Spark.	Arc.	Wehnelt.		Wave-length.	Spark.	Arc.	Wehnelt.	
3175.0	2	3	2		3821.5	. .	. .	1	heads ?
3249.0	. .	. .	4		3837.5	. .	. .	2	
3262.5	15	10	20		4046.5	. .	. .	1	
3273.5	. .	. .	3		4307.5	. .	. .	1	
3283.5	20	. .	3		4382.5	. .	. .	2	
3330.5	3	10	30		4512.0	. .	. .	1	
3352.0	30	. .	8		4525.5	8	40	100	
3392.0	. .	. .	1	heads of flutings ?	4586.0	10	. .	1	
3416.5	. .	. .	1		4617.3	. .	. .	2	heads ?
3446.5	. .	. .	1		4651.5	. .	. .	1	
3472.5	. .	. .	1		4722.0	. .	. .	1	
3487.3	. .	. .	4		4810.0	. .	. .	1	
3571.5	. .	. .	1		4858.0	10	. .	5	
3589.0	. .	. .	4		4923.0	2	. .	. .	
3619.0	. .	. .	1		5100.5	8	. .	2	
3639.0	. .	. .	8		5147.5	. .	. .	1	
3655.5	1	8	10		5223.0	4	. .	. .	
3693.2	. .	. .	1	heads ?	5243.5	. .	. .	1	
3723.4	. .	. .	2		5290.0	2	. .	. .	
3734.9	. .	. .	2		5332.0	50	. .	7	
3746.0	1	. .	1		5562.0	100	. .	20	
3801.0	15	30	80		5588.0	100	. .	15	
					5632.0	. .	10	10	
					5798.0	50	. .	10	

Spark. Tin electrodes.
Arc. Metallic tin on carbon electrodes.
Wehnelt. Tin wire in hydrochloric acid.

of the atmosphere of mixed hydrogen and oxygen is suggested, though other cases (see aluminium and magnesium) seem to indicate the preponderence of the action of the oxygen freed by the dissociation of the water vapor.

Some evidence in the matter is given by the research of Crew * on the arc spectra of metals in hydrogen. The results which seem to have bearing in this case are: —

"I. All lines in the arc spectrum which are affected by hydrogen, whether enhanced or diminished, belong to the spark spectrum also.

"II. Lines which belong to Kayser and Runge's series are unaffected by the change from air to hydrogen."

These conclusions cannot be applied as a whole to the spectra of the Wehnelt, but something very similar to the second of them seems to be true for zinc, tin, and lead. (Vid. lead.)

LEAD. [Plate 1, Fig. 6.]

Wave-length.	Spark.	Arc.	Wehnelt.	Wave-length.	Spark.	Arc.	Wehnelt.
3573.0	20	20	25	4245.0	100	. .	6
3640.0	20	25	40	4387.0	100	. .	5
3671.5	10	20	8	4478.0	. .	10	. .
3683.5	50	20	30	5005.5	6	. .	3
3726.0	. .	20	. .	5045.0	8	. .	. .
3740.0	30	. .	25	5201.5	. .	5	. .
3854.0	15	. .	. .	5372.5	10	. .	1
4019.5	10	20	8	5546.0	10	. .	1
4058.0	100	40	50	5588.0	. .	3	. .
4062.5	10	8	6	5607.0	10	. .	. .
4168.0	10	12	5	6002.0	. .	4	. .
4242.0	10	. .	. .	6020.0	. .	. .	10

Spark. Lead electrodes.
Arc. Metallic lead on carbon electrodes.
Wehnelt. Lead wire in hydrochloric acid.

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* Ibid., **12**, 167 (1900).

The absence of several of the strong lines of the spark spectrum is of interest. (λ 3854.0, λ 4242.0, etc.) The strong pair of spark lines at λ 4245.0 and λ 4387.0, which is entirely absent from the arc spectrum, is present in the Wehnelt, but with greatly diminished intensity. This pair is one like those already referred to under zinc and tin, differing from the other lines of the spark spectrum in appearance by being broader and more diffuse.

The spectrum of a carbon point in various acids has also been photographed. (See Plate 1, Fig. 1.) None of the plates show any lines or bands which are not directly traceable to impurities in the carbons used. About 90 lines of iron, 12 of calcium, and 2 of aluminium were measured in one case. These lines were superimposed on a continuous spectrum of considerable intensity, but no bands were observed.

Preliminary tests on salts containing complex ions have given results of interest. A carbon or platinum point in a solution of potassium ferrocyanide gives lines of potassium and iron and no new lines. Potassium chromate as electrolyte absorbs all but a small strip of actinic light, but of a dozen lines measured in the vicinity of the D-lines, two were lines of potassium and the rest were lines of chromium, the triplet λ 5204–λ 5208 being strong.

Besides the metals described the following have been photographed in the Wehnelt, both in the first and third spectrum: — copper, nickel, iron, gold, palladium, platinum.

As has been already stated, no effect of the anion has been observed at any time. The breaking up of the chromate ion and the appearance of the lines of metallic chromium in the case of potassium chromate, and the fact that all the salts of the same metal give identical spectra, show this clearly. The facts all point to the high temperature as the cause of the luminescence, and to the probability that the electrolysis plays a purely secondary part.

Although no effect of the anion is observed, the spectra produced by this arrangement are in many cases compound spectra, if we are to accept the sharp distinction of *compound* and *metallic* or elementary spectra as evidenced by the presence or absence of *bands*. Aluminium, magnesium, calcium, barium, give the bands which are ascribed to their oxides, and zinc and tin give fluted spectra which appear to be of the same general type. The presence of free oxygen and hydrogen at a temperature above the point of dissociation of water affords the possibility of a strong oxidizing action on any metal and the opportunity for the exhibition of an oxide spectrum when the dissociation temperature of the oxide in question lies at a point higher than the dissociation temperature of water. That the free hydrogen in the mixture of gases could under these conditions form metal-hydrogen compounds * seems more than doubtful, yet the variations between Wehnelt and spark and arc in air bear a striking resemblance to the variations in the arc spectra of the same metals in air and hydrogen, and in the latter case the necessity for the assumption of such metal-hydrogen compounds seems to be felt.†

Although the spectra under discussion are in many ways similar to those produced by allowing the spark to pass from a metallic point to a solution of a salt, many of the characteristic bands of compounds produced by the latter method are absent in the Wehnelt. Compound spectra, varying from salt to salt, are the rule in the more volatile metals ‡ when the spark passes to a solution of one of their salts. The point used gives, in the Wehnelt, all the strong lines of the metal, even of platinum, while in the other method the lines of the electrode are but faint.

The tables and plates show the following general points of especial interest: —

1. The lines of the Wehnelt spectra include some which have been usually ascribed to the spark and some which have been ascribed to the arc; usually the spectrum is closely allied to that produced in the spark, but often *some of the strongest lines are missing.* Other researches which give evidence as to the effect of varying conditions on spectrum lines serve to strengthen the conclusion which must be drawn: — that there is no sharp boundary between arc and spark, and that the transition from "arc" spectrum to "spark" spectrum is a gradual and continuous one.

2. Under constant conditions a spectrum may contain the strongest lines of the condensed spark and at the same time lines usually ascribed to the flame. §

3. Under the same conditions the "oxide bands" may be present at the same time as the strong spark lines. §

4. Under the same conditions the spectrum of zinc contains, beside lines belonging to both spark and arc spectra, a band spectrum not

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* Vid. Liveing and Dewar, l. c.

† Vid. Basquin, l. c., and Porter, l. c.

‡ Vid. Lecoq. Spectres. Lum., atlas.

§ See on these points Liveing and Dewar, l. c.; also Proc. Roy. Soc., **32**, 189 (1881).

hitherto observed under any conditions. The same is true of the spectrum of tin in the Wehnelt interrupter.

5. Certain lines which are of broad, diffuse appearance in the spark and are absent from the arc spectrum of the metal, are present in the Wehnelt spectrum, but much decreased in intensity. The metals which show this phenomenon most sharply are zinc, tin, and lead.

The first thought is, of course, that we are dealing with a complicated process involving an entire series of temperatures from that sufficient to give the band spectrum of a compound to that necessary to produce the strongest lines of a difficultly fusible metal like platinum or chromium, and that the point and the space about it pass through this series of temperatures at each interruption of the current. The result would be a spectrum which was the sum of the spectra corresponding to the various temperatures through which the system passes. Such a conclusion should be evidenced by the presence of all the lines of all the spectra, and if we are to couple each spectrum with a definite temperature we should expect the resulting composite to be complete. But in none of the cases examined are all of the lines of the various spectra present. Some strong lines are and some are not present in normal intensity. So this conclusion, though perhaps the simplest consistent with our present knowledge of temperature and spectra and the relation between them, seems not to explain the facts.

The interrupting action in the Wehnelt is undoubtedly caused by the intense heating due to the high current density about the small point. The heat vaporizes and then dissociates the water into hydrogen and oxygen, as has been shown by the analyses of Voller and Walter* and others. That spectra like those of the oxyhydrogen flame should result from the explosion of the mixed gases by the hot metal in the cooler parts of the gaseous envelope is not surprising, it is rather remarkable that not more of the characteristic flame spectra were observed. At the same time the great affinity for oxygen of such metals as aluminium, magnesium, calcium, and barium might explain the appearance of the bands belonging to the oxides of these metals.

The one point which is difficult of explanation by any reasoning based on the assumption of a varying temperature is the absence of some of the characteristic strong lines, while others usually ascribed to the same temperature are present.

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* L. c.

It seems probable that the environment of the point does pass through a very great range of temperature with each interruption of the current. There seems to be no reason for believing that the same is not true of the spark in air and of the arc. In the spark in air the discontinuity of succeeding oscillations is quite complete, and the necessity for an explanation of the absence of lines ascribed to lower temperatures seems as evident as in the case at hand. It seems probable that all three of these spectra, spark, arc, and Wehnelt, are composites, though this is not so evident in the usual methods of spectrum production.

In the Wehnelt, reversals were not observed in any case. This is remarkable since Hale,* and Hale and Kent,† and Lockyer, ‡ and others who have examined the spectra of the arc under liquids have shown that a very strong tendency toward reversal may exist under these circumstances. The mechanism of spectrum production in the two cases must be very different.

Lockyer gives as the result of his investigations and as the basis of many deductions four distinct stages of temperature corresponding to the spectra of metals as they are produced in the laboratory. They are:

"1. The flame spectrum, consisting of a few lines and flutings only, including several well-marked lines, some of them arranged in triplets.

"2. The arc spectrum, consisting of many lines.

"3. The spark spectrum, differing from the arc spectrum in the enhancement of some of the short lines and the reduced relative brightness of others.

"4. A spectrum consisting of a relatively very small number of lines which are enhanced in the spark. This latter spectrum is produced at a temperature which is that of the very centre of the spark."

Lockyer has also made frequent use in his writings of a diagram showing a series of furnaces of increasing temperature, spectra being used to designate the temperature in each. The flame is taken as the lowest, the arc next, and then the spark, followed in application to astrophysical comparisons by the spectrum of a typical cooler star, then hotter ones, and so on. We have in the Wehnelt interrupter a "furnace" in which under constant experimental conditions a banded (compound) spectrum like that found for aluminium, and a pure line spectrum like that found for platinum may be produced. The amount of current passing in the two cases is the same, the other factors of the experiment (composition of electrolyte, size of electrodes, etc.) remain constant, and there is every reason to believe that the maximum temperature reached is the same in the two cases. Any criterion of the temperature corresponding to either of these spectra would appear to be wanting unless they correspond to the *same* temperature. Nor does the fact of the absence of some of the strong lines of a metal appear to offer any measure, however qualitative, of the temperature, since other strong lines usually belonging to the same temperature are present.

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* Astrophys. J., **15**, 190 (1902).

† Ibid., **17**, 155 (1903).

‡ Ibid., **15**, 190 (1902).

It is certainly very easy to be led into reasoning in a circle in a case where much of our knowledge of temperatures has been deduced from observations on spectra, and much of our theory as to the correspondence of certain spectra with certain temperatures has been drawn ultimately from the same source.

JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY, Jan. 15, 1904.

PLATE 1.

FIG. 1. Spectrum of carbon point in hydrochloric acid. (Lines of impurities, which are iron, calcium, and aluminium.)

FIG. 2. Spectrum of carbon point in solution of sodium chloride.

FIG. 3. Spectrum of carbon point in solution of potassium chloride.

FIG. 4. Spectrum of platinum point in solution of barium chloride.

FIG. 5. Spectrum of platinum point in solution of strontium chloride.

FIG. 6. *a.* Spectrum of spark in air between lead terminals.
b. Spectrum of arc in air. Metallic lead on carbon terminals.
c. Spectrum of lead point in hydrochloric acid.

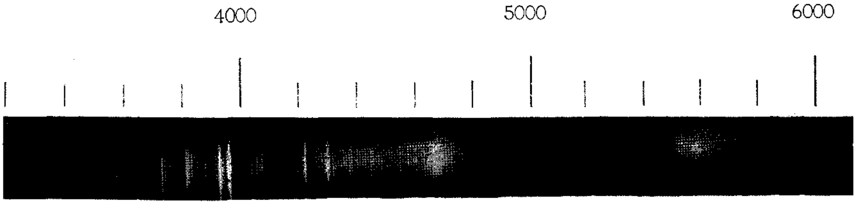
4000
5000
6000

Fig. 1

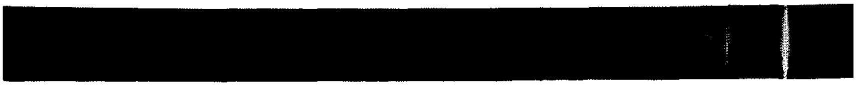

Fig. 2

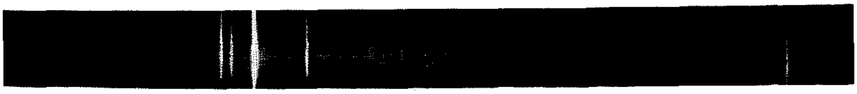

Fig. 3

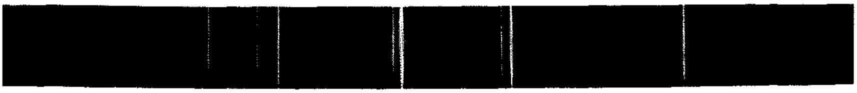

Fig. 4

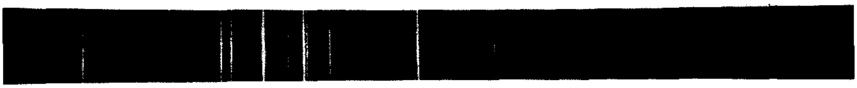

Fig. 5

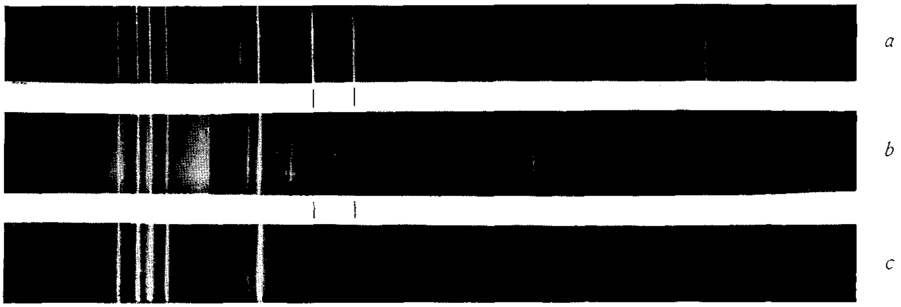

Fig. 6

PLATE 2.

FIG. 7. *a.* Spectrum of arc in air. Lithium chloride on carbon terminals.
b. Spectrum of spark in air. Lithium chloride on carbon terminals.
c. Spectrum of carbon point in solution of lithium chloride.

FIG. 8. *a.* Spectrum of spark in air between magnesium terminals.
b. Spectrum of arc in air. Metallic magnesium on carbon terminals.
c. Spectrum of magnesium point in hydrochloric acid.

FIG. 9. *a.* Spectrum of spark in air between aluminium terminals.
b. Spectrum of arc in air. Metallic aluminium on carbon terminals.
c. Spectrum of aluminium point in hydrochloric acid.

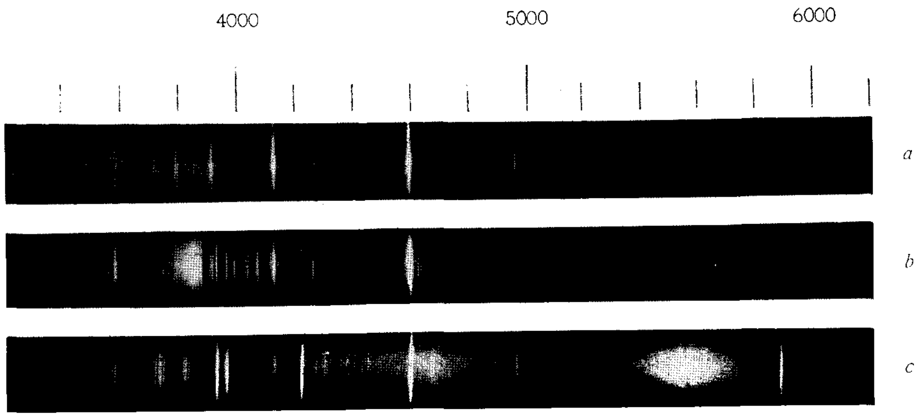

Fig. 7

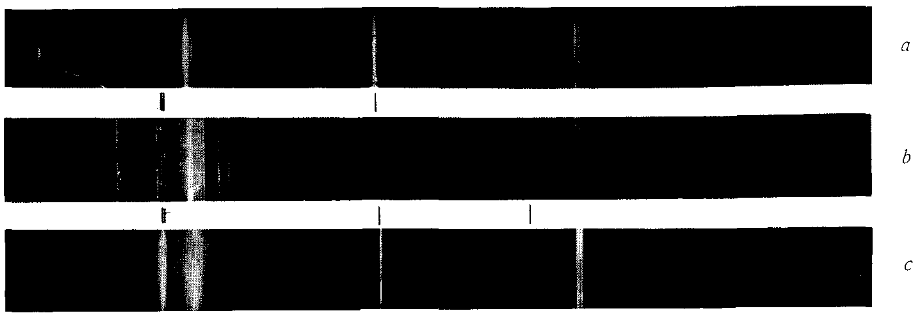

Fig. 8

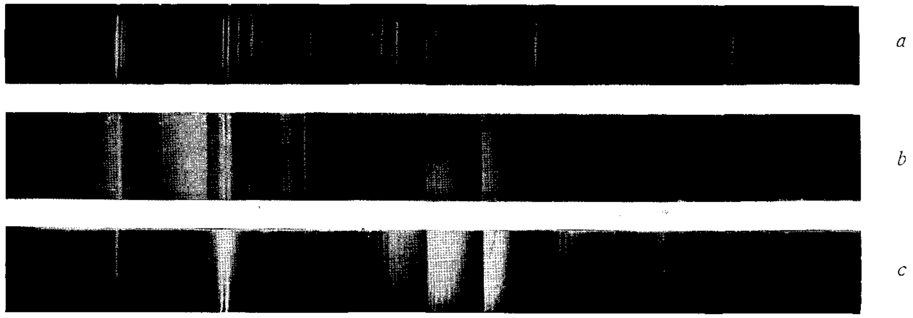

Fig. 9

PLATE 3.

FIG. 10. *a.* Spectrum of spark in air between zinc terminals.
b. Spectrum of arc in air. Metallic zinc on carbon terminals.
c. Spectrum of zinc point in hydrochloric acid.

FIG. 11. *a.* Spectrum of spark in air between amalgamated copper terminals.
b. Spectrum of arc in air. Metallic mercury on carbon terminals.
c. Spectrum of platinum point in solution of mercuric nitrate.

FIG. 12. *a.* Spectrum of spark in air between tin terminals.
b. Spectrum of arc in air. Metallic tin on carbon terminals.
c. Spectrum of tin point in hydrochloric acid.

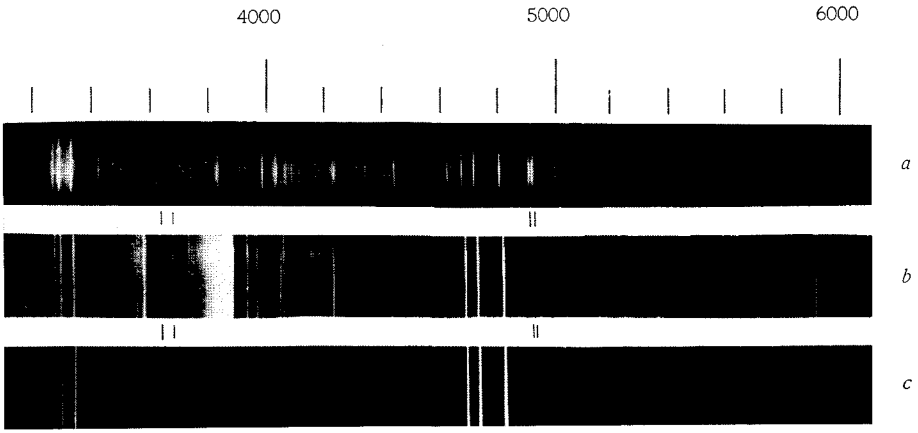
4000
5000
6000
a
b
c

Fig. 10

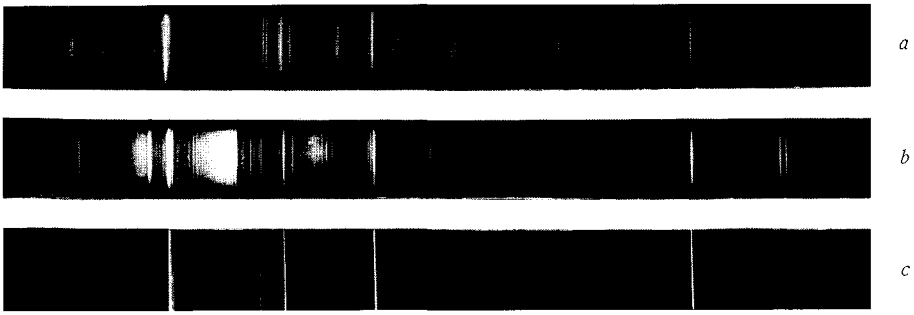

Fig. 11

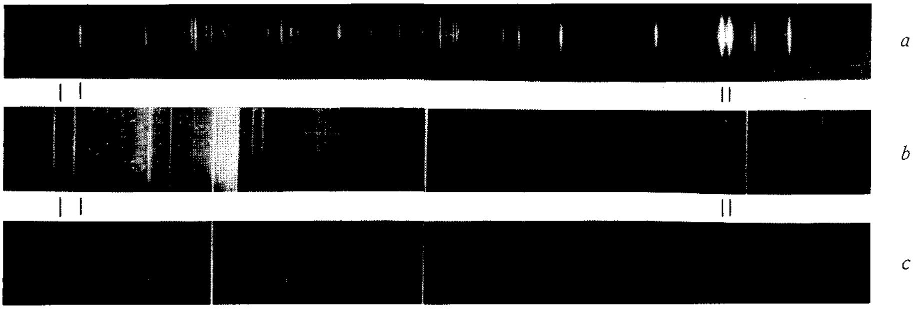

Fig. 12

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STUDIES ON THE TRANSFORMATIONS OF SATURNIAN MOTHS, WITH NOTES ON THE LIFE-HISTORY AND AFFINITIES OF BRAHMAEA JAPONICA.

By Alpheus S. Packard.

STUDIES ON THE TRANSFORMATIONS OF SATURNIAN MOTHS, WITH NOTES ON THE LIFE-HISTORY AND AFFINITIES OF BRAHMAEA JAPONICA.

By Alpheus S. Packard.

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This paper is the continuation of one presented to this Academy in February, 1893. It contains descriptions of the larval stages of some interesting genera of the family of silk-worm moths, Saturniidae, not hitherto known. Of these genera some are among the most highly specialized, while others are more primitive. It is a matter of some interest to have worked out the transformations of *Callosamia calleta*, an annectant form between Callosamia and Philosamia. The early stages of the species of Rothschildia, which represent in the New World the Asiatic Attacus, have now been discovered after several years of effort to secure the cocoons and eggs. We are now in a much better position than before to work out the phylogeny of the family and its relations to the Ceratocampidae.

Callosamia calleta.

A lot of 25 eggs kindly given me by the Sydney Ross Co., New York, through Mr. C. B. Riker, had hatched several hours before their receipt on June 6, from eggs laid May 21, 1903.

Egg. — Oval cylindrical, somewhat flattened; chalky white; the surface seen under a powerful lens to be minutely pitted. Length 2⅖ mm.; breadth about 1⅗ mm. The hole eaten by the larva for its exit is situated at the end of the shell, but sometimes encroaches on one side.

Larva. — *Stage I.* Length 6 mm. The body tapers slightly toward the end. The head is as in *P. cynthia*, being smaller than in *C. promethea*, and with no pale band; velvety black-brown; clypeus-posterior a little paler; clypeus-anterior pale gray or dull livid whitish; the head bears a few scattered black hairs. The body is blackish brown and all the setae are black-brown, being of the same hue all over the body.

The first thoracic segment bears six tubercles, of which the lowest one is a little larger than those of the row above, and the two median dorsal ones are slightly smaller than those on each side. They are well developed, higher than in *C. promethea*, and nearly as in *P. cynthia*. The tubercles of the three thoracic and 1st abdominal segments of the same size and height, those of abdominal segments 2–7 a little smaller; all are about twice as long as thick; the longer setae are fully twice as long as the tubercles bearing them; those on the 2d thoracic to 7th abdominal segments are a little curved. Each dorsal tubercle bears 10–11 setae, several arising some distance below the crown; all are of unequal length and thickness; those on the mid-abdominal segments 7–8 in number. The median dorsal tubercle on 8th abdominal segment is twice as thick and considerably higher than those next to it.

Suranal plate with two tubercles higher than broad; this and segments 9 and 10 and all the legs, both thoracic and abdominal, are brown-black.

The body is black-brown; all the segments are ivory white on the posterior edge (the 1st and 2d thoracic less so than those behind), and the front edge of the white transverse band makes two scallops on each side. Front edge of each segment dark, but the base of each tubercle is surrounded by white.

It moulted June 11, all the individuals casting their skins simultaneously, just as all were observed to hatch at the same or nearly the same moment.

Stage II. Length 11–13 mm. The body is rather thick and the thickness uniform. The head is not nearly so wide as the body. It is peculiar in shape and color, rather deeply bilobed; the surface soft velvety black-brown, apex of the clypeus pale greenish; between this triangular spot and the eyes is a deeper yellow roundish spot, and behind each group of ocelli is another long yellow mark; clypeus-anterior white, labrum black.

The body is armed with large, rather high tubercles, all nearly of the same size, though those of the 2d and 3d thoracic segments are very slightly larger than those on the abdominal ones, and that on the middle of the 8th abdominal segment is nearly twice as thick as those on the 7th segment.

On the prothoracic segment are four large high tubercles of equal size and height, but slightly smaller than those on the 2d and 3d thoracic segments, each with four stout spines and four smaller ones. All the tubercles are fleshy and stand up like columns, and are about twice as high as thick. The median tubercle on the 8th abdominal segment bears twelve setae arranged six on each side of the crown; those on the 9th segment with four large setae.

Suranal plate with two large erect tubercles from a third to a half smaller than those on the 9th segment, and bearing five large, long, and five finer setae; farther back near the end are two small low tubercles bearing from two to three coarse, and two or three finer bristles. Anal legs with setiferous tubercles on the hind edge.

The body is uniformly deep ochreous yellow. The 1st thoracic segment is prussian or dark steel blue, but yellow on the hinder edge, with a yellow spot on the side, or the segment may be all yellow except around the base of the tubercles; the other segments are yellow. Suranal plate and anal legs of a rich dark steel blue. On the front edge of each segment (thoracic 2–3 and abdominal 1–8) there are three dorsal triangular black-brown spots which are usually connected by a black line along the suture; behind these spots on each segment is a similar black line dilating into three triangular spots which vary in being either connected or separate. Spiracles dark, on a dark ground. Both thoracic and abdominal legs black; under side of the body blackish.

In this stage the markings are much as in *C. promethea*, i. e. the two transverse black bands, but they are farther advanced or more specialized, and the head is without the transverse pale band in front; the prothoracic and other tubercles are the same. They are of the same general proportions, but in color *C. calleta* is darker, especially beneath, and on the suranal plate and legs *calleta* has longer bristles.

End of Stage II. When about to moult (May 26) it is 23 mm. in length. The head in my example was very small, and is just as described in Stage II, but with traces of another yellow spot. At this time the body is much less spotted and marked with black-brown. The 1st thoracic segment is ochre-yellow, with four irregular black spots along the hinder edge, while the area around the base of the four tubercles is blackish. All the tubercles are smaller in proportion to the body, but preserve the same proportion to each other as in the beginning of Stage II; the body being larger, the tubercles are wider apart.

Each segment behind the 1st thoracic with two series of three blackish triangular spots, the middle one in front triangular and longer than the lateral ones (as in the early part of the stage). The one corresponding to it on the hinder line (which is half-way between the dorsal tubercles and the hinder edge) is transversely linear; the spots vary in being either separate or connected; they are smallest on the 8th and 9th abdominal segments. All the tubercles are uniformly dark prussian or steel blue. Suranal plate yellow, except the black subtriangular patch giving rise to the tubercles. Along the side below next to the base of the legs is a series of detached greenish yellow spots, the two on the 2d and 3d segments centred by a steel-blue minute tubercle, this being the fourth on each side. Abdominal legs dark, with a yellow spot at the base on the outside.

Stage III. Described May 29. Length 35 mm. The larva is in this stage a very different creature from what it was in the previous stages. It is now gorgeously colored, and a very conspicuous caterpillar. The ground color of the body is pea-green, while the large swollen base of each tubercle is deep reddish orange approaching vermilion, contrasting with the dark steel-blue of the spines surmounting the base.

The head is very small, not one-half as wide as the body, and one-third narrower than the 1st thoracic segment. The clypeus-posterior is now pale turquoise-blue, and there is a broad red-orange stripe on each side of the clypeus.

The three black spots on each segment are still large and conspicuous, while those on the hinder edge are minute. The centre of the suranal plate and of the anal legs is stained deep red-orange, as also the outside of the mid-abdominal legs. The thoracic legs are also painted with two vermilion-red rings.

There is the same proportion between the size of the dorsal tubercles.

The setae are scarcely as long as the spines themselves.

Stage IV. Length 38–40 mm. Differs but little from the previous stage in armature and coloration. In this stage most of the larvae have the tubercles entirely black; in a few, however, the dorsal ones on the 2d and 3d thoracic, 1st, 2d, 3d, and 8th and 9th abdominal segments are pale blue at the tips; and sometimes those on a less number of segments are tipped with blue. The six tubercles on the prothoracic segments are still well developed. The suranal plate is pale red; the sides of the anal legs deep bright red, as also are the ends of the mid-abdominal legs; the thoracic legs are black ringed with red.

Stage V. Length 65–70 mm. Head and body pea-green. Shape of the body nearly as in Philosamia, but the tubercles very much thicker and larger. Those on the 3d thoracic segment and 1st abdominal a little larger than the pair on the 2d thoracic segment. The dorsal abdominal tubercles somewhat unequal in size, those on segments 2–7 about one-half smaller than those on the 1st abdominal segment, those on segment 6 being a little smaller than the others. The median tubercle on the 8th abdominal segment nearly three times as thick as that on the 6th segment. All the tubercles bright red on the swollen base; beyond, turquoise-blue; the short spinules black. Suranal plate, the side of the anal legs, and the thoracic legs yellow. Mid-abdominal legs yellow, black at base.

There are two subdorsal rows of black spots, one on each side; a black band on the front of each segment passes down to a point near the spiracles. The larvae vary in the amount of black on the body, some having more, others less.

The larva differs from Stage IV in the part of the tubercles above the red swollen base being much thicker, the spinules shorter, and there are no tubercles on the prothoracic segment, these being in the last stage represented by black dots.

The larva feeds on the wild cherry.

This is an unusually conspicuous larva, and as usual in the group the bright hues and spots may be regarded as warning colors, the insect being protected by what are probably venomous spines.

In its markings (the six dorsal black spots) and armature it closely resembles *Philosamia cynthia;* the tubercles are all large and of nearly equal size and shape as in the 2d stage of *P. cynthia.* There is no such inequality as is to be observed in the 3d stage of *C. promethea* where the 2d and 3d thoracic dorsal tubercles and that in the middle of the 8th abdominal segment are three times as large as all the other dorsal abdominal ones.

Judging by the larval characters *Philosamia cynthia* is a more primitive or generalized form than Callosamia, since Philosamia retains the equality in the size and shape of the tubercles all through its larval life.

C. calleta up to the end of Stage III is really nearer to Philosamia than the genus to which it is generally referred, and this suggests that phylogenetically Philosamia is the older genus, and that from it directly sprang the species of Callosamia; *C. calleta* being the oldest species.

Mode of casting its skin. — Before moulting the abdominal legs are folded together and the crown or rings of both are contracted.

One was seen in the act of ecdysis; the skin had split open at various places from one end of the body to the other. The thoracic legs become free or are drawn out of the old skin before the head is cast off.

Rothschildia jorulla.

(*Attacus cinctus.*)

Larva. — Beginning of Stage II. It was hatched in New York from a few eggs laid by examples bred from the cocoon and mated by Mr. Joutel, and sent me May 8, and described May 9. Length 7 mm. Body unusually short and thick, and now much wider than the head, which is of the usual shape (as in Telea and Philosamia), smooth, with long scattering irregular hairs; antennae white; it is black and yellowish-white, the sides black, with a black V on each side of the yellow clypeus-posterior; clypeus-anterior with a whitish transverse line.

Body and legs dull livid brownish. The tubercles are very large and crowded together; they are full, globular-conical, and a little higher than thick, and all pale straw-yellow.

There are four dorsal prothoracic tubercles, but the two on each side are united at base, making a large double transverse tubercle higher than any of those behind, and all four connected by a ridge; yet they are shorter than in Stage I.

All of the tubercles behind the prothoracic segment are of the same size and height, the meso- and the metathoracic ones not differing from each other, nor are the abdominal ones smaller than those on the thoracic segments, nor do they differ in height or size among themselves, except the median tubercle on the 8th abdominal segment, which is quite broad and one-fourth larger and higher than the others, and about twice as wide as the others; there are 6–7 setae on each side. Those on the sides of all the segments are of the same size.

The dorsal tubercles are each armed with eight stiff setae, which are pale brown, the longer ones of which are about a third longer than the tubercle itself, one being in the centre and the others arranged around it.

On May 17, after being forced by a sojourn in the kitchen, it was ready to moult; its length was 13 mm. The body had now become livid greenish-yellow; the tubercles all straw-yellow, and of the same proportions. The spiracles are dark, rather conspicuous. There is a transverse blackish line on the prothorax behind the four tubercles.

Stage III. It moulted May 18, and was described on the 19th. Length 20 mm. Head yellow, black behind, with a broad V in front connecting with the black base of the clypeus anterior. Body thick, head only one-half as wide as the body in the middle. First thoracic considerably narrower than the 2d thoracic segment. The tubercles are still all similar, of the same size and color. The two dorsal ones on each side of the prothoracic shield still united. The median dorsal tubercle on the 8th abdominal segment is nearly twice as thick and somewhat higher than those on each side. Each dorsal tubercle bears 8 or 9 setae, while on each infraspiracular tubercle are from 8 to 12 setae or spinules.

The body is now black, the sutures between the segments white; the hinder edge of each segment, beginning with the 2d thoracic, also white. Suranal plate straw-yellow with an irregular longitudinal band, widening behind the two tubercles. The tubercles orange-yellow, straw-yellow at base. Anal legs entirely black. Thoracic legs and antennae pale at the end; abdominal legs all black. Spiracles black, surrounded by black. It moulted May 22–23.

Stage IV. Length 23 mm.; at end of the stage 35 mm. The body is now thick and stout, leaf-green. The head is pale straw-yellow, black behind, enclosing a distinct oblong black spot; clypeus broadly edged with black, making a V. Antennae and clypeus-anterior whitish.

The dorsal tubercles are all of the same size, and of the same orange color. The spines on those of the three thoracic and 1st abdominal segments somewhat dusky; those on the tubercles of the segments behind are all pale, while those on the infraspiracular tubercles are a little dusky, as also those of the two dorsal tubercles on the suranal plate. The plate is rather smooth, pale straw-yellow, and bearing along and just outside of the edge a hollow triangular black mark with the corners well rounded. Anal legs green, with a similar but larger hollow black subtriangular ring; the legs below and all the abdominal legs are black with a green spot on the outside of the middle, and bearing long white hairs of even length.

At the end of this stage, May 28–29, it was 35 mm. in length. Now the tubercles are all deep reddish from the top to the base, greatly contrasting with the deep green of the body. The lower edge of the 9th and 10th abdominal segments are now white, forming a broad line.

The front edge of abdominal segments 3 to 7 whitish. The green spot on the outside of the mid-abdominal legs is now yellow.

For a day after moulting it nibbled its cast skin.

It moulted for the fourth time about June 1, and was described June 3.

Stage V. Length 48–50 mm. Head and body deep pea-green, the head green, with two black lines behind, while the clypeus is lined with black. The head is still small in proportion to the body, being about one-third the width of the latter, and only two-thirds as wide as the prothoracic segment. The prothoracic shield is now smooth, with four median minute groups of 4–5 minute short setae, and two rounded lateral small tubercles. The *tubercles of the 2d thoracic segment are smaller than those on the 3d, the latter not being so high and prominent* as those on the 1st abdominal segment; those on the thoracic segments are flattened, button-like, not so high as thick; all orange-yellow, and bearing from 4 to 7 short black stout setae, the longest ones no longer than the tubercle is high.

The *tubercles on abdominal segments* 2–7 are smaller *than those on the* 1*st abdominal segment,* and gradually become smaller towards the 7th segment. The setae in general are a little longer than the tubercles are high.

The median tubercle on the 8th segment is low, not so high as broad, and in outline seen from above is transversely oval, bearing 4 principal larger setae and three minute ones besides. (The originally double origin of the tubercle is not evident.)

There are on the 9th abdominal segment very minute green vestiges of two tubercles.

Suranal plate green, with a conspicuous white stripe on the edge, and on top of the plate is a subtriangular black mark made by a narrow black line, and a triangular black ring on the outside of the anal legs.

Front edge of abdominal segments 3–7 *with a broad, conspicuous pale purple-madder stripe, which is widest below and narrow above; this is succeeded by a white band, widening dorsally and conspicuous when seen from above.* Spiracles yellow ochre. Thoracic legs green, irregularly ringed with black, the tips black; the four pairs of mid-abdominal legs black-brown, with an external conspicuous yellowish-green patch, and armed with rather long curved setae. The plantae of the anal legs black-brown.

Cocoon. — On the morning of June 13 my larva, which was a little under the normal size, had begun to spin a cocoon, and the handle or stem of the cocoon had been spun before it was nearly finished; the rather large opening for the exit of the moth had been left open.

Number of moults and habits. — There are four castings of the skin and five stages, as usual in the bombycine moths, and the family Saturniidae. In captivity in a northern State (New York and afterwards Rhode Island), the ecdyses occurred at about every ten days, the eggs being laid near the end of April and the larvae hatching out the first of May.

Food plant. — The larvae were fed on the ash and wild cherry, but preferred the latter. Its native food-plant is unknown to us.

Rothschildia orizaba.

Larva. — *Stage I.* Hatched July 8–9. Described July 12, 1902. Length 8–10 mm. Body cylindrical, rather slender, slightly more so than in *T. luna.* Segments not convex, more as in Callosamia than in Telea or Tropaea. Head rather small, rounded, much as in Callosamia, a little more than half as wide as the body in its thickest part; pale whitish, surface very finely granulated; edge of clypeus dark brown, and a transverse line across the front edge; a large brown round spot on each side.

Prothoracic segment with no definite plate, but with six long slender high greenish-yellow tubercles, about one-third as thick at base behind; six setae in each dorsal tubercle; the longer setae about twice as long as the tubercles. All the tubercles on the second thoracic segment to the end of the body much swollen at base, being conical, large, and pale straw-yellow, and *all of the same size and height*, but those of the 2d and 3d thoracic segments are fused together at their base, while those of the abdominal segments are distinctly separate.

The median tubercle of the 8th abdominal segment is only a little larger and a little broader than those on the 9th segment; those on 9th abdominal segment fused at base.

Body above black, except the thoracic segments, which are greenish, with two black spots on each segment above; the sutures livid greenish.

Suranal plate with two large, slender, distinct tubercles, about one-half as large as the supraspiracular one on the 9th segment; it is greenish yellow, but behind the tubercles dusky. Anal legs blackish in the middle. Infraspiracular row of tubercles arising from the lateral ridge pale greenish-livid, as is the under side of the body and the abdominal legs. Thoracic legs black.

Each tubercle with six dusky setae, the longer ones a little longer than the tubercle itself.

A very beautiful caterpillar, which resembles that of Callosamia in coloration, but the tubercles are larger, conical, and all alike in size.

Larva. — *Last stage.* Head rounded, about one-half as wide as the body is thick, and not so wide as the cervical plate; the surface of the head smooth, polished, with scattered fine hairs on the clypeal region and about the ocelli; in color pea-green. Body thick, cylindrical, tapering a little towards each end; how convex the segments are cannot be determined in blown examples. Cervical plate smooth, with no vestiges of tubercles. On the prothoracic segment below the edge of the cervical plate is a slight vestige of a tubercle in front of but a little below the spiracle; farther down, near the base of the leg, is a decided but small low tubercle bearing three setiferous spinules; those in the same relative position on the 2d and 3d thoracic segments are of the same size and shape.

On each of the two hinder thoracic segments there are four small dorsal tubercles in a transverse row. Each tubercle is crowned by a ring of 5–6 spines, with one in the centre. These and the dorsal ones on abdominal segments 1–7 are so closely alike in size and armature that it is difficult to see any difference in size between the thoracic and abdominal dorsal ones. The four dorsal ones on each of the two last thoracic and the first seven abdominal segments are alike in size, number, and arrangement of the spinules; there not being the usual distinction which obtains in the Attacinae (Samia, Callosamia, Telea, etc.) between the submedian (dorsal) and supraspiracular series. Those of the 7th abdominal segment are, as far as I can see, just like those on the 2d and 3d thoracic and 1st abdominal segments.

The median spine on the 8th abdominal segment is scarcely higher (longer), but is about one-quarter thicker than the one on each side; it is slightly broader than long (seen in section from above) and bears five spinules on each side of the median line. There is no infraspiracular row of tubercles (such as are present in the Citheroniinae and Attacinae). There also seems to be no difference in color between any of the tubercles.

The suranal plate is green, a little rough on the surface, and there are traces along the hinder edge of piliferous spines; the plate is edged with black. The anal legs are large, green, the triangular area edged with black, and on the edge are scattered small black piliferous warts. The thoracic legs are pea-green; the abdominal legs green, with irregular rows of black warts bearing white hairs above the plantae, which are, with the hooks, black. Along the side of the body extends a broad lateral yellow infraspiracular line, from which and below which arise long fine white hairs. The spiracles are sienna brown-yellow. The body is covered with fine white short clavate hairs. Length 80 mm.

An inflated example from Mexico in the U. S. National Museum, collection of Dr. H. G. Dyar; two inflated examples, in bad condition, from Tacubaya, Mexico (Barrett).

In the species of this genus the degree of specialization of the tubercles in the larva is very slight, as they show a tendency to reduction and atrophy, which reaches its greatest perfection in *R. betis*, in which there are, according to Burmeister's figure and description, no tubercles at all, while the body is blackish, with conspicuous transverse bands.

R. orizaba, as a larva, in form and markings most nearly approaches that of *R. aurota*, while the most generalized species is *R. speculifer*, in whose larva there are the longest tubercles, most nearly approximating those of Samia and Philosamia.

Food plant. — Specimens raised from the egg by Mr. Joutel fed on the white ash. Its native food plant is unknown to us.

Rothschildia jacobeae.

The materials on which our life-history of this species is based were obtained by the American Museum of Natural History from Buenos Ayres. It consisted of the eggs, inflated larvae in three stages, the pupa, cocoon, with the ♂ and ♀, all in an excellent state of preservation, and I am indebted to Dr. H. C. Bumpus, the Director of the Museum, and Mr. William Beutenmüller, the assistant in charge of the Department of Entomology, for the loan of the specimens.

Eggs. — Oval, flattened, chalky white; surface shining, seen under the lens to be very finely pitted; greatest diameter about 2 mm.

Larva. — *Stage II.* The blown larva in the 3d stage had retained on the last half of the body the skin of what we suppose to be of this stage. It shows, what is not present in the next stage, *two parallel rows of* 6 *linear black spots passing across the segments.* On the suranal plate, on each side, is *a low flattened green tubercle, bearing six dark spine-like setae.* There are faint traces of a reddish spot on the plate, and on the side of each anal leg. *All the tubercles black.* The median tubercle on the 8th abdominal segment slightly bilateral, bearing four setae on each side. Suranal plate with a yellow band on the edge.

Stage III. What is probably of this stage is a blown larva. Length of body 34 mm.; width of head 2.7 mm. Head pale yellowish brown (probably green in life), clypeal sutures black, and on each side is a black line extending from the eyes up to the centre of the vertex on each side.

Body green; cylindrical, with prominent tubercles as in the corresponding stage of other Attacine larvae. Prothoracic plate with four well-developed green tubercles, which are low, rounded, about half as large as those on the 2d thoracic segment, and each bearing six dark setae. Tubercles of the 2d and 3d thoracic segments each with fairly large green warts or tuberculets, each bearing a black seta longer than the main tubercle, at least a third longer. Those of the 1st abdominal segment a little larger than those on segments 2–7.

The median tubercle on the 8th abdominal segment double, and apparently as large as the thoracic ones (this is covered by the loosened skin of the previous stage). Legs as in the last stage, but the mid-abdominal ones not so much black in extent at the end. Spiracles black. The yellowish rings of the later stage are in this period faint.

Stage next to the last. Length of body 70 mm.; width of head 4 mm. Head and body pea-green, as in the last stage. On the prothoracic segment the position of the primitive dorsal tubercles (seen in Stage III) is indicated by minute setiferous warts. Dorsal tubercles on the 2d and 3d thoracic segments a little (about one-fourth) larger than those on abdominal segments 1–7; those on the latter-named segments slightly larger than those on the intermediate segments, all bearing six radiating and a central erect black seta; the tubercles are pale green (in the blown larvæ examined they have turned yellowish like the dorsal ones).

The median tubercle on the 8th abdominal segment of the same size as those of the dorsal on the 2d thoracic segment; it shows a slight bilateral arrangement, there being six black setae on each side.

Suranal plate and each of the anal legs with a pale bright red spot as in the last stage. Spiracles black; legs as described in the last stage.

The larva in this stage differs from the full-grown caterpillar in the larger tubercles, all of which are armed with well-developed spine-like setae; in the nine (one more than in the last stage) yellow bands, there being an additional one on the 2d thoracic segment, the bands or incomplete rings beginning at the infraspiracular row of tubercles and passing over the back from one side to the other.

Last stage. Length of body 110 mm.; width of head 4½ mm. Body cylindrical. Head small rounded green, smooth, less than one-half as wide as the prothoracic segment; the clypeus narrowly edged with black, and two diverging black lines extending from near the centre of the vertex and passing down inside of the ocelli.

Prothoracic segment with no traces of dorsal tubercles, but a green pyriform tubercle in front of and a little below the spiracle, and a larger, more prominent one still lower down, bearing six short setae, five around the edge of the crown and one in the centre.

Dorsal tubercles small pyriform, present on the 2d and 3d thoracic segments; those on the 1st abdominal segment about a third smaller than the thoracic ones, and with no setae. On abdominal segments 2–7 the dorsal tubercles decrease in size to the 7th. The median tubercle on the

8th abdominal segment dark green, fully as large as the thoracic ones, and bearing five distinct peripheral and one central black seta (their stumps being present, probably broken off while being inflated; they are faintly seen in the penultimate stage). Tubercles of the supraspiracular row minute, smaller than the dorsal ones; those of the infraspiracular row larger, and still armed with 7–8 black setae. It differs from the larva of *R. orizaba* from Mexico in having no definite hairy lateral yellow ridge.

Suranal plate small, rounded behind, with a few small fine hairs and bearing a large central bright red rugose spot; the lower edge yellow; on the sides of the anal legs a little larger triangular spot of the same color. Spiracles black, very conspicuous. Segments smooth, and with a pale yellowish band or incomplete ring (8 in all) in front of each spiracle.

Thoracic legs pale, black at the end, 1st and the 3d joint entirely black. Mid-abdominal legs green, a yellow patch above the planta, broadly bordered with black, and the planta black.

Described, as are the other stages, from blown specimens.

It differs in this stage from the larva of *R. orizaba* in the abundant hair-like setae on the lateral ridge, and in the red spots on the suranal plate and side of the anal legs. In its armature it differs from that species.

Cocoon. — Oval, not much larger at one end than at the other; the stalk very slightly developed. Length 53 mm., thickness 21 mm.

Pupa. — Of the form of Cynthia, but not so thick; of the usual chestnut brown color. Length 28 mm., thickness 13 mm.

Rhodia fugax.

The eggs, received from Japan, were hatched April 27, and the larvae described on the 28th. It feeds on the poplar and dwarf willow.

Larva. — *Stage I.* Length 6 mm. Head large, wider than the body, with long scattered hairs; brown-black with a transverse whitish patch on the clypeus-anterior, while the base of the antennae is of the same color, so that across the front of the head extends a broken whitish inconspicuous line. The body, as usual in freshly hatched Saturnian larvae before the larva has taken food, tapers to the end.

The dorsal tubercles are distinct, a little longer (or higher) than thick, those on the thoracic segments larger than the abdominal ones; those on the prothoracic segment about half as large as those on the 2d thoracic segment, which are very slightly larger and longer than those on the 3d thoracic segment. The prothoracic dorsal tubercles each bear four curved setae, while the 2d and 3d thoracic dorsal tubercles each carry six setae. The abdominal tubercles are nearly as long as the thoracic ones, but much slenderer, and they are rather high, contracting in diameter in the middle; they each bear 2 or 3 long curved black stiff bristles, and two minute ones, the long ones being curved backwards, and before the larva has begun to feed they are longer than the body is thick, so that it appears to be rather thickly clothed with fine black bristles.

The single median dorsal tubercle on the 8th abdominal segment is a third thicker than the others in front, bearing three long black setae on each side.

The two dorsal tubercles on the 9th abdominal segment are quite large, larger than those on abdominal segments 1–7. Two dorsal tubercles arise from near the middle on each side of the suranal plate; they are only a third smaller than those on the 9th segment; around the edge of the suranal plate arise long grayish stiff straight hairs, and the bristles on each lateral tubercle are grayish and directed downward.

The body is dull straw-yellow on the sides and black above. The wide black dorsal band is irregular in width, varying at the end of the stage.

It gives off on each side behind the middle a straight black rectangular line forming a black half-ring extending down to near the spiracles; the hinder edge of each segment is black. The color variations of the larva in this stage are marked; in the most extreme the entire upper part of the body is black below the spiracles, the black pigment extending down from the main mass to the infraspiracular tubercles, the under side only of the body being yellow. There is low down on the side of the body a narrow worn somewhat broken black line, situated below the infraspiracular tubercles. Spiracles small, inconspicuous, black. Thoracic legs black; abdominal legs in general yellow, when freshly hatched black, the planta slightly purplish livid. In this stage the larva is very pretty and interesting, but with the true Saturnian shape of body and tubercles. They moulted May 24 or 25.

Stage II. Length 12 mm.; toward end of stage 18 mm. Now very different from Stage I. The body is pale straw-yellow, and no black on it in my examples (while one of those reared by Mr. Joutel is black on the sides and on the 1st, 3d thoracic and 8th abdominal segments, and he found that there are all stages from the wholly yellow to the dark form).

The head is smooth, pale-green, black behind the eyes, the black extending up towards the back of the vertex; it is nearly as wide as the 1st thoracic segment, which is considerably narrower than the two following. The tubercles are now more prominent and are green and pale turquoise-blue. The four dorsal ones on the 1st thoracic segment are well developed and higher than before, the two middle ones nearly a quarter larger than the outer ones, each bearing six stiff straight black setae. The spiracular one is pale turquoise-blue, as all the others of this series to the end of the body. The two dorsal tubercles on the 2d and 3d thoracic segments considerably (about one-sixth) larger than those on the abdominal segments and those on the 3d thoracic considerably larger than those on the 2d segment; they each bear eight black stiff setae, seven on the crown with an eighth in the centre; these are yellowish at base, beyond turquoise-blue; in the examples I bred the tubercles on the 2d thoracic segment were green. The setae on the thoracic tubercles are not more than a quarter larger than the tubercle itself. All the dorsal abdominal tubercles are green (or yellowish green, Joutel), bearing each four black stiff and straight setae, one of them being usually small; the longest setae are about one-third longer than the tubercle itself. The dorso-median tubercle on the 8th abdominal segment, unlike all the other abdominal ones, is turquoise-blue; it is fully twice as thick as those on the 7th segment. It does not show its double origin as there is a central seta, and one behind it on the median line, with three on each side of the two; they are not arranged on each side of the median line.

On the side low down are two or three minute black dots below each of the turquoise-blue tubercles of the lowest series.

Suranal plate and its tubercles pale straw-yellow, concolorous with the rest of the body, with a large black roundish spot in the centre. In those reared by Mr. Joutel, the two tubercles on this plate are turquoise-blue. Spiracles black, fairly conspicuous. Anal legs green, centred on the outside by a rather large and conspicuous black spot; the mid-abdominal legs are green or yellowish green; thoracic legs black.

On June 2d the larva had become 18 mm. in length, otherwise it did not differ from its appearance directly after hatching.

Mr. Joutel's figures illustrate the wide colorational differences exhibited in the same brood of caterpillars. Some represent an entirely yellow individual, except the eleven turquoise dorsal tubercles, and those of the infraspiracular row; and the black lateral line, which is wanting in some of his specimens and mine. He found all variations between these and the very dark larva. In the 2d stage of the larva the body and tubercles are black, with the exception of the yellow dorsal portion of the 2d thoracic and abdominal segments 1–7, and the under side of the body, including the mid-abdominal legs. He tells me that other specimens have as much black pigment on the body, but that the dorsal tubercles on the 2d and 3d thoracic and 8th abdominal segments are as blue as in the light-colored caterpillars.

The larva I had in Providence moulted for the second time about June 4 or 5.

Stage III. Length 20 mm. The head is now wholly pea-green. The two median prothoracic, and the two dorsal tubercles on the 3d thoracic segment, and those of the lateral row are turquoise-blue, but the two dorsal tubercles on the 2d thoracic segment are greenish. The two 3d thoracic dorsal tubercles are thicker than those on the 2d thoracic segment, and about twice as thick as those on the 1st abdominal segment. The medio-dorsal tubercle on the 8th abdominal segment is much thicker than those near it. The proportionate size of all the dorsal tubercles is as in the previous stage. Spiracles pale brown, not conspicuous.

The black dots low down below the lateral ridge are now nearly obsolete.

Anal legs with no external black spots, and entirely green. The thoracic legs green, with a few black spots.

Mr. Joutel, who had more larvae under inspection than I, says, "All the larvae are gradually losing the black pigment; the dark ones have only the base of the dorsal tubercles on the 3d thoracic and 8th abdominal segments black, with the tips blue; also the lower half of the face is black."

It was found to have moulted for the third time June 23.

Stage IV. Length 23 mm. (when underfed) – 42 mm. The body is now thicker at the 3d thoracic segment, and in fact somewhat so throughout, while the tubercles are smaller, and all but those on the back of the 3d thoracic and 8th abdominal segments are decidedly reduced in size. *The skin is now covered with minute rounded warts or granulations.* The head and body are pale greenish straw-yellow (more yellow than green). The tubercles are of the same hue as in Stage III. The six tubercles on the front edge of the 1st thoracic segment are turquoise-blue.

The two dorsal ones on the 3d thoracic segment are turquoise-blue. The medio-dorsal one on the 8th abdominal segment, and the four on the side of the 9th segment are turquoise-blue, but the two dorsal ones are yellowish, of the same hue as the skin. The lateral series of tubercles are turquoise-blue, as before. On all the tubercles the setae are black, and shorter than the tubercles themselves are high.

The two dorsal tubercles on the front edge of the 1st or prothoracic segment are as large as those on the 2d thoracic segment, *while those on the 3d thoracic segment are nearly twice as high as thick*, also slightly smaller than in Stage III. Each bears six black setae, and an odd 7th bristle in the centre. The setae are not quite so long as the tubercles are thick.

The dorsal tubercles on abdominal segments 1–7 are now small, slender, scarcely half as high as the two on the 3d thoracic segment, and evidently undergoing reduction; each bears three setae of unequal length, two of which are short, and one generally longer, though they vary in length, the longest one being about as long as the tubercle is high. There are six setae on the medio-dorsal tubercle of the 8th abdominal segment.

Suranal plate with a rather stout turquoise-blue tubercle on each side near the middle, each bearing seven short black setae; end of the plate armed with five small setiferous tubercles, and an irregular row of black setae on the edge of the anal legs.

The tubercles of the supraspiracular row are minute, yellowish, and not quite twice as long as thick.

In the few reared by Mr. Joutel the larvæ were all alike, no matter what their color was in the previous stages.

Stage V. and last. Unfortunately my single larva died before assuming its final stage of growth, and I shall have to depend on Mr. Joutel's drawing for the following description. Up to this stage the caterpillar was in all its stages a remarkably striking and exquisitely beautiful one, especially in Stages III and IV, where the tubercles were of such a soft and delicate turquoise-blue, contrasting so markedly with the yellow or green hues of the body.

Length, when at rest, head down, 63 mm.; when extended in the act of walking, 78 mm. The body is now thick and fleshy, destitute of tubercles with the exception of the two medio-dorsal ones on the 3d thoracic segment, and the medio-dorsal one on the 8th abdominal segment, otherwise the body is entirely unarmed, with apparently no traces of the tubercles present in the previous stages. The tubercles on the 3d thoracic segments are (united into one), stout conical, nearly twice as high as thick at the base, and with a conspicuous dark spot in front. When at rest, head down, this horn projects up in a very conspicuous way. The posterior tubercle (medio-dorsal) is small, low, button-like, not conspicuous; on neither of the tubercles are there any setae.

The body is now greenish yellow, with the sutures bathed with yellow. The lateral ridge is also yellowish, and along this ridge is a series of small blackish spots, one to each thoracic and abdominal segment, there being one on the side of the suranal plate. The head, all the legs, both thoracic and abdominal legs, and the under side of the body are pea-green.

Such differences between the last and the preceding stages as are seen in this caterpillar are very unusual. It is evident that the tubercles and bristles were undergoing reduction in the 4th stage, but one was hardly prepared for the loss, without apparent vestiges, in the last ecdysis, of all except the dorsal ones on two of the segments. The cause, if it could ever be explained, would be most interesting.

This case reminds us of a member of the same family, *Rothschildia betis*, but in that form all the tubercles are atrophied, and the change from the penultimate to the last is otherwise probably slight. A more striking resemblance is that to the larva of *Cercophana frauenfeldi* of Chili, of which there is a colored drawing in the British Museum. In this case the 3d thoracic segment is prolonged into a long curved fleshy horn which projects over the head when the latter is retracted. It has no tubercles, and a yellowish stripe extends from the tip of the horn along the side of the body to the end of the acutely prolonged suranal plate.

Its resemblance to the last (4th) stage of *Aglia tau* should also be noticed, though the latter at its final moult discards every trace of the armature of its earlier stages. This larva has what Poulton believes to be a terrifying spot on the side of the 1st abdominal segment. The question arises whether the spot on the front of the thoracic tubercles of *Rhodia fugax* is of this psychological nature, and also whether the dark patch on the front of the body under the horn of Cercophana is deterrent to other animals. All three of these larvae have a conspicuous yellowish or (on Aglia) reddish lateral stripe.

Sound produced by the larva. — Mr. Joutel informs me that this larva in its last stage "makes a squeaking noise by moving its head up and down on the prothorax."

Caligula japonica.

A large number of the eggs of this species were received through the kindness of Prof. C. Sasaki of the Agricultural College, Imperial University, at Tokio. They began to hatch March 31. Many of the eggs contained Pteromalid parasites in the pupa state; these emerged at the same time as the larvae, and probably stung them, since they all died before completing their transformations, though several moulted five, and one six times.

It being too early for the natural food, which Professor Sasaki informs me is the chestnut (though the buds were given them), they ate sparingly at first of willow buds, and afterwards the leaves, so that after Stage II the larvae were apparently undersized, and the measurements here given will probably not apply to those raised in the air in their native country.

Egg. — Cylindrical, more so than usual; each end obtuse and well rounded; the shell is of a peculiar dull whitish-gray hue, irregularly mottled with blackish. The surface is seen under a strong lens to be finely pitted. Length 2 mm.; diameter 1 mm. The hole eaten by the larva for its exit is situated at the extreme end.

Larva. — *Stage I.* Length when hatched 5–6 mm.; width of head 1.01 mm. The head is nearly as wide as the body, jet black. The larva is of the usual Saturnian type, and appears much as in that of Telea and Tropaea; the body cylindrical, rather thick, though a little slenderer than in Telea, etc., and with two separate dorsal tubercles; and is armed with large tubercles, which give rise to radiating dark setae. The tubercles are black like the body.

On March 31 all were black, but on the next day (April 1) in three examples the tubercles were livid white, as also the mid-abdominal legs; while the hairs on the back from the dorsal and supraspiracular tubercles are black, those on the side of the body are pale. Yet these individuals had not freshly hatched, as there were no egg-shells in the box with them.

The tubercles and their size relative to the body much as in other Attacine larvae in their first stage; they are high, well developed. Those of the tergum of the prothoracic segment are minute, arising from a pale plate or area, and are about a fifth to a quarter as large as those on the segments behind. On thoracic segments 2 and 3 and on the abdominal segments all the tubercles of the two dorsal rows are a little higher than thick, giving rise to 8–9 dark radiating unequal hairs, one or two of which are longer, sometimes nearly twice as long as the others; they are on the average about one-half as long as the body is thick. They are minutely barbed, the barbs being short. The two dorsal tubercles on the 8th abdominal segment are no higher than those in front, all being of uniform height. There are hairs on the supraspiracular tubercles.

At the end of this stage (April 16–17) the larvae became 8–12 mm. in length, though the head is no larger than at first.

The body is livid brown-black, but paler on the under side, almost glaucous-green, with a yellowish tinge, so that the lateral or pleural fleshy ridge becomes *a faint greenish-yellow line.* The tubercles are jet-black, and the hairs black, with gray ones intermixed.

Other examples before moulting have a pale dull whitish band extending across the labral region of the head, and *a lateral row of livid pale whitish spots.*

Stage II. Moulted April 16–17. Length of body 11–12 mm., width of head 1 mm., about twice as great as before. Directly after moulting the head, legs, and tubercles are *all* white. The lateral broken line or series of pale greenish-yellow lateral linear spots is *much more* distinct than before, being now conspicuous, while the body is dull livid blackish. Now there are *scattered over the body long white hairs, and similar ones form the central one or two setae in each tubercle;* they are about one-third as long as the whole body, but in the lateral or infraspiracular row of tubercles all the radiating hairs are long and whitish. The white hairs are most abundant on the prothoracic plate and head. The short hairs on the body are still black. Under side of the body livid.

Stage III. Moulted the second time May 1. Length 24 mm.; head larger, width 2.5 mm.

The most decided change is in the prothoracic hairs and those on the sides of the body, which are straw-yellow, while the anal legs are edged with straw-yellow. The longest dorsal hairs are about a quarter to a third as long as the entire body. The body is still rather long and slender, and much more hairy than in the larva of Antheraea. The tubercles are still more prominent, black, the ends and the tuberculets livid. The hairs are long, not stiff, but a little crinkled, and they partially conceal the tubercles. The latter are still all of the same size.

Stage IV. Third moult occurred May 17–20.

Length of body 22 mm.; width of head 3 mm.

There is now a decided change in coloration and markings. The head is nearly as wide as the body, black, the clypeus-anterior white; labrum yellowish; the head armed with long uneven yellow hairs.

The body is still only moderately thick, and is long and slender, not at all like Antheraea in shape; the segments are not convex, but flat, and of the same thickness to the end of the body, but the anal legs are large and triangular.

The tubercles are larger and more prominent than before, and very conspicuous, as they are white or yellowish white. The prothoracic plate bears long hairs, and on this segment there are no decided dorsal tubercles, the vestiges being low and flattened, while the lateral ones are fairly prominent and black.

The 2d and 3d dorsal thoracic tubercles are no larger than the abdominal ones; those of the 3d thoracic segment a little larger, however, than those on the 2d thoracic segment; and those on the 6th abdominal segment a little larger than those on the 7th and 8th segments (one on the 6th is larger than its mate).

The two dorsal tubercles on the 8th abdominal segment are separate, like the others, and show no signs of coalescence; they are slightly larger than those on the 9th segment, which are also separate.

Each dorsal tubercle gives rise to about four radiating stout, sharp, slightly dusky setae, while from the centre rises a very long yellow hair, or more often two or three, usually three such hairs; the skin between the tubercles bears on the top and sides of the body long fine white hairs. The longest yellow hairs are about one-third longer than the body is thick. The area around the base of the tubercles is white, due to crowded flattened snow-white warts, which are crowded around the base of the tubercles and scattered between so that the dorsal region is white, though interrupted; the sides of the body are black, enclosing the dull livid spiracles. The oval flattened tubercles are scattered along just above the pleural swollen yellow line and beneath on the under side, where they become straw-yellow. The tubercles of the 9th abdominal segment bear setae which are mostly black or dark. The tubercles of the supraspiracular row are prominent, but small, pale, and bear 8–9 short pale setae, with a very long yellow one (sometimes two) arising from the centre. Those of the infraspiracular row are about twice as large as those of the supraspiracular series, and bear about nine setae arising from prominent warts, while the setae assume the shape of long uneven yellow hairs.

The thoracic legs are black; the mid-abdominal ones pale greenish-yellow with the plantae reddish; anal legs large, broad, triangular, black, spotted in the centre with yellow and with yellow hairs around the edge, but none arising from the central portion. Suranal plate broad, triangular, a little shorter than broad, roundish at the end, with setiferous white warts, the hairs around the edges yellow; two groups of 7–8 setae, one on each side near the edge, arising from distinct warts; each group, representing a primitive tubercle, is situated a little nearer the base of the plate than the end.

The setae on the supraspiracular tubercles of the 8th abdominal segment are also dark.

In this stage it is a rather conspicuous larva, and a very beautiful one.

Stage V. It moulted again June 3. It is now a most beautiful larva.

Length of body 35–40 mm. Width of the head 4–4½ mm.

Head nearly as wide as the prothoracic segment, greenish straw-yellow, spotted with brown, the sides brown; a V-shaped dark line in front, each side of the clypeus being black.

The body of a beautiful tint of pearly greenish-white; the top of the body being broad, uniformly flattened, the sutures between the segments being shallow. The tubercles are distinct, but of uniform size, and about as high as thick, bearing seven rather stiff setae, one rising from the centre; the setae of the lateral tubercles are twice as long as the dorsal ones. Behind the tubercles each segment is crossed by two transverse impressed lines. On the 9th abdominal segment a few of the radiating setae are blackish or entirely black.

The body is very hairy for a Saturnian, being clothed with long pale whitish-yellow uneven hairs, the longest nearly twice as long as the body is thick; those on the side of the body bright straw-yellow. The lateral ridge and the tubercles of the two lateral rows are straw-yellow. Around the base of each infraspiracular tubercle is a reddish ring.

The spiracles are large, conspicuous, of a rich delicate turquoise-blue. Around each spiracle are several irregular small linear black spots and streaks, which are directed more or less upward and broken up into dots. The thoracic segments are black on the sides low down and on the under side. The separate warts on the sides of the body and on the outer sides of the mid-abdominal legs are rather large.

The suranal plate is rounded, as in the earlier stages, and with the anal legs spotted with black, and clothed with short curly pale yellow hairs.

Thoracic legs brownish; mid-abdominal legs yellow ochre, like the under side of the body, but above the planta greenish-yellow.

Stage VI. It moulted for the fifth time June 12–13.

Length of body 40 mm.; width of the head 4½–5 mm.

The head is now entirely black, except the base of the labrum. The sides of the body (what before was straw-yellow) are now *all black, this* *tint extending far above the blue spiracles.* The upper edge of this black region is irregularly dotted with white; the under side of the body is also black.

The top and middle of the prothoracic plate is white, divided into two halves by the median line. *The dorsal or upper side of all the segments to the end of the 8th abdominal segment is cream-white.* The tubercles are also greenish-white; each bear 6–7 long setae and one or two central ones, which are greenish and as long as the body is thick.

The suranal plate and anal legs greenish-yellow, with black dots and streaks. Thoracic legs chestnut brown: mid-abdominal legs dark green with dark plantae. Mr. Joutel tells me that some of his larvae had an extra stage.

Antheraea yama-mai.

I am indebted to Professor Sasaki for the eggs, which hatched at the end of March.

Larva. — *Stage I.* Freshly hatched. Length 6 mm. Head large, chestnut red. Body thick, of the general saturnian shape of the larva of Telea, etc., at the same age. Body straw-yellow, with a narrow black and a lateral or subdorsal black stripe, the latter a little narrower than the dorsal stripe; this dorsal stripe stops at the median tubercle on the 8th abdominal segment, while behind this tubercle is a transverse black line or suture between the 8th and 9th segments.

The setae or hairs are at first brown, very soon becoming black, but the tubercles at first are all yellow; afterwards they begin to turn black at the base, the ends remaining yellow; at the end of the stage they all become black. Thus, when from 24 to 36 hours old, the two dorsal tubercles on the 3d thoracic tubercle become black, as also the median one on the 8th abdominal segment. There is also a black triangular patch on the suranal plate, and one on the side of each anal leg.

The tubercles of the 2d and 3d thoracic segments are very large and conical; the median tubercle of the 8th abdominal segment is double, high; the two dorsal ones on the 9th segment are high, and the supraspiracular tubercles are prominent.

In the freshly hatched larva the black dorsal or heart line is heavier and more distinct than in the larva at the end of this stage.

Cervical plate of the prothoracic segment gamboge-yellow, the front edge around each submedian tubercle whitish.

The hairs above are brown-black, while low down on the sides of the body, on the head and prothorax, they are whitish.

The dorsal setae are rather stout, with slightly marked short spinules, while the setae of the lateral tubercules (supraspiracular and infraspiracular series) are slenderer, with longer delicate spinules, and longer than in *Telea polyphemus*, Stage I, being about three times as long as the tubercle itself.

Brahmaea japonica Butler.

A number of pupae from Japan of this interesting monogeneric type were obtained by purchase, and Mr. Joutel kindly mated the moths, thus obtaining a supply of the eggs, from which the larvae were reared by him and myself in all stages. The eggs were laid in April, and the larvae hatched April 27, while I received them alive (four examples) April 28. They fed on the privet and lilac, and throve well in Providence on the latter plant.

Eggs. — Rather high, somewhat conical, with a broad, flattened base and the apex a little depressed. It thus differs from the somewhat flattened eggs of Ceratocampidae, and from the ovo-cylindrical eggs of the Saturniidae; apparently presenting striking family differences. The shell is seen under a strong lens to be finely pitted.

Larva. — Stage I. Length 9–12 mm. Width of head 1⅕ mm. Head small, no wider than the body, smooth, jet-black, polished and shining; with scattered long hairs. Body elongated, rather slender compared with Ceratocampid larvae of this stage; the body does not taper behind. On the trunk segments (thoracic and abdominal) are six rows of small black tubercles scarcely higher than they are thick, and giving rise to from 7 to 8 black hairs about as long as one-third the diameter of the body. The tubercles on the 1st thoracic segment are no larger than those on the abdominal segments; on this segment they are situated on a black cervical shield, forming a transverse black band, somewhat contracted in the middle.

Four large, long, thoracic dorsal horns, all of equal length, a pair on the 2d, and another pair on the 3d thoracic segment. They are each about one-fifth to one-quarter as long as the body, all of the same shape and thickness, scarcely tapering to the end, which is blunt, black-brownish at the end, flexible, not stiff, rigid and chitinous, and furnished with numerous long fine hairs, which are of nearly equal length; those at the end almost forming a pencil or brush; each hair very finely spinulated.

The horns themselves are of uniform width, not varying in diameter, and with a twisted appearance, as in Stages II. to IV.

A large erect caudal horn on the 8th abdominal segment, a little stouter and not quite so long as the thoracic ones, but like them thin-skinned, hairy, not tuberculated or twisted as in the penultimate stage.

The dorsal tubercles on the 9th abdominal segments are of the same size and shape as those of segments 1–7.

Suranal plate small, triangular, black.

On each side of the base of the plate are two high erect horns, like those on the 2d and 3d thoracic segments, but about one-half as long, though of the same shape, thin-skinned, flexible, and with similar black hairs.

The spiracles are black and inconspicuous.

Coloration: The body is dull black, except the thoracic segments and end of the body. The thoracic segments are livid yellow, with black patches on the sides; in some individuals the first thoracic segment is black, with a small yellow spot above. There are two dark roundish dorsal spots on the hinder edge of the 2d and 3d thoracic segments. Along the body is a faint yellowish spot (sometimes reddish or rusty yellow) on each side above the legs. The 8th and 9th abdominal segments are pale ochreous yellow; the base of the caudal horn and suranal horns black.

The distinctive colorational feature is the pale steel-blue ring or band passing around the body in the middle of each segment and the dark sutures, making about thirteen bluish rings in all. Of these dark bands, that on the 1st abdominal segment is the widest, the corresponding ones behind, known by enclosing the spiracles, being a little narrower. The narrow bands are on the sutures between the segments.

The thoracic and abdominal legs are dark, except the front of the anal legs, which is yellowish.

In this stage the caterpillar is a very conspicuously marked and colored one, with a formidable armature in appearance, but the high slender horns are not stiff and spiny, only hairy. It remains to be seen whether this caterpillar is inedible by reason of some poisonous or nauseous secretion.

The larvae at Providence moulted May 8 or 9, having been in Stage I about 10–11 days.

Stage II. Length 12–15 mm. Width of head 1¼ mm. It does not now much differ in the general proportions and length of the horns from the previous stage, though these are now *hairless and twisted* and somewhat longer, *but the body is now of a porcelain white, and the bands, now brown-black, are broken up into black spots.* All the horns are now longer, smooth, without the hairs of Stage I, while they appear as if slightly twisted, being enlarged at irregular intervals and there giving off a minute seta. The head is still black; there are two white longitudinal bands, quite irregular in length and width, on each side of the head. The antennae are black and the region around their base is black. The clypeus-posterior is black with two yellow spots in the centre. The labrum and the region on each side is black. The head is fully as wide as the body.

First thoracic segment straw-yellow, with four dorsal black spots whitish in the middle above; two black spots on each side. On the 2d thoracic segment are two larger black spots, and two larger ones on the 3d segment. On each of abdominal segments 1–7 are five black dorsal spots, the black bands now being divided into spots; on segments 8 and 9 two dorsal black spots, and the front edge is black. The 8th segment is orange-reddish, becoming deeper on the sides. The sides of the abdominal segments below the spiracles are orange-ochre, on the thoracic segments the corresponding region is yellowish. On the 9th segment are two dorsal black spots, and on the front edge five black spots.

Suranal plate pale straw-yellow, black at the end; there are two small black dots on the front edge. Each abdominal segment with eight black spots on each side of the median spot, making seventeen spots in all.

Anal legs white, with five black unequal vertical stripes, the edge of each stripe black all around. Mid-abdominal legs dark brown, with a black ring at base, and another just above the planta.

The skin is provided with very fine short scattered dark hairs, those on the mid-abdominal legs much longer than those on the body.

Stage III. Length 17–27 mm. Width of head $2\frac{2}{5}$ mm. The head is mostly black, but with white lines so disposed as to break it up into five irregular black lines or spots on a side; the two on the vertex, one on each side of the median line or suture of the head, above the apex of the clypeus, each forming a short curved band; the two lowest black spots down on the side near the ocelli are triangular. Clypeus black, the head being white on each side next to it. The anterior division of the clypeus is white, as also the base of the antennae. Toward the end of the stage the white lines of the head become wider, so that the black areas are somewhat reduced in size.

The body is pearly white, with a larger portion white, the black spots being smaller and more numerous than in Stage II.

Prothoracic segment with only two dorsal spots instead of the four black spots of Stage II, the two dorsal ones conspicuous and elongated, diverging from each other.

The 2d and 3d thoracic and 8th abdominal segments, and the suranal plate at the base of the two horns, are now distinctly enlarged or swollen, remaining so through Stage IV, when they are more conspicuously inflated, much as in *Arsenura xanthopus* as figured by Peters. All the horns are much as in Stage II, being swollen where a seta is given off, and more or less crumpled, kinky, or twisted. Length of thoracic horns 10 mm., being fully one-half as long as the entire body; length of caudal horn 6 mm.; of the two suranal horns 3½ mm.

The black patches more numerous and smaller; those on the tergum are now broken up into separate spots. There are now on each abdominal segment 1–7 three rows of lateral black spots, and a conical one just behind the middle of each segment, or seventeen spots on each segment. Of these three rows the lowest one is the spiracular one, the black spiracle being situated in the centre of the middle one of the three spots forming the line.

On the 8th abdominal segment are four black spots on each side, one enclosing the spiracle, and a median one behind the caudal horn. On the 9th segment are six black spots on each side and one in the middle, or thirteen in all. On the front edge of the 2d and 3d thoracic segments are two black spots.

The suranal plate is smooth, white, unarmed, no setae, only the two black horns; the end of the plate is black.

There is a group of about a dozen minute short fine setae on each side of the tergum of abdominal segments 1–7, marking the probable site of tubercles in the ancestors of the group.

Above each mid-abdominal leg is a black spot, while the legs themselves are blackish at base and near the planta, the basal spot enclosing a white patch; the hinder part of the leg is marked with obscure greenish yellow. Anal legs black, enclosing three unequal vertical white stripes. Thoracic legs black.

Under side of the body livid pale, with a wide black lateral and a black middle line.

This description will almost equally well apply to Stage IV, the two stages being much alike in coloration and armature.

Stage IV. Described May 16. The length of this stage was 4–5 days. Length 42 mm.; width of head 2⅗ mm.

The body is now still long and slender, and of the same width throughout. On the dorsal side it is of a shining bluish white with a yellow tinge. Four black dorsal dots on the prothoracic segment arranged in a square, and two larger black spots in front of each pair of horns; on each abdominal segment three or four minute dorsal black dots. On the side of each segment are 7–8 black spots which are arranged in two rows, one (containing the larger number of spots) straight, including the black spiracles, the other broken up into an oblique row of about three spots on one side of each segment and passing up the front edge of each segment. Further down on each side is a large black spot over the base of each mid-abdominal leg.

The head is striped with black, with five irregular unequal short black lines on each side, while there are two black patches next to the antennae.

The horns are now long and slender, a little tapering, but blunt at the tips; those of the 3d a little longer than those of the 2d thoracic segment, *and between a third and* a half as long as the body; *the caudal horn* is about one-fourth as long as the body, while those of the pair on the 9th segment are slightly more than half as long as the caudal one. The horns are all more decidedly curled or twisted than before, and the fine spinules more developed.

On abdominal segments 1–7, in the place where tubercles should be, is a group of ten minute very short somewhat scattered setae, indicating that they may be the vestigial remains of a tubercle; there are two similar groups on the 9th abdominal segment, of six similar fine setae.

End of the suranal plate black; the surface smooth, not finely tuberculated.

Under side of the body pale greenish, with a median interrupted black line, and on each side higher up is a heavier black stripe, more or less undulating, and on the abdominal leg-bearing segments they are slightly oblique, ascending toward and reaching the front edge of the segment.

It moulted about May 20–21.

It is noteworthy, that on the front edge of the prothoracic segment there are six vestigial tubercles, indicating the descent of this genus from ancestors with six tubercles. The two dorsal ones form a very small, nearly obsolete, slightly marked transverse rough area; but in front of the spiracle is a large low rough slightly raised round area with very short fine setae; below, above the base of the legs, is a more prominent brown one. The corresponding ones on the 2d and 3d thoracic segments are still less marked, being only faintly visible.

Stage V. Length 63–76 mm., width of head 6⅔ mm. *The body is now without horns, and is smooth*, cylindrical, a little flattened and naked, with *minute tubercles in place of the horns.* The head is flattened in front, a little angular on the sides; the surface rough and straw-yellow, with five black lines of uneven width and length, a black band ending on the eye-area, there being in all eight lines and spots. Clypeus whitish, with two semicircular cross bands. There is a whitish patch in the middle of the vertex, and a whitish area on the sides.

The body is less cylindrical than in the previous stages, being a little flattened, and has the general appearance of a hornless Sphingid caterpillar.

The ground color is a delicate greenish pearly or porcelain white, bathed on each side of each segment and along the hinder edge of each segment with pale straw-yellow. On the smooth prothoracic segment are four black dorsal spots, two round ones in front, and two large ones behind; a lateral long stripe behind the spiracle. There are no marked dorsal spots; those of the oblique lines are minute, and the black marks are chiefly confined to the spiracular series.

On the 2d thoracic segment in place of the two horns are two low, rather large fleshy tubercles which are a little larger but not so high and prominent as those on the 3d thoracic segment, though the two latter are less broad at the base.

On the 8th abdominal segment, in place of the atrophied caudal horn, is a low inconspicuous tubercle, at the apex of which is an indistinctly double (?) little wart.

On the 9th segment are two slightly full, low swellings with seven or eight short minute setae, indicating that they were the site in the ancestral form of this genus of two horns, warts, or tubercles.

Suranal plate black at the end, which is somewhat rugose. Surface of plate whitish; on the sides and behind pale straw-yellow. On the site of the two horns of the previous stages are two rounded conical black tubercles which are more solid and chitinous than those in front, and more like vestigial horns.

On the side of the segments is a black spiracular very irregular and much broken line, the spiracles being black; above this line on each segment are three black spots forming an oblique line, the most anterior spot (forming the upper end of the line) being near the front edge of each segment. On each abdominal segment except the 10th (suranal plate) are two black dots situated near the front edge of the segment. Below the spiracular line the body is deeper, almost orange-yellow.

Thoracic legs pale, with black wings and spots, mid-abdominal legs dull livid green with a black narrow wing at the base, and another one above the planta.

Under side of the body deep salmon red, with an irregular black line on each side, and on this line and in towards the middle of each segment are small setiferous black tubercles, white at the apex, and giving rise to a very short seta. Anal legs fairly large, and sphingiform.

Habits. — The larva is active in its movements; when rudely poked or handled it will suddenly jerk its head as if offended, and eructate a portion of its partially digested food. It assumes a decided sphinx-like attitude.

Its larval life extends over about five or six weeks or 35–45 days. It moults four times; the first stage occupies about 9 or 10 days, the second about 6 or 7; the third and fourth stages about 4 or 5 days; and it remains in the last stage about a week (7–9 days); the eggs hatched May 27, and one larva pupated June 11–12.

Acceleration of characters after Stage I. — It is to be observed that the armature and coloration are the same in Stages II–IV. It is a rule in Ceratocampidae and Saturniidae that the characters of the last stage are not assumed until after the 2d moult, but here those of the penultimate stage are assumed after the first moult, and in Stage IV there is no sign of the atrophy of horns, and of the color differences of Stage V.

The larva malodorous and its colors warning. — Larvae of the last stage on being handled were observed to emit a peculiar strong odor, a little like musk, and it did so on several occasions when disturbed or handled.

This fact, that the larva throws off a bad smelling or repugnatorial odor, and that it also vomits its food on being disturbed, should be coupled with the fact that it is a conspicuously marked caterpillar in all stages of existence, both when ornamented with long horns at each end of the body, and when after its last moult it is smooth-bodied, without even a caudal horn. The caudal horn of the earlier stages shows no sign of a bituberculous origin.

Analogies to Ceratocampidae, and to Sphingidae. — On a first glance at the moth, one would naturally suppose that it had no relationship to either of these two families; the shape and peculiar markings of the wings are so unlike any of them, but a knowledge of the early larval stages, and of the pupa, with its subterranean habits, led us to examine its structures and affinities, and at first we supposed that this monotypical group had descended from the Ceratocampidae and forms a side branch.

The larva in its third and fourth stages closely resembles that of Arsenura in the same stages, while the larvae of the two genera are similar in the final Stage V, but these resemblances are only analogies, not true affinities.

A pair of locomotive tubercles or claspers on the 2d abdominal segment. — Mr. Joutel has called my attention to a pair of soft tubercles or claspers on the 2d abdominal segment "which is thrust out and seems to be used in walking as though it were a clasper."

On examination of alcoholic specimens of Stages IV and V, I find on the 2d abdominal segment a pair of small tubercles situated in a place exactly corresponding to that of the mid-abdominal legs. Each one is about one-eighth as large as a mid-abdominal leg; about as high as broad at the base, rounded, soft, and giving rise to 6 or 7 black setae of uneven length, but with no definite arrangement; the tubercles are situated in the path of the latero-ventral black line. On the 1st abdominal segment is a minute soft low flattened tubercle situated in the same relative position, but wanting in Stage IV. Are these processes to be compared with the supernumerary legs of Megalopyge? Are they the survivals of primitive abdominal legs?

Pupa. — In shape and size closely similar to that of *Eacles imperialis.* It agrees with the head-characters, i. e., the pupae, eyes, antennae, legs, and wings, differing only in lacking the short stout spines which beset the head and thorax of Eacles, the body being smoother and somewhat polished; the hinder edge of the free segments (abdominal) is smooth, not spinose, as in Eacles. The chief difference consists in the shape of the cremaster, which instead of being narrow, slender, and flattened, is very stout, somewhat conical, much contracted at its base, and with coarse pits, some of which are confluent, forming deep smooth furrows or channels. The cremaster ends in two smooth cylindrical spines. Length 40 mm.; thickness of body 13 mm.

Variation in the terminal spines of the cremaster. — In one example there is but a single median spine at the end of the cremaster, with no vestige of its mate, the spine being central.

When one comes to the moth one would hesitate at classifying Brahmaea with the Ceratocampidae or the Saturnians: its general appearance forbids this. The wings of the Asiatic species are not falcate, and the mode of coloration of the Brahmaeidae is marked by the extraordinary development of undulating lines or bars, which remind one of the geometrid *Scotosia undulata.* Evidently its style of protective coloration, the green, black and gray tints and lines, enable it while resting on the bark of a lichen-covered tree to elude observation.

The venation (♂ and ♀), however, while presenting some important Ceratocampid characters, indicates that Brahmaea should be assigned to an independent family. The first subcostal vein (III_1) is very short, arising very near the apex of the wing, far beyond the origin of the 3d subcostal (III_3). The 2d median (IV_2) forms an independent vein, as in Ceratocampidae. The discal cell is very small, and the discocellular veins form a reëntering angle. In the hind wing the discal cell is remarkably short, not much longer than wide. The subcostal vein (II), instead of being remote from the subcostal as in all Ceratocampidae, is very close to the subcostal, nearly touching it a little beyond the discocellulars.

The ♂ genital armature is of the Ceratocampid type; the suranal plate being large, broad, slightly bilobed at the end, while the claspers are very broad, rounded at the end, and not mucronate.

While *B. certhia* is regarded as the type of the genus, it, or any of the Asiatic species, scarcely appears to be the stem-form. For this we shall have to turn to the West African *B. lucina*, in which the fore wings are produced towards the apex which is somewhat falcate, and the markings suggest the South American Arsenura; the apical oval dark spots recall those of that genus.

Although there are certain striking superficial resemblances in the larva and pupa to the Ceratocampidae, the antennae are, however, as in *Bombyx mori* and Lasiocampidae, there being but a single pair of pectinations to a joint. The larva before the first moult is as in *B. mori* and Endromis, the flattened tubercles giving rise to several (6–10) setae. The head is as in *B. mori*, the epicranium short and broad, the clypeus sunken; the suranal plate as in *B. mori* and Lasiocampidae. The Brahmaeidae should be associated in a group for which we propose the name *Symbombycina*, with the families Bombycidae, Endromidae, Lasiocampidae, Liparidae, and Eurypterotidae.

We have, from our studies, been led to infer that the original home of the Brahmaeidae may have been in Africa south of the Sahara region, and that the Asiatic and southeastern species are derived from African forms. They are certainly rather more modified, the wings shorter and broader, and the markings more specialized than in the African *B. lucina*.

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY
OF HARVARD COLLEGE.

THE SIGNIFICANCE OF CHANGING ATOMIC VOLUME.

IV. — *THE EFFECTS OF CHEMICAL AND COHESIVE INTERNAL PRESSURE.*

By Theodore William Richards.

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THE SIGNIFICANCE OF CHANGING ATOMIC VOLUME.

IV.—THE EFFECTS OF CHEMICAL AND COHESIVE INTERNAL PRESSURE.

By Theodore William Richards.

Presented February 10, 1904. Received February 3, 1904.

In the previous papers[*] upon this subject, it has been shown that there are many facts pointing toward the existence of a causal relationship between the volume occupied by solids and liquids and their internal pressures of cohesion and chemical affinity.

If a relation between the volume and internal pressures really exists, the volume occupied by the gram-equivalent of a solid becomes as important and essential a property of material as any other property.

In attempting to explain the facts with the help of the atomic hypothesis, there are two possible alternatives, — either a relatively incompressible atom exists within a compressible space, or else the atom itself is compressible, and in a solid or a liquid comes into contact with its neighbors. The former hypothesis is the one usually accepted; but it seems to me to be hampered by several insuperable difficulties. The nature of the supposed empty space between the molecules of a solid is not easy to imagine; for if it is due to the impact of the thermal vibrations of a small, hard particle, the difference between a solid and a gas is not sufficiently explained. Moreover, even at the lowest attainable temperatures, where the thermal vibration must be scarcely perceptible, ice occupies only about half the volume of the solid oxygen and hydrogen from which it may be made.[†] Such a large change in volume seems incredible, if it is imagined to be due to a restriction of thermal vibrations already almost infinitesimal.

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[*] Richards, These Proceedings, **37**; 1 (1901), 399 (1902); **38**, 293 (1902); also Zeitschr. phys. Chem., **40**, 169, 597 (1902); **42**, 129 (1902).

[†] Dr. Frederick Soddy has kindly called my attention to this circumstance, the facts having been reported by Professor Dewar in an Address at the British Association, 1903.

These facts add strong support to the many-sided inference drawn from other facts that some repulsive tendency other than heat-vibration must be the cause of the permanent volume of solids and liquids — some tendency which is an intrinsic property of the atoms themselves. If so, this property must exist in the atoms of a gas as well as in those of a solid and liquid, and must also in a gas prevent the actual collision of the atomic centres, and thus take part in the "covolume" of the equation of van der Waals. This conclusion is not in any way inconsistent with the kinetic theory of gases, — in a gas the compressible molecules must still be supposed to be separate and to produce pressure by the momentum of *their impact. In this connection* it might be well to call attention to other evidence to be drawn from the viscosity of gases, that the bulk of polyatomic gas-molecules must be of the same order as that of monatomic gas-molecules. Noyes and Goodwin * found that the viscosity of mercury vapor seems to show its molecule to be about the same bulk as the molecule of carbon dioxide, and they sum up their investigation with these words: "These results indicate that the *atoms and molecules are of the same order of magnitude*, and that the spaces between the atoms within the molecule, if any exist, are not large in comparison with those occupied by the atoms themselves."

In short, the "atomic environment" is really the volume which comes into consideration in all cases where the atomic volume is to be considered at all, and hence should be considered as the true volume of the atom. The conception of a hard, central particle is unnecessary, and assumptions concerning the supposed volume of the atomic centre must be of the most vague and hypothetical character, since this supposed volume never comes within the range of direct relative measurement. Since the measurable atomic volume is compressible, it is reasonable to assume that the atom is compressible, and that this atomic elasticity affords the possibility of heat vibration in solids and liquids composed of contiguous particles.

Some of the many other arguments in favor of this point of view have been already recounted in the three previous papers, and need not be recapitulated here. The object of the present paper is to bring a greater degree of numerical accuracy into the interpretation of the facts, and to show that the more closely the data are scrutinized, the more emphatic is their support of the somewhat revolutionary hypothesis of atomic compressibility. The fact that this should be the case is in itself a strong support of the hypothesis; for fallacious arguments lose weight on closer scrutiny instead of gaining weight.

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* A. A. Noyes and Goodwin, These Proceedings, **32**, 235 (1897). Professor Noyes has kindly called my attention to this work. See also Lord Rayleigh, Phil. Trans., 186 A, 187 (1895). Schultze Drude's Ann. (4) **5**, 140 (1901).

As has been already pointed out, the chief omission in all other work on the subject of changing atomic volume * has been the lack of attention to the compressibilities of the substances concerned. If the pressure of affinity causes a compression of the atomic volume, it is clear that a given affinity will produce a greater change of volume when the atomic volume is easily compressible than when it is only slightly compressible. This matter will be considered at greater length in this paper than has ever been possible before, since a number of appropriate compressibilities have just been determined in this laboratory for the first time, as well as several especially important specific gravities.

Another question, which must also receive consideration, is the varying intensity and nature of the affinities causing combination and cohesion. Do *both* of the two affinities, the chemical and cohesive, which together determine the stability of solids and liquids, exert internal pressure; and what effect does each of these have upon the volumes of substance?

These questions are so closely related that they may best be discussed in a single thesis. Indeed, owing to the complexity of the subject and the scarcity of exact data, the verdict of each answer is needed to sustain that of the other.

In the first place, the question of compressibility may be taken up.

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* It is a pleasure to call attention here to the entirely independent work of J. Traube in this direction. By an interesting coincidence he published in Drude's Annalen, **5**, 550 (June 20, 1901), the following statement: "Das Atomvolumen eines Elementes ändert sich vielmehr von Stoff zu Stoff; es ist um so kleiner, je grösser die Anziehung zu den benachbarten Atomen ist," while on June 15 of the same year there appeared in the Proceedings of the American Academy, **37**, 17, my version of the same relationship: "*The atomic volume is not constant, but a function of pressure and temperature and probably of electric stress.*" By *pressure* was meant the internal pressure caused by affinity. In his pamphlet, "Uber den Raum der Atome" [Ahrens's Sammlung. chem. und chem.-techn. Vortage IV. 256 (Stuttgart, 1899)] Traube anticipated several points contained in his paper of 1901, and in my work in the same year, but this pamphlet was wholly unknown to me at the time. The fabric of Traube's reasoning is complicated by his hypothetical assumptions of "Covolumen" and "Kernvolumen," "geburdener" and "freier Aether;" but nevertheless he deserves the credit of having appreciated the importance of many of the facts. The question of priority is of little consequence, especially since Müller-Erzbach (1881) had priority over both.

The effect of varying compressibility is best studied in series of elements possessing large compressibilities, for in these cases the variations are more evident. Hence there were determined for this purpose the compressibilities of chlorine, bromine, and iodine by Richards and Stull, and of lithium, sodium, and potassium by Richards and Bonnet.*

The values thus found, expressed in fractional decrease in volume for an increase of pressure of a kilogram per square centimeter, are given below: —

		Specific Gravity at 20°.	Atomic Volume.
Chlorine	[0.00011]	1.412	25.0
Bromine	0.000061	3.121	25.6
Iodine	0.000013	4.940	25.7
Lithium	0.000011	0.552 †	12.7
Sodium	0.000018	0.972 †	23.7
Potassium	0.000032	0.870 †	45 0

The value for chlorine is only an approximation, being computed from that of chloroform, carbon tetrachloride, and other analogous compounds, but it is sufficiently exact for the present purpose. These values justify the previous choice of chlorides, bromides, and salts of potassium in order to show in as marked a fashion as possible the effects of affinity in causing volume change, since the three elements, chlorine, bromine, and potassium, are unusually compressible. In a previous paper ‡ the choice of these three elements was made on the basis of their large coefficients of thermal expansion, for this property is more or less closely associated with compressibility, as the rule of Dupre § indicates.

A cursory study of the above table of compressibilities leads one immediately to predict that, with a given affinity, a greater change in volume would be caused in the formation of a salt of potassium than in one of sodium, and a greater change in one of sodium than in one of lithium. If we use heats of formation as indices of the free-energy change, or the work which may be done by affinities (a proceeding which is permissible in the case of these simple binary salts because of the small change of heat-capacity which occurs when they are formed from the elements*), we find that this prediction is fully

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* Of these investigations the first has already been printed in Publication No. 7 of the Carnegie Institution of Washington, and the second will soon appear in the same series.

† The specific gravities of these three alkali-metals were determined by Richards and Bonnet, and will soon be published.

‡ Richards, These Proceedings, **37**, 399 (1902).

§ See, *for example*, Ostwald, Lehrbuch, **1**, 394 (1891).

	Specific Gravity of Salt.	Sum of Atomic Volumes of Elements.	Molecular Volume of Salt.	Contraction, C.	Heat of Formation, H.	Quotient, $\frac{C}{H} \times 100$.
LiCl	2.07 †	37.7	20.9	16.8	383 ‡	4.3
NaCl	2.14	48.7	27.2	21.5	399	5.4
KCl	1.94	70.0	37.8	32.2	427	7.6
LiBr	3.46 †	38.2	25.2	13.0	334	3.9
NaBr	3.08	49.2	34.2	15.0	359	4.2
KBr	2.72	70.5	44.2	26.3	398	6.6
LiI	4.05 †	38.4	33.1	5.3	257	2.1
NaI	3.65	49.4	41.4	8.0	289	2.8
KI	3.07	70.7	53.8	16.9	335	5.1

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* Richards, These Proceedings, **38**, 293 (1902). In the last paper upon this subject, it should have been made clear that the statement concerning the possible application of the heat-capacity relation to electrolytic dissociation is applied to very dilute solutions only by extrapolation, since no data exist for the heat-capacities of solutions more dilute than half normal. Professor Nernst has kindly pointed out to me that in case this relation really holds in very dilute solutions the coefficients obtained by him and by Jahn would acquire a special significance, their application to the facts remaining unchanged. This circumstance has nothing to do with the cases considered in the present paper; it is alluded to now only because I regret that in the previous discussion a possible inconsistency was suggested, which does not really exist.

† These results are due to careful experiments very kindly made in this laboratory by Dr. G. P. Baxter. The results of F. W. Clarke are undoubtedly too low, since the salts are hard to prepare pure. The value of the specific gravity of lithium, 0.552, is due to experiments made here by Richards and Bonnet.

‡ Because the heats of formation of chlorides as given in the text-books always refer to gaseous chlorine, there must be subtracted from them the latent heat of evaporation of liquid chlorine. The molecular latent heat of evaporation of chlo-

borne out by the facts concerning either chlorides, bromides, or iodides of these three metals. In every case the value of the fraction

$$\frac{\text{change of volume}}{\text{heat of formation}}$$

grows smaller as the compressibility of the metal is less. In other words, a given energetic effect is shown to produce a smaller change of volume whenever the compressibility is smaller. The facts are given in the tables below. The first table compares the different metals combined with a given halogen.

As has been said, the prediction is fully verified. The quotient of the contraction divided by the heat of formation is always smaller when the compressibility is smaller. Exactly the same phenomenon is to be observed if the several halogens be varied, in combination with a given metal.

	Contraction, C.	Heat, H.	Quotient, $\frac{C}{H} \times 100$.
LiCl	16.8	383	4.3
LiBr	13.0	334	3.9
LiI	5.3	257	2.1
NaCl	21.5	399	5.4
NaBr	15.0	359	4.2
NaI	8.0	289	2.8
KCl	32.2	427	7.6
KBr	26.3	398	6.6
KI	16.9	335	5.1

rine has been found by Knietsch to be 18.6 kilojoules (Z. elektrochemie, **9**, 847, 1903), a number not far from that (20.5) calculated in the record of the present series of papers, and this amount is subtracted from all the commonly accepted heats of formation of chlorides. In the case of single gram atoms of chlorides only half of this quantity is subtracted.

It is worthy of note that these thermal quantities also contain the "heat of cohesion" or latent heat of liquefaction and solidification of the products. If the compounds did not condense, the heats of formation would be less. This cor-

Thus, the diminution of the quotient always accompanies a diminution in compressibility, as would be expected on the basis of the hypothesis. The nature of the case prevents a more exact mathematical treatment of these expected variations, but the unanimous verdict of this approximate treatment of them advances the agreement of fact and hypothesis to another order of approximation, in which not only the qualitative succession of magnitudes possess significance, but also the deviations from direct proportionality. Thus is afforded a strong presumption in favor of the important significance of changing atomic volume.

Many other cases of the same kind might be cited, but occasionally an apparent exception appears. For example, silver can hardly be much more compressible than zinc, and yet during the union of a mol of chlorine with the former metal a contraction of 19 milliliters occurs, while with the latter the contraction is only 13 milliliters, although the heat of formation in the latter case is almost twice as great as in the former.*

At first such an exception as this seems a serious impediment to the acceptance of the underlying hypothesis of compressible atoms, but further consideration shows at once the reason of the seeming anomaly, and, instead of being an exception, the case becomes a supporting example.

It will be apparent below that such cases are easily explained by the plausible assumption that more than one kind of internal pressure exists in a solid compound, — not only are elements probably compressed by their chemical attraction or affinity, but also the smallest particles of the resulting compound exert pressure in cohering under the influence of a cohesive attraction of less intensity.

In endeavoring to imagine the mechanism of the action of these forces, it is necessary to amplify the ordinary conception of atoms, and endeavor to imagine the behavior of each single atom under the action of the stresses to which it must be subjected. Probably each atom in such a solid as argentic chloride would be bound on one side by chemical

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rection applies so nearly equally to all that it may be neglected for the present, however.

* The molecular volume of argentic and zincic chlorides are respectively 25.8 and 46.9, and those of the metals 10.3 and 9.5, while the respective heats of formation (corrected), are 114 and 384. Since the contraction of the chlorine is probably the chief factor in the question, equivalent quantities of the two materials must be compared; therefore, in the calculation double the data of silver should be taken to correspond to one of zinc, because zinc is bivalent.

In the previous paper where this was discussed (the second of the series) a careless mistake in calculation of the silver chloride exaggerated the discrepancy.

affinity — thus forming the smallest particle or molecule of the compound — and on its other sides by a weaker affinity, causing the cohesion to the atoms belonging to other molecules — thus forming a solid mass of many molecules. Every one will grant that there are many reasons for believing that this cohesion is often much feebler than chemical affinity. For example, the former is often far more easily overcome by the introduction of heat energy than the latter. This is the case with zincic chloride, for example, a salt which boils unchanged at a temperature as low as 710°, giving a vapor which seems to possess the simplest possible formula, $ZnCl_2$.

To those who are indisposed to consider seriously any hypothesis concerning the different sides of a single atom, it is only necessary to point out the benefit which has already been derived from the stereochemical hypothesis of van't Hoff and Le Bel. The present discussion carries the logic of three dimensions into another division of chemical argument.

If one side of an atom is more firmly bound than another, it is only reasonable to suppose that the more firmly bound portion is subjected to the greater pressure, and therefore compressed into the smaller volume or more flattened. It seems probable that this compressed portion must be less compressible than the portion less firmly bound, because universal experience shows that with a given substance at a given temperature, the greater the volume the greater is the compressibility. In other words, that part of an atom which is under less pressure must be more compressible than the part which is under greater pressure. Hence, a small development of energy by a change of a given small cohesive pressure for another cohesion slightly greater would correspond to a large contraction, equal to the change of volume caused by a great change in a pressure of chemical affinity already large. Thus the molecular contraction caused by the compression of cohesion may at times quite mask the atomic contraction caused by the far greater pressure of chemical affinity, because the latter is concerned with that portion of the atomic volume which has been previously compressed. There is nothing in this argument to show that the shape of the atom is *permanently* deformed or irregular, as some have supposed; it seems much more probable that in the free state every atom is spherical, and that the inequalities in the atomic surface occur only under the influence of unequal pressures, such as those which form the subject of this paper.

These considerations enable us to interpret at once the seeming anomaly concerning zincic and argentic chlorides. Zincic chloride is easily volatile, boiling, as has been said, at 710°, while argentic chloride was found by Biltz and Victor Meyer* to be non-volatile at 1400°, and only slightly volatile at 1700°. The high boiling point of argentic chloride is one of the indications of great molecular cohesion,† and the pressure of this cohesion must produce great compression of the chlorine. Therefore we may refer the unusually great contraction taking place during the formation of argentic chloride chiefly to the physical intermolecular pressure and not so much to the essentially chemical interatomic pressure.

Even this, however, is not a complete treatment of the matter, since the metals themselves have different boiling points, and hence are probably under different internal pressures. Silver itself must be more compressed by its own cohesion than zinc is, for Nernst has shown that the former metal probably boils at a point above 1950°‡ This condition in silver must tend to counterbalance in part the effect of the great internal pressure in argentic chloride, for if silver is already more greatly compressed, it could not be so much changed as zinc by additional pressure. The much greater compressibility of chlorine is undoubtedly the reason why the great cohesion of argentic chloride diminishes the volume of a salt to a greater extent than the great cohesion of the silver diminishes its own volume.

The complication of these various conflicting tendencies is no argument against the validity of the reasoning; on the contrary, any pertinent hypothesis which does not consider them is incomplete. The volume-change occurring when two solids or liquids combine is obviously an average or composite change, depending upon the extent of both the chemical and physical compression of the factors which must be overcome, as well as upon both the chemical and physical compression of the product, — just as the heat of reaction concerns the heat absorbed in disintegrating the factors, as well as the "chemical heat" and the "latent heat" given out in the act of combining and solidifying. These considerations were neglected in comparing above the cases of haloid salts of the three alkali-metals, since these salts are very similar, and their boiling points probably not far apart.§ Moreover, the metals, too, are very similar. Hence the effect of cohesion was largely eliminated and the remaining effect could be safely ascribed to action of the pressure of chemical affinity on the different compressibilities of the elements compared in each series.

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* Ber. d. deutsch. ch. Ges., **22**, 727 (1889).

† Ostwald gives a table illustrating Trouton's rule (Lehrbuch, **1**, 354), which shows that with nearly related substances the boiling point is very nearly proportional to the molecular latent heat of vaporization, and that even with very different substances this is approximately true.

‡ Z. Elektrochem., **9**, 62 (1903).

§ The boiling points of these salts have not been accurately determined, but qualitative experiments of my own in tubes of fused quartz showed that all the

The thorough understanding of this matter is so important that it is worth while to cite other cases illustrating it. A particularly illuminating series of examples is that of the halides of zinc and cadmium. The bromides and chlorides of both these metals, as well as the metals themselves, may be easily volatilized, and their boiling points are well known; hence more is known in these cases than in almost any other concerning the relative intensities of the internal physical pressures. The work of chemical compression is again taken as equal to the heat of formation, because only a small part of the heat of formation is usually due to the heat of molecular cohesion, and the change of heat-capacity is small.

Below are given the necessary data concerning zinc and cadmium. The data are all reasonably accurate, the specific gravities being especially so, thanks to the care of Dr. Baxter and his assistants, who kindly determined three of them for use in the present paper.

From these results it is seen that in each case the contraction which takes place during the formation of the zinc halide is less than that which takes place during the formation of the cadmium salt by about five milliliters per mole, although their heats of formation are nearly equal. A part of this difference is undoubtedly to be ascribed to the cause which gives the cadmium salt a higher boiling point in each case, namely, a greater cohesive pressure in the case of the cadmic salts. A part may also be expected to be due to the fact that metallic zinc, having a higher boiling point (920°)* than cadmium (778°) — and therefore a greater molecular latent heat of evaporation or greater internal physical pressure — must be supposed to be in the more compressed state. Other indications also point to the same conclusion; for example, zinc has the smaller coefficient of expansion, the smaller molecular volume, the greater hardness, and the larger "energy quotient," (a quantity having the dimensions of pressure which is obtained by dividing the atomic heat by the atomic dilation).† Since each of these latter qualities is not a function

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chlorides boil considerably over 1200°, and not far apart. Lithic chloride seemed to be slightly the more volatile of the three. The melting points are all near together, — of the bromides and iodides as well as of the chlorides, — and in similar salts of this kind it is not very dangerous to infer similar boiling points from their known melting points.

* Zeitschr. phys. Chem., **42**, 118 (1902); Compt. Rend., **131**, 380 (1900).

† These Proceedings, **37**, 1 (1901).

Salt.	Boiling Point.	Specific Gravity.	Molecular Volume.	Sum of Atomic Volumes.	Contraction.	Heat of Format'n. (kj.)
$ZnCl_2$	730 *	2.91 ‡	46.9	9.5 + 50 = 59.5	12.6	388
$CdCl_2$	910 †	4.05 §	45.4	13.0 + 50 = 63.0	17.6	371
Difference					5.0	
$ZnBr_2$	650 *	4.22 ‖	53.4	9.5 + 51 = 60.5	7.1	318
$CdBr_2$	810 †	5.20 §	52.2	13.0 + 51 = 64.0	11.8	315
Difference					4.7	

of the average internal pressure alone, but also of other variables, none of them alone furnishes certain evidence; but when all point in the same direction the probability of their combined verdict is great. If then, zinc is in a more compressed condition than cadmium, it should not contract so much as cadmium upon further compression. Here the difference in cohesion of the two metals affects the result in the same direction as that of the salts, instead of in the opposite direction, as in the case of silver and zinc.

These two probable circumstances — namely, the greater internal compression of the salts of cadmium and the less internal compression of metallic cadmium — seem to me quite enough to afford an explanation of the fact that greater contraction occurs during the formation of cadmic salts than during the formation of zincic salts.

On comparing the chlorides with the bromides the result is essentially similar. The contraction on forming bromides is less in proportion to the energy-change than that on forming chlorides, chiefly because bromine is less compressible than chlorine, but partly also because the resulting bromides are less compressed than the chlorides, judging by their lower boiling points.

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* Freyer and V. Meyer, Ber. d. deutsch. ch. Ges., **25**, 622 (1892).

† Carnelley and Carleton-Williams, J. Chem. Soc., **37**, 125 (1880).

‡ Baxter and Lamb, Am. Chem. Journ., **31**, 229 (1904).

§ Baxter and Hines, Am. Chem. Journ., **31**, 220 (1904). The previous data, obtained by Knight under the direction of F. W. Clarke, were much too low. Sill. J., (3) **16**, 202 (1878); Am. Chem. J., **5**, 240 (1883).

‖ Richards and Rogers, Z. anorg. Chem., **10**, 6 (1895).

Thus, all the variations in the zinc and cadmium series seem to be not only explicable but necessary.

In order to make the statement of possible types more complete, it is worth while to cite the circumstances attending the formation of an easily volatile endothermic compound from two elements less volatile. Here we should expect the compound to occupy much *more* space than the elements, since both the chemical and physical internal-pressures are less after combination than before. Such a case is that of carbon disulphide, whose formation involves the absorption of 93 kilojoules of heat, accompanied by an *increase* of volume of 25 milliliters (from 35 to 60 milliliters) per gram molecule. This very large increase of volume is too great to be referred to the decrease in either the chemical or the physical pressure alone; its magnitude, therefore, shows that both these causes are working together to cause a change of volume, as the hypothesis predicts. It must be borne in mind that a given output of energy in the change of physical compression caused by cohesion would have a more marked effect on the volume than the same output in the change of chemical affinity, because the cohesive attraction acts on the expanded and easily compressible part of each atom — namely, the outside surface of the molecule, which is compressed only by comparatively slight pressure of its cohesion with other molecules. In short, when the existing pressure is small, the change of volume for a given additional change of pressure will be large, with a given expenditure of energy.

That exothermic non-volatile sulphides behave in the usual way, and hence that this behavior of carbon disulphide does not depend upon any peculiarity of sulphur, is seen by comparing the contraction of 55 milliliters which occurs during the formation of potassic sulphide with that of 2 milliliters in the case of argentic sulphide, the respective heats of formation being 423 and 14.

It is easy to show that these principles generalize and correct a great part of the irregularities in the curve giving the relation between heats of reaction and decrease in volume. The majority of the other irregularities which still remain are undoubtedly due to inaccurate data. The published values of specific gravities are notoriously untrustworthy, often because they have been based on salts only partially dehydrated. For examples, the specific gravities of zincic and cadmic bromides are usually supposed to have the values 3.64 and 3.79, instead of the true values 4.22 and 5.20 respectively, — an error of over twenty-five per cent in the latter case. Again, heats of formation are often untrustworthy. For example, the value for carbon tetrachloride is given by Thomsen as

76 kilojoules, and by Berthelot and Mantignon as 226 kilojoules.* In such cases one is at a loss as to the true value, without new experimental data, and is unable to apply intelligently the principles under consideration.

It is not to be expected, however, that even when all the data have been correctly determined the agreement should be mathematically precise, especially when solids are concerned, on account of the complexity of the problem, and the fact that a given substance often assumes different solid forms. In nearly all cases, it is true, the relation of the different solid phases of a given substance accords qualitatively with the principles above: — the formation of the more compact phase evolves heat, and hence this phase gives a less heat of reaction with other substances,† indicating a state of greater compression. This is true, for example, in the case of sulphur, phosphorus, and carbon. In such cases, the output of heat of allotropic modification has the same sign as the contraction in volume, and these quantities add respectively to the heat evolved, and to the change of volume during any reaction, not affecting the qualitative relation.

In some cases, however, the act of polymerization or of solidification seems to involve an anomalous expansion of volume, an apparent inconsistency which needs further discussion. There can be no doubt that the explanation given in a previous paper‡ affords the most probable interpretation of this fact. The explanation runs as follows, — it will be amplified below: —

"In those cases where there is a transition from a more compressible union to a stabler, less compressible one, involving more work of compression, solidification would involve increase of volume, as in the case of water."

This possibility might be discussed as an abstraction; but it becomes much clearer when a specific case is considered. Of course, such a consideration presupposes a knowledge of the nature of the molecular change which takes place in the substances, and but little is known concerning such changes. Hence, the evidence must in most cases be vague and rather unsatisfactory. It seems to me, however, that enough is known concerning at least a single case, that of the solidification of water, to allow its interpretation to serve as a type; and if the discussion is viewed as a possible concrete example of a general principle, rather than as a definite statement of fact, this risk of partial mistake in some of the details can do no serious harm.

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* See Ostwald, Lehrbuch, **2** (1), 175 (1893).

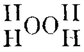
† Petersen, Zeitschrift für physikalische Chemie, **8**, 611 (1891).

‡ Richards, These Proceedings, **37**, 407 (1902).

In the first place, there are many well-known reasons for believing that liquid water is partly polymerized, and that its polymeric form is more bulky than its simple form. The admirable experimental work of Ramsay and Shields on surface tension* and the interesting logic of van Laar† may be cited as among the work affording a strong presumption of this partial "association." The determination of even the average extent of the polymerization is a more difficult matter, as may easily be seen by studying the papers of van der Waals‡ and Ramsay§ upon the subject. A conservative estimate is perhaps that given by Ramsay in his last mentioned paper — namely, the estimate that water at 0° has an average molecular weight of 1.7×18, and at 140° an average molecular weight of 1.3×18. Thus the liquid under ordinary conditions seems to consist of a mixture of $(H_2O)_n$ and H_2O, the latter preponderating at high temperatures.‖ Presumably ice consists wholly of $(H_2O)_n$.

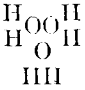
The determination of the magnitude of n in the above polymer is even more difficult than the determination of the exact average extent of polymerization; but fortunately for the present discussion neither of these points is crucial as far as it is concerned. Usually n is taken as 2; but there seem to me several reasons for believing it to be 3, an assumption which would correspond to other familiar polymers and with the well-known trigonal symmetry of ice.¶ In this case, of course, less of the polymeric form would be needed to account for the anomalous properties of water, than if its polymerized formula were $(H_2O)_2$.

In any event, because of the well-marked univalence of hydrogen, every one will concede that the polymer is probably formed by the union of oxygen to oxygen. The two most probable structural forms will then be



$${}^{H}_{H}OO^{H}_{H}$$

and



$$\begin{array}{ccc} H & & H \\ & OO & \\ H & & H \\ & O & \\ & HH & \end{array}$$

Either assumption will satisfy the requirements of the present argument.

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* Ramsay and Shields, Zeitschr. phys. Chem., **12**, 433 (1893).

† Van Laar, ibid., **31**, 1 (1899).

‡ Van der Waals, Zeitschr. phys. Chem., **13**, 714 (1894).

§ Ramsay, Zeitschr. phys. Chem., **15**, 106 (1894).

‖ This conclusion does not exactly agree with the highly hypothetical assumptions of Sutherland. Phil. Mag., (5) **50**, 460 (1900).

¶ The crystal form of ice is cited by Sutherland as a reason for assuming $(H_2O)_3$.

Having thus decided that the existence of a polymeric form is probable, the next step is a study of the probable mechanism of the act of polymerization. In a simple molecule of water, for reasons given above, it is usually assumed that each hydrogen atom is attached to the oxygen, HOH. When a molecule thus constituted coheres with other molecules to form a liquid, there are two typical situations attainable by the atoms of neighboring molecules. The atoms of each kind may be in juxtaposition with the same kind of atoms, or hydrogen of one molecule may be in the neighborhood of oxygen of another. If neither of these two situations involved a great excess of cohesive attraction over the other (the normal condition of an unassociated liquid), both situations would be present to about equal extents. From what has been said before, this may be supposed to be the case in liquid water at high temperatures.

What now happens when the water is cooled? Since there is every reason to believe that the act of polymerization evolves considerable heat, this act will tend more and more to occur as the temperature becomes lower, according to the theorem of Le Chatelier. All our experimental evidence points in this direction. But this increasing polymerization will involve an increasing shift of the cohesive attraction from the condition of equal distribution to that in which oxygen is in juxtaposition with oxygen, and hydrogen with hydrogen.

The effect of such a shift as this upon the volume will depend entirely upon the relative compressibilities of the elements concerned, under the peculiar circumstances to which they are exposed. There is good reason to believe that under conditions as parallel as possible, oxygen is less compressible than hydrogen. For example, pentane (C_5H_{12}, boiling at 38°), containing much hydrogen, is half again as compressible as sulphur dioxide (SO_2, boiling at −10°) if the figures are compared at corresponding temperatures (0° and 50° respectively, each a corresponding number of degrees above the boiling point).* Since sulphur in combination is probably at least as compressible as carbon, the difference here is to be referred primarily to that between hydrogen and oxygen. The study of many other, indeed all the known, compressibilities of compounds rich in hydrogen, leads to the conclusion that the compressibility of hydrogen is great, and that of oxygen smaller. This is not indeed surprising, since the average density of the oxygen in combination is about eight times as great as that of hydrogen, if Kopp's atomic volumes be taken as an approximate criterion, and density seems to be an important factor in determining compressibility.*

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* Landolt and Börnstein Tabellen, p. 268 (1894). The values were found by Grimaldi and by Colladon and Sturm.

Assuming, then, that oxygen is less compressible than hydrogen, and that the act of polymerization would involve the severing of cohesive attraction of oxygen to hydrogen, and the substitution of the attraction of oxygen for oxygen, the hydrogen, on being freed and but little attracted by its neighboring hydrogen, would expand more than the oxygen could contract. Thus an increase of volume would result, even although the energy developed by the oxygen in combining with itself might be considerable. Here, as before, it must be borne in mind that a given change of volume in a compressible substance corresponds to a smaller amount of work than the same change of the same volume of a slightly compressible substance.

In the act of solidification the oxygen-hydrogen cohesion would all be severed, and the system would suffer a sudden expansion in consequence. Thus may be explained the irregular coefficient of expansion of water, and the abnormal expansion on solidification; as before, the apparent exception really supports the argument.

Another although essentially similar case is the somewhat unusual one in which an *elementary* substance on cooling assumes the form of a phase of less density. The most striking case of this anomaly is that of tin, which possesses at least two different forms with specific gravities of 7.3, and 5.8 respectively, the more dense being stable at high temperatures, and the lighter at low temperatures.† Even such a case as this is not inexplicable, however; for if the denser form consists of atoms compressed nearly equally on all sides, an expansion accompanied by evolution of heat might occur through a polymerization involving a greater compression on one side of each atom, with the partial release and consequent expansion of the other side. Because the compressibility diminishes with decreasing volume, the expansion on the sides of the atom partially released might easily exceed the contraction on the compressed side of the atom. If this is really the case with tin, one would expect the irregularly constituted gray tin to possess a smaller tenacity than the evenly compressed white tin, because tenacity must depend upon the weakest cohesive attraction existing in a substance, just as the strength of a chain is the strength of its weakest link. As a matter of fact, gray tin seems to possess far less tenacity than white tin, for the former is always described as crumbling to powder; hence the fact is consistent with this interpretation. The situation is far too complicated, involving the simultaneous action of too many intensities of pressure to attempt at present a precise mathematical solution, or to afford a basis for the discovery of the reason why tin in particular acts in this peculiar way. The explanation above is given only to show that this kind of case, instead of offering insuperable objection to the theory of compressible atoms, accords with it and affords a means of predicting an entirely different property of the anomalous form — namely, its slight tenacity.

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* According to Dupre's rule, compressibility is inversely as the square of the density. This relation only holds strictly true with analogous compounds, but nevertheless serves as an index of the importance of density.

† See Cohen, Zeitschr. phys. Chem., **30**, 601 (1899), etc.

The other possible varieties of tin may be explained by arguments similar to these. See Rammelsberg, Jahresbericht, 1870, pp. 358, 724; Shepherd, J. Phys. Chem., **6**, 519 (1902).

Besides the changes of volume which may thus be effected by the polymerization of a solid or liquid, changes of volume are undoubtedly occasioned by the exigencies of crystal form. These will be considered in a subsequent paper. Taking into account all these possible irregularities, it is not at all surprising that the changes of volume do not exactly correspond to the heats of formation in chemical reactions; indeed, it would be incredible that the correspondence should be exact. Nevertheless, in spite of these expected and reasonable irregularities, enough obvious regularity still persists to show the general tendency, and to make highly probable the fundamental statement that the atomic volume is not a constant, but is a function of the total pressure (internal and external) and the temperature.

It has been suggested above that in general the lower the boiling point, the greater should be the compressibility, — because a low boiling point indicates slight molecular cohesion; and when the pressure upon the surface of the molecules is small, these molecules should be in an expanded condition, and therefore easily compressible. This postulate is supported not only by the facts given above, but also by many other data of the same kind. Of course the relation is sometimes partly hidden by the specific nature of the elements concerned; hence it is shown most clearly by the comparison of isomeric organic substances. In any complex molecule of this kind, it is true that the compressibility must be an average value; for the application of an outside mechanical pressure must add not only to the pressure of cohesion between the molecules, but also to the various affinity-pressures which exist between the atoms. Nevertheless, the change of volume produced by the former of these effects should be so much greater than that produced by the latter (because the affinity-pressure must be already so great) that usually the former alone would be expected to become manifest by its effect. Data illustrating these inferences are given below.

Another property, going hand in hand with this relation of compressibility and boiling point, is density. It is to be expected that of two similar substances, that with the higher boiling point and the less compressibility, being subject to greater internal pressure, should be the more dense.

A few figures, representing several classes of compounds, are given below to show that in general these considerations are supported by fact.

Relation between Boiling Points, Densities, and Compressibilities of Isomeric Compounds.

Substance.	Boiling Points.	Density at 0°.	Compressibility × 10⁶.
Ethyl butyrate	120°	0.899	93
Methyl valerate	127°	0.909	91
Isopropyl alcohol	83°	0.788	103
Propyl alcohol	97°	0.804	97
Isopropyl chloride	37°	0.874	
Propyl chloride	46°	0.916	
Paraxylol	138°	0.880	
Orthoxylol	143°	0.893	

In each of these cases, the higher boiling point accompanies greater density and less compressibility. The last-named property has been determined in so few cases that the material for generalization is small; but as far as they go the data all point in the same direction.* On the other hand, so many densities have been determined that the usual parallelism of this property with the boiling point is easily verified. Among many interesting groups of compounds, that having the common formula $C_7 H_{14} O_2$ is especially complete and comprehensive, the boiling points ranging from 224° with normal heptylic acid to 121° with isopropyl isobutyrate, while the densities range from 0.938 * to 0.879. Among the seventeen compounds in this group whose data are to be found in Beilstein's invaluable handbook, the density of only one (methyl caproate) deviates more than 0.005 from its place in the parallel series, the density steadily diminishing as the boiling point diminishes. This is shown in the following diagram, where centigrade boiling points are recorded as abscissae, and densities at 0° as ordinates. Clearly the correspondence of density to boiling point is too close to be the result of accident.

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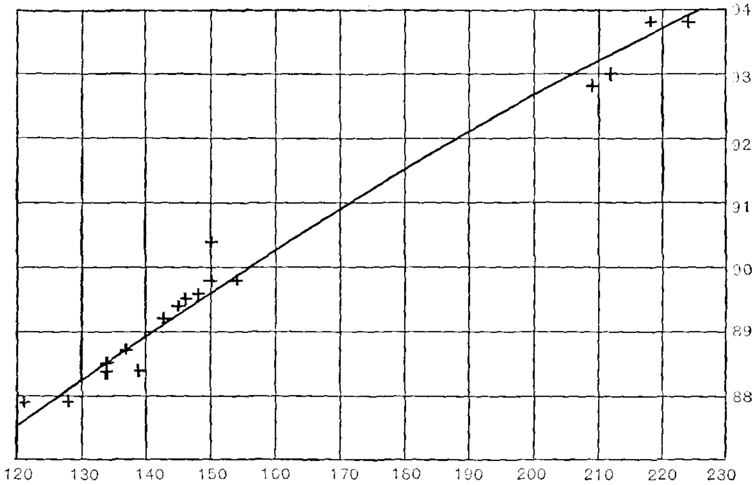
* Gartenmeister Liebig's Annalen, **233**, 309 (1886), found that esters with highest boiling point have the least coefficient of expansion — a property that very usually varies in the same direction as compressibility.

Boiling Points and Densities of Substances $C_7 H_{14} O_2$.



The divergent point on the ordinate 150° represents methyl caproate, and that less divergent on the ordinate 139°, isoamyl acetate.

The other substances, named in order, are isopropyl isobutyrate, isopropyl butyrate, propyl isobutyrate, ethyl isovalerate, butyl propionate, propyl butyrate, ethyl valerate, isobutyl propionate, amyl acetate, methyl isobutyl acetate, hexyl formiate, isomylacetic acid, isoheptylic acid, iso-onanthic acid, and heptylic acid.

This regularity is so obvious that it must have been noticed before; but nevertheless, the plausible hypothetical explanation of it, depending

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* This value is Franchimont's (Ann. Chem. Phys., **165**, 242 (1873) corrected to 0° with the help of Zander's value for the coefficient of expansion. Ibid., **224**, 70 (1884). Franchimont's value is chosen because his substance was the purest, having a melting point of −8°.

upon the theory of compressible atoms, has probably not before been emphasized.

From these examples it is clear why the density must usually be taken at about the boiling point in order to obtain additive values for the molecular volumes of organic substances, as Kopp found. Those substances which possess a high boiling point, or by inference great pressure of cohesion, would be expected to be in a highly compressed state. When compared at the same temperature, the molecular volumes of the more compressed material will therefore be smaller than the molecular volumes of the less compressed material; but by heating the more compressed material it may be expanded until the thermal expansion at a definite point replaces the volume lost by this cohesive compression. The higher the boiling point, the greater must be the cohesive compression, and the higher will be the rise of temperature needed to counteract it. This corresponds exactly with the fact. In many cases (those of fat acids, esters, and ethers) it happens that the required rise of temperature not only varies in the same direction as the boiling points, but also closely corresponds in magnitude with their difference. In others, especially the alcohols, aldehydes, halides, and aromatic compounds, the deviations are considerable. In the case of butyl alcohol, for example, the liquid must be heated under pressure to a point about 35° above its boiling point in order that it may attain the volume required by Kopp's rule, being five per cent too small in volume* at its boiling point.

Deviations of this kind seem to indicate clearly that although in general a substance possessing a higher boiling point must be heated to a higher temperature in order that the molecular volume may become a precisely additive quantity, the exact temperature needed is determined by the concerted action of too many variables to be precisely determined. The fact that in a minority of cases the boiling point itself is a suitable point of comparison seems to be accidental.

The existence of irregularities in the above described relation of density to boiling point might have been predicted by the logical application of the idea of compressible atoms; for evidently *chemical affinity* causing atomic rearrangement within the molecule might cause a change of atomic compression, and hence of density, without greatly affecting the *cohesive* attraction, thus leaving the boiling point unchanged. Accordingly, the usual relation between boiling point and density might be marred in some cases.

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* See Ostwald's Lehrbuch, **1**, 369 (1891).

In seeking for quantitative verification of such an effect, the irregularities themselves are the best guide to their cause, hence the alcohols, already quoted as abnormal in their behavior, serve as useful examples. Fortunately, too, the heats of formation of enough alcohols are known, in order to provide a clue as to a change of the internal stress within the molecule. Below is given a brief table typically illustrating the point.

THE EFFECT OF INTERNAL ATOMIC REARRANGEMENT ON DENSITY.

Alcohols.	Boiling Point.	Specific Gravity at 20°.	Heat of Combustion of Liquid.* (Ostwald Calories.)	Latent Heat of Evaporation calculated by Trouton's Rule. (Ostwald Calories.)	Heat of Combustion of Vapor. (Ostwald Calories.)	Heat of Formation of Vapor from the Elements. (Kilojoules.)
Propyl	97°.4	0.804	4803	96	4899	312
Isopropyl	82°.8	0.789	4783	92	4875	322
Isoamyl	131°	0.825	7939	105	8044	376
Dimethyl ethyl carbinol	102°	0.825	7885	97	7982	402

In the case of the propyl alcohols given in this table, the usual relation exists, — the density is greater where the boiling point is greater, — but in the case of the amyl alcohols, on the other hand, an anomaly appears. In the latter case the densities of the two compounds given in the table are identical, although their boiling points are 29° apart.

The most plausible explanation of this circumstance lies in the assumption of increased internal atomic compression, due to the peculiar structure of dimethyl ethyl carbinol; a compression which has nothing directly to do with the cohesive attraction and the boiling point, because these are concerned only with the superficial attraction of the molecules. By causing decrease in volume this internal stress serves to conceal the increase in volume due to a lesser pressure of cohesion in the lower-boiling liquid. This hypothetical inference is supported by the heats of formation of these compounds. Dimethyl ethyl carbinol has a distinctly greater heat of formation than isoamyl alcohol, while in the case of the propyl alcohols there is no great difference between the heats of formation of the isomers. Since a greater heat of formation usually signifies greater affinity, since greater affinity must cause greater pressure, and since greater pressure must diminish the volume, it appears highly probable that in the case of the amyl alcohols a greater stress within the molecule in one case may cause an increase in the density which is enough to compensate for the decrease of density on the molecular surface due to a smaller pressure of cohesion between different molecules. Thus, the apparently anomalous behavior of the alcohols not only offers no argument against the principles involved, but rather supports the other arguments in favor of those principles.

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* These figures are due to Louguinine (Ann. chim. phys., (5) **21**, 139 (1880)). Thomsen's figures show similar differences, although in a more exaggerated degree. Qualitatively they would lead to the same conclusion.

These cases may be taken as typical; and probably most of the variations in density, compressibility, and boiling point may be explained in this way. As before, however, it is not to be expected that every case could exactly correspond, for the densities, compressibilities, and heats of formation of complex compounds of this kind represent average values, and much would depend upon the constitution of the body, and the arrangement of the atoms in space. As in the case of water and tin, cited above, it might easily happen that a considerable excess of heat of formation in a given case might be evolved by a shift from a more compressible union to a less compressible one, involving the expenditure of more chemical energy without causing a decrease in volume.

It is evident upon consideration that all these typical cases are really dependent for their interpretation upon the following very simple conceptions: *first, that internal pressure, due to both cohesive attraction and* chemical affinity, must tend to diminish the volume of condensed systems (*i. e.* solids and liquids); secondly, that the greater the pressure, the greater is this tendency; and thirdly, that, other conditions being equal, a substance already much compressed by internal pressure suffers less contraction upon subjection to additional pressure than one but little compressed. The circumstance that the facts above cited are all consistent with this obvious logic affords a strong presumption that it is correctly applied.

The existence of these different intensities of internal pressure affords a basis also for explaining the partial parallelism and frequent apparent irregularity of the various physical properties of material, such as tenacity, hardness, ductility, malleability, etc., and the relation of these properties to boiling points. For example, in order that a liquid may vaporize monatomically as zinc does, even the strongest tie binding each atom must be broken; on the other hand the *tenacity* of the metal could be no greater than the cohesion of the weakest tie, as has already been pointed out, in the case of tin. If all the atomic ties were nearly equal in each of two cases, that is, if the atoms were equally compressed on all sides, tenacity and boiling point would be expected to vary in a parallel manner. In a similar way the other physical properties might be discussed in relation to the varying intensities of cohesive and chemical attraction; but the complete discussion involves more knowledge concerning the extent and nature of polymerization in solids than is at present available; hence it must be postponed.

In conclusion, it may be noted that the hypothesis of compressible atoms is consistent with either of the two alternative hypotheses concerning the ultimate nature of material — namely, the corpuscular conception so popular to-day, and the hypothesis assuming the atom to be an ultimate continuous although compressible aggregation.

Moreover, it is not a necessary consequence of the facts discussed above that the atom should have a very sharply defined surface, — the concentration of its substance may increase gradually as the centre is approached. On the other hand the fact that a definite volume is attained, and that the compressibility decreases greatly with diminishing volume, seems to prove that the distending effect increases much faster than the law of inverse squares governing the usual forms of attraction. The discussion of these questions is best deferred until some of the more immediate consequences of the hypothesis have been studied.

SUMMARY.

In this paper the following conclusions, based upon new data of known accuracy, are attained: —

(1) Among compounds of lithium, sodium, potassium, chlorine, bromine, and iodine, it is shown that in every case the compound of a more compressible element is formed with a greater decrease of volume than the compound of a less compressible element, other conditions being equal.

(2) From a study of data concerning the chlorides of silver, the chlorides and bromides of zinc and cadmium, and carbon bisulphide, it is shown that the more volatile the substance (i. e. the slighter its cohesive attraction), the greater is the molecular volume, other conditions being equal.

(3) It is pointed out that a given change in chemical energy produces a smaller change of volume than the same change of cohesive energy produces.

(4) These typical facts are explained by the assumption that both chemical affinity and cohesive attraction exert a compressing effect upon solid or liquid substance, and that the chemically tied part of each atom is more compressed than that which is subject only to cohesive pressure.

(5) Polymerization and crystallization are shown to be capable of causing irregularities. The cases of water and tin are discussed in detail as types, and it is shown that there is nothing in these cases contradictory to the explanation given above.

(6) From the comparison of many isomeric organic compounds, it is shown that the explanation covers also their relations of densities, compressibilities, and boiling points, including even some of the apparent irregularities in those relations. Thus is shown the reason why additive molecular volumes are obtained only when, of two liquids, the less volatile (i. e. the more cohesive) is heated to a higher temperature.

(7) It is suggested that these varying intensities of internal pressure are adequate to explain the variety of other physical properties, such as tenacity and malleability.

(8) These considerations, by accounting numerically in detail for most of the irregularities in the more obvious facts discussed in previous papers, afford increased evidence as to the significance of changing atomic volume, and increased support for the theory of compressible atoms.

CAMBRIDGE, MASS.
January 25, 1904.

Proceedings of the American Academy of Arts and Sciences.

Vol. XXXIX. No. 24. — June, 1904.

RECORDS OF MEETINGS, 1903–1904.

REPORT OF THE COUNCIL: BIOGRAPHICAL NOTICES.

- Horace Gray. By Francis C. Lowell.
- William Sumner Appleton. By Charles C. Smith.
- James Elliot Cabot. By T. W. Higginson.
- Alfred Perkins Rockwell. By R. H. Richards.
- Horatio Hollis Hunnewell. By C. S. Sargent.

OFFICERS AND COMMITTEES FOR 1904–1905.

LIST OF THE FELLOWS AND FOREIGN HONORARY MEMBERS.

STATUTES AND STANDING VOTES.

RUMFORD PREMIUM.

INDEX.

(Title Page and Table of Contents.)

RECORDS OF MEETINGS.

Nine hundred and forty-first Meeting.

JUNE 17, 1903. — SPECIAL MEETING.

The RECORDING SECRETARY in the chair.

The Chair announced the death of Karl Gegenbaur, Foreign Honorary Member in Class II., Section 3.

Letters were read from William F. Ganong, accepting Fellowship; from W. F. Osgood resigning Fellowship; from the Royal Academy of Sciences of Turin, announcing the death of its Vice-President, Barnardino Peyron.

Mr. Rotch moved that a Committee be appointed by the Chair to confer with a Committee of the Boston Library Society relative to leasing a portion of its house, 114 Newbury Street, and to report in the autumn.

The Chair appointed the President, Librarian, and Treasurer.

On the motion of Professor Cross, it was

Voted, That the Recording Secretary be an additional member of this Committee.

The vacancy occasioned by the resignation of W. F. Osgood was filled by the appointment by the Chair of

Harry M. Goodwin, member of the Committee on the Library.

On the motion of Professor Wright, it was

Voted, To proceed to ballot for a Treasurer in place of Mr. Samuel Cabot, who had been elected at the annual meeting but had declined to serve.

The result of the balloting was the unanimous election of

Francis Blake, Treasurer.

The following papers were presented by title: —

"On the Prolongation of Spectral Lines." By Theodore Lyman.

"The Athenian Vase-Painter Brygos, and his Characteristics." By Oliver Samuel Tonks. Presented by John H. Wright.

Nine hundred and forty-second Meeting.

OCTOBER 14, 1903. — STATED MEETING.

The PRESIDENT in the chair.

The Chair announced the following deaths: —

J. Peter Lesley, Class II., Section 1; George S. Morison, Class I., Section 4, Associate Fellows.

Luigi Cremona, Class I., Section 1, Foreign Honorary Member.

The following gentlemen were elected members of the Academy: —

Charles Émile Picard, of Paris, to be a Foreign Honorary Member in Class I., Section 1 (Mathematics and Astronomy), in place of the late Hervé-Auguste Étienne Albans Faye.

Joseph Larmor, of Cambridge, to be a Foreign Honorary Member in Class I., Section 2 (Physics), in place of the late Sir George Gabriel Stokes.

The President announced the resignation of William L. Richardson as Auditor, and appointed Frederick J. Stimson in his place.

The following papers were presented by title: —

Quantitative Studies in the Evolution of Pecten. III. "Comparison of Pecten opercularis from three Localities of the British Isles." By C. B. Davenport.

"The Electrical Conductivity of Aqueous Solutions at High Temperatures." I. Description of the Apparatus. Results with Sodium and Potassium Chloride up to 306°. By Arthur A. Noyes and William D. Coolidge.

"Metabolism and the Reaction of Division in Protozoa." By Amos W. Peters. Presented by E. L. Mark.

"The Laws of Heredity of Galton and Mendel and some Laws Governing Race Improvement by Selection." By W. E. Castle.

"A Revision of the Atomic Weight of Iron." By Gregory Paul Baxter. Presented by T. W. Richards.

"The Color Changes in the Skin of the so-called Florida Chameleon, Anolis carolinensis Cuv." By Frank C. Carlton. Presented by E. L. Mark.

"On the real Automorphic Linear Transformation of a real Bilinear Form." By Henry Taber.

"On the Lines of Certain Classes of Solenoidal or Lamellar Vectors Symmetrical with Respect to an Axis." By B. O. Peirce.

The following report of Mr. Rotch on library accommodations, read by the Recording Secretary, was accepted and placed on file.

Mr. Rotch, Chairman of the Committee appointed at the Special Meeting of June 17, 1903, to confer with a Committee of the Boston Library Association, relative to leasing a portion of its newly acquired house, 114 Newbury Street, for the use of the Academy, reports that this was done in the early summer. The Recording Secretary, Treasurer, Librarian, and the late Assistant Librarian visited the premises and decided that without expensive alterations they would not meet the requirements of the Academy, and Mr. Rotch therefore informed the Secretary of the Society that the offer was declined.

The Treasurer is of the opinion that by capitalizing the rental of the present rooms in the Historical Society's building and adding thereto the Building Fund, land might be purchased in the vicinity and a suitable building erected for the use of the Academy.

Professor Goodwin, after expressing his appreciation of the honor conferred upon him by being chosen President of the Academy, proceeded to give some interesting reminiscences of meetings of the Academy which he attended thirty years ago.

He then gave the communication of the evening, "A Visit to the Palace of Minos at Gnossos in Crete."

Nine hundred and forty-third Meeting.

NOVEMBER 11, 1903.

The Academy met at the house of President Goodwin.

PRESIDENT GOODWIN in the chair.

The Chair announced the death of Theodor Mommsen, Foreign Honorary Member in Class III., Section 3.

The Corresponding Secretary read a letter from T. de Azcárate announcing his appointment as director of the Marine Observatory of San Fernando, Cadiz, Spain.

The following papers were read: —

"Medicine and Surgery in Ancient Rome." By Morris H. Morgan.

Remarks on this paper were made by Professors Bowditch, Lanman, and Dr. William Everett.

Biographical notice of the late William Sumner Appleton, by Charles C. Smith.

Nine hundred and forty-fourth Meeting.

DECEMBER 9, 1903.

The PRESIDENT in the chair.

Letters were received announcing the deaths of Professors Theodor Mommsen and Karl Gegenbaur; letters of acknowledgment of election from K. A. R. von Zittel, Émile Picard, and Joseph Larmor, Foreign Honorary Members. From the Royal Academy of Sciences of Sweden, two letters, one from the Nobel prize committee on Physics and one from the Nobel prize committee on Chemistry, soliciting names of competitors.

On the motion of the Corresponding Secretary these two letters were referred to the Rumford and C. M. Warren Committees.

The Corresponding Secretary announced that an invitation from the Schlesische Gesellschaft für Vaterländische Cultur in Breslau to attend its jubilee on the 17th of December had been received too late to name a delegate from the Academy to take part in the celebration, and that the Society would be informed of the regrets of the Academy at the delay.

The Chair announced the death of W. E. H. Lecky, Foreign Honorary Member in Class III., Section 3.

The following papers were presented by title: —

"Edge Corrections in the Calculation of the Absolute Capacity of Condensers by the Schwarzian Transformation," by Joseph G. Coffin. Presented by A. G. Webster.

"On Generalized Space Differentiation of the Second Order," by B. O. Peirce.

The following communication was made by Professor Raphael Pumpelly, illustrated with lantern projections: "Explorations in Turkestan."

Nine hundred and forty-fifth Meeting.

JANUARY 13, 1904. — STATED MEETING.

The Academy met by invitation at the house of Mr. Francis Bartlett, 236 Beacon Street.

The PRESIDENT in the chair.

The Chair announced the death of General Alfred P. Rockwell, of Boston, Resident Fellow in Class I., Section 4.

On motion of Professor C. R. Cross, Chairman of the Rumford Committee, and seconded by Professor A. G. Webster, it was

Voted, To grant the sum of five hundred dollars ($500) from the income of the Rumford Fund to Professor Edward W. Morley, of Adelbert College, in aid of his research on the nature and effect of ether drift.

The following paper was presented by title: —

"Contributions to our Knowledge of Heredity," by Alexander Petrunkévitch. Presented by E. L. Mark.

The following communication was given by Dr. Francis H. Williams: "Some of the Properties and Uses of Radium Salts."

Nine hundred and forty-sixth Meeting.

FEBRUARY 10, 1904.

VICE-PRESIDENT TROWBRIDGE in the chair.

The Chair announced the death of Karl A. Ritter von Zittel, of Munich, Foreign Honorary Member in Class II., Section 1.

The following paper was read by Dr. W. E. Castle: "Mendel's Law of Heredity, illustrated by Breeding Experiments with Guinea Pigs."

The following papers were presented by title:

Contributions from the Chemical Laboratory of Harvard College: "The Significance of Changing Atomic Volume. IV. The Effects of Chemical and Cohesive Internal Pressure." By Theodore William Richards.

Contributions from the Jefferson Physical Laboratory: "Spectra Produced by the Wehnelt Interrupter." By H. W. Morse. Presented by J. Trowbridge.

"The Green Schists and Porphyries of Rhode Island." By B. K. Emerson.

Contributions from the Gray Herbarium of Harvard University. New Series. — No. XXVIII.

I. "Revision of the Genus Sabazia." By B. L. Robinson and J. M. Greenman.

II. "Revision of the Mexican and Central American Species of Trixis." By B. L. Robinson and J. M. Greenman.

III. "Revision of the Mexican and Central American Species of Hieracium." By B. L. Robinson and J. M. Greenman.

IV. "Synopsis of the Mexican and Central American Species of Alnus." By M. L. Fernald.

V. "Diagnosis and Synonymy of Mexican and Central American Spermatophytes." By J. M. Greenman.

VI. "Some Mexican and Nicaraguan Dicotyledons." By M. L. Fernald.

Presented by B. L. Robinson.

Nine hundred and forty-seventh Meeting.

MARCH 9, 1904. — STATED MEETING.

VICE-PRESIDENT TROWBRIDGE in the chair.

The Corresponding Secretary read a circular from the Organizing Committee of the International Congress of Americanists, announcing that the fourteenth session of the Congress would be held at Stuttgart, in August, 1904.

The Chair announced the following deaths: H. E. von Holst, of Class III., Section 3, Associate Fellow; Sir Leslie Stephen, of Class III., Section 4, Foreign Honorary Member.

On motion of the Corresponding Secretary, it was

Voted, To send the following message to Sir Henry Enfield Roscoe, on the fiftieth anniversary of his graduation as a Doctor of Philosophy of the University of Heidelberg. "On this memorable anniversary of the beginning of a life-work rich in fruitful research and helpful service in the advancement of science, The American Academy of Arts and Sciences sends warmest greetings and enthusiastic congratulations."

The Chair appointed the following councillors to serve as Nominating Committee: —

George F. Swain, of Class I., Robert DeC. Ward, of Class II., Denman W. Ross, of Class III.

On motion of the Recording Secretary, it was

Voted, To meet on adjournment on the second Wednesday in April.

A biographical notice of the late J. Elliot Cabot was read by Colonel T. W. Higginson.

Remarks on the life and character of Mr. Cabot were made by Professor Barrett Wendell.

Nine hundred and forty-eighth Meeting.

APRIL 13, 1904. — ADJOURNED STATED MEETING.

The CORRESPONDING SECRETARY in the chair.

The Corresponding Secretary read letters from F. C. Thorpe acknowledging the receipt of a copy of the resolution passed by the Academy at its meeting on March 9; from the Naturhistorische Verein of Bonn, announcing the death of Dr. August Huyssen; from the Permanent Committee of the International Congress of Botany, requesting the Academy to appoint a delegate to the Congress to be held at Vienna, in June, 1905.

On motion of Professor Webster, for the chairman of the Rumford Committee, it was

Voted, To appropriate from the income of the Rumford Fund the sum of two hundred dollars ($200) to Professor Dewitt B.

Brace, in aid of an investigation of double refraction in gases in an electric field.

The following gentlemen were elected members of the Academy: —

Edward Stevens Sheldon, of Cambridge, to be a Resident Fellow in Class III., Section 2 (Philology and Archæology).

Herbert Weir Smyth, of Cambridge, to be a Resident Fellow in Class III., Section 2 (Philology and Archæology).

Eugene Woldemar Hilgard, of Berkeley, Cal., to be an Associate Fellow in Class I., Section 3 (Chemistry), in place of the late Ogden N. Rood.

James Duncan Hague, of New York, to be an Associate Fellow in Class I., Section 4 (Technology and Engineering), in place of the late George S. Morison.

Israel Cook Russell, of Ann Arbor, to be an Associate Fellow in Class II., Section 1 (Geology, Mineralogy, and Physics of the Globe), in place of the late Joseph Le Conte.

Abraham Jacobi, of New York, to be an Associate Fellow in Class II., Section 4 (Medicine and Surgery), in place of the late J. W. Powell.

Theophil Mitchell Prudden, of New York, to be an Associate Fellow in Class II., Section 4 (Medicine and Surgery), in place of the late J. P. Lesley.

Felix Klein, of Göttingen, to be a Foreign Honorary Member in Class I., Section 1 (Mathematics and Astronomy), in place of the late Luigi Cremona.

Adolf Harnack, of Berlin, to be a Foreign Honorary Member in Class III., Section 3 (Political Economy and History), in place of the late Theodor Mommsen.

Pasquale Villari, of Florence, to be a Foreign Honorary Member in Class III., Section 3 (Political Economy and History), in place of the late William Edward Hartpole Lecky.

Marie Louis Gaston Boissier, of Paris, to be a Foreign Honorary Member in Class III., Section 4 (Literature and the Fine Arts), in place of the late Gaston Bruno Paulin Paris.

The following communication was given by Colonel W. R. Livermore:

"On the Political Changes of Europe, especially from 1100 A.D. to the Present Time," illustrated by a series of colored maps, showing the boundaries of each tribe and nation for every ten years.

The following paper was presented by title: —

"On a New General Theory of Errors." By W. E. Story.

Nine hundred and forty-ninth Meeting.

May 11, 1904. — Annual Meeting.

The President in the chair.

The Corresponding Secretary read the following letter from Sir Henry E. Roscoe, in reply to the congratulatory resolution passed by the Academy March 9, 1904:

"To my friends and brothers in Science living and working in New England, New York, and Pennsylvania, I, from Old England, send my affectionate greetings and best thanks.

"It is pleasant to receive, as I have done to-day, assurances of appreciation from those speaking other tongues than English and living in other environments than those to which one is accustomed, but it is a still greater pleasure to hear in familiar accents from people of the same blood having aims and sympathies closely akin to one's own, words of encouragement and good will. Such words you have sent me.

"English men of science look with brotherly admiration, sometimes perhaps not unmixed with envy, on the wonderful progress which America is wisely making in the provision for teaching and research of the highest kind, thanks to the open-handed liberality of her wealthy citizens.

"Nor with you is the prime mover wanting. You are rich in men willing to devote themselves to a scientific career, men gifted with the true scientific spirit. With this union of mind and matter who can place a limit to the work which America will produce?

"Henry E. Roscoe.

"April 22, 1904."

The Corresponding Secretary also read letters from E. S. Sheldon, accepting Fellowship; I. C. Russell, A. Jacobi, acknowledging election as Associate Fellows. He also announced that Professor B. L. Robinson had been appointed a delegate to the International Congress of Botany, to be held at Vienna, in June, 1905.

The Chair announced the death of Charles S. Storrow, Resident Fellow in Class I., Section 4.

The annual report of the Council was read.

The annual report of the Treasurer was read, of which the following is an abstract: —

General Fund.

Receipts.

Balance, April 30, 1903		$29.13
Investments	$5,877.20	
Assessments	905.00	
Admission fees	40.00	
Sale of publications	234.64	7,056.84
		$7,085.97

Expenditures.

General expenses		$2,896.17
Publishing	$2,251.34	
Library	1,496.37	
Catalogue	37.00	3,784.71
Balance, April 30, 1904		405.09
		$7,085.97

Rumford Fund.

Receipts.

Balance, April 30, 1903		$ 197.79
Investments	$2,705.94	
Sale of publications	16.00	2,721.94
		$2,919.73

Expenditures.

Researches	$1,525.00	
Publishing	587.47	
Library	234.58	
Miscellaneous	15.00	$2,362.05
Balance, April 30, 1904		557.68
		$2,919.73

WARREN FUND.

Receipts.

Balance, April 30, 1903		$1,161.57
Investments	$330.48	330.48
		$1,492.05

Expenditures.

Investigations	$1,000.00
Balance, April 30, 1904	492.05
	$1,492.05

BUILDING FUND.

Receipts.

Balance, April 30, 1903	$ 104.92
Investments	384.98
	$ 489.90
Income invested and transferred to capital account	$ 100.00
Balance, April 30, 1904	$ 389.90

The following reports were also presented: —

REPORT OF THE LIBRARIAN. 1903–4.

The library sustained an irreparable loss in the death of the Assistant-Librarian, Dr. Austin Holden, which occurred suddenly at his home in Newton, Sept. 30, 1903, at the age of forty-nine. Dr. Holden had been connected with the Academy for thirty-two years, having first come to the library in the autumn of 1871, to aid Dr. Dana, then Assistant-Librarian, Professor Asa Gray being then President, and Mr. Edmund Quincy, Librarian of the Academy. Upon the death of Dr. Dana in 1872, Dr. Holden succeeded to the office, and to the time of his death performed the duties with great fidelity and discrimination, always having the interests of the Academy at heart. His general knowledge of books and his genial courtesy were much appreciated by all who had occasion to use the library. His familiarity with the library as well as with the customs and rules of the Academy, was invaluable to the Librarian and the Secretaries, and the former takes this opportunity to express his high appreciation of the character and ability of Dr. Holden and of the long and faithful service which he has rendered to the Academy.

Mrs. Austin Holden, who temporarily took her husband's place, has been appointed Assistant-Librarian by the undersigned.

The card-catalogue was continued by Dr. Holden up to the time of his death, he having written 925 cards, making the total number of cards 7,225. Although of the appropriation made last year only $37 was spent, the usual appropriation of $100 from the income of the General Fund, and $50 from the income of the Rumford Fund is requested to continue this work of cataloguing.

The accessions during the year have been as follows: —

	Vols.	Parts of Vols.	Pams.	Maps.	Total.
By gift and exchange	439	2,568	548	12	3,567
By purchase — General Fund .	44	592	5		641
By purchase — Rumford Fund .	32	320			352
Total	515	3,480	553	12	4,560

17 volumes on light and heat, recommended by the Rumford Committee, were bought at an expense of $58.41.

229 books have been borrowed from the library by 27 persons, including 15 Fellows, one Fellow having taken out 140 of that number. All have been returned for the annual examination except 11. Of the 12 volumes reported not returned a year ago, 2 are still unaccounted for.

The expenses charged to the library are as follows: — Miscellaneous, which includes expenses in no way related to the library, $631.63; Binding, $452.90, General, and $42.35, Rumford Funds; Subscriptions, $411.84 General, and $152.43, Rumford Funds; making a total of $1,496.37 for the General and $194.78 for the Rumford Funds. In explanation of the excess of the expenditure for miscellaneous items over the appropriation of $500, it may be said that a larger amount of printing is now done than formerly, and that this year some office furniture was purchased. The increase in the subscriptions paid, over those of the preceding year, is due to including some that cover next year.

An appropriation of $1,400 from the income of the General Fund is requested, in addition to the customary appropriation of $150 from the income of the Rumford Fund for the purchase of books and periodicals.

A. LAWRENCE ROTCH, *Librarian.*

REPORT OF THE RUMFORD COMMITTEE.

The following grants have been made by the Committee from the sum of $1000 which was placed at its disposal at the last Annual Meeting of the Academy.

Oct. 14, 1903. Three hundred dollars to Professor W. J. Humphreys of the University of Virginia, in aid of his research on the shift of spectral lines due to pressure.

Oct. 14, 1903. Two hundred and fifty dollars to Professor N. A. Kent of Wabash College, in aid of his research on the circuit conditions influencing electric spark lines.

Jan. 13, 1904. Two hundred dollars to Mr. J. A. Dunne of Harvard College Observatory, in aid of his research on fluctuations in solar activity as evinced by changes in the difference between maximum and minimum temperatures.

Jan. 13, 1904. Two hundred dollars to Professor Carl Barus of Brown University, in aid of his research on the study by an optical method of radio-actively produced condensation nuclei.

At the meeting of Nov. 11, 1903, it was voted to request the Academy to appropriate the sum of five hundred dollars from the income of the Rumford Fund to Professor Edward W. Morley of Adelbert College, in aid of his research on the nature and effects of ether drift; and at the meeting of Feb. 10, 1904, it was voted to request the Academy to appropriate from the same a sum not exceeding two hundred dollars to Professor Dewitt B. Brace of the University of Nebraska, in aid of his investigation on double refraction in gases in an electrical field.

The Academy granted the sums of five hundred dollars and two hundred dollars to the respective applicants as recommended by the Committee.

At the meeting of April 13, 1904, it was voted to appoint a sub-committee consisting of Professor Webster, Mr. Leavitt, and Professor Thomson to report upon the desirability of a new determination of the specific heat of super-heated steam.

At the same meeting the following votes were passed:

1. To request the Academy to appropriate the sum of one thousand dollars from the income of the Rumford Fund for the immediate needs of the Committee in furtherance of research.

2. To request the Academy to appropriate from the income of the Rumford Fund the sum of one hundred and fifty dollars for the purchase and binding of the usual periodicals.

3. To recommend to the Academy to appropriate from the income of the Rumford Fund the sum of fifty dollars for the purchase of books relating to light and heat.

At the meeting of Feb. 10 it was voted unanimously for the first time, and at the meeting of April 13 for the second time, to recommend to the Academy the award of the Rumford Premium to Professor

Ernest Fox Nichols of Columbia University, for his researches on Radiation, — particularly on the pressure due to radiation, the heat of the stars, and the infra-red spectrum.

Reports upon the progress of researches aided from the Rumford Fund have been received from the following persons: C. Barus, F. L. Bishop, A. L. Clark, J. A. Dunne, E. B. Frost, G. E. Hale, E. H. Hall, W. J. Humphreys, N. A. Kent, F. A. Laws, R. S. Minor, E. W. Morley, E. F. Nichols, A. A. Noyes, E. C. Pickering, T. W. Richards, F. A. Saunders, S. D. Townley, R. W. Wood.

Since the last Annual Meeting papers as follows have been published in the Proceedings at the expense of the Rumford Fund: —

Lyman, T. An Explanation of the False Spectra from Diffraction Gratings.

Wood, R. W. The Anomalous Dispersion, Absorption, and Surface Color of Nitroso-dimethyl-aniline.

Noyes, A. A., and Coolidge, W. D. The Electrical Conductivity of Aqueous Solutions at High Temperatures. I. Description of the Apparatus. Results with Sodium and Potassium Chloride up to 306°.

Morse, H. W. Spectra from the Wehnelt Interrupter.

CHAS. R. CROSS, *Chairman.*

REPORT OF THE C. M. WARREN COMMITTEE.

The C. M. Warren Committee begs leave to report that letters have been received from Professor Mabery, from Professor H. O. Hofman, and from Dr. S. A. Mulliken, reporting satisfactory progress in the work aided by grants voted by the Academy at the last Annual Meeting.

The Committee recommends that the income of the C. M. Warren Fund for the past year, $342.05, be placed at the disposal of the Committee for use in the furtherance of Chemical Research.

LEONARD P. KINNICUTT, *Chairman.*

REPORT OF THE COMMITTEE OF PUBLICATION.

Since the last Annual Report there have been published three numbers of Volume XXXVIII. of the Proceedings, and the first twenty numbers of Volume XXXIX., amounting to 627 pages and 10 plates. Five biographical notices have also been printed. On the second and third pages of the cover of each article in the Proceedings is now printed a price-list of the articles contained in the current volume and the one immediately preceding it. This was begun with No. 6 of the current volume.

One number of Volume XXXVIII. (No. 25), and three numbers of Volume XXXIX. (Nos. 3, 4, 7), were printed at the expense of the Rumford Fund, and one number, having been printed elsewhere before it appeared in the Proceedings, was paid for by the author.

The unexpended balance of appropriations from the General Fund for publications on April 30, 1903, was $211.18; the appropriation for the year now ended, $2,400; and the receipts from sales of publications, $234.64, making the total amount available for publications, $2,845.82. The total expenditures to April 30, 1904, exclusive of those paid from the Rumford Fund, have been $2,251.34, leaving an unexpended balance of $594.38.

E. L. Mark, *Chairman.*

May 11, 1904.

On the recommendation of the Committee of Finance, it was

Voted, To make the following appropriations from the income of the General Fund for the expenditures of the Academy during the ensuing year: —

For general expenses		$3,000
For publishing		2,400
For the library		
Books and binding	$900	
Miscellaneous	500	
Catalogue	100	1,500
		$6,900

On the recommendation of the Rumford Committee, it was

Voted, To make the following appropriations from the income of the Rumford Fund: —

For the immediate needs of the Rumford Committee in furtherance of research, $1,000.

For the purchase and binding of periodicals, $150.

For the purchase and binding of certain specified books on light and heat, $50.

On the recommendation of the C. M. Warren Committee, it was

Voted, To make the following appropriation from the income of the C. M. Warren Fund: —

For the use of the C. M. Warren Committee in furtherance of chemical research, $342.05.

On the motion of the Recording Secretary, it was

Voted, That the annual assessment for the ensuing year be five dollars ($5).

The annual election resulted in the choice of the following officers and committees: —

William W. Goodwin, *President.*
John Trowbridge, *Vice-President for Class I.*
Henry P. Walcott, *Vice-President for Class II.*
John C. Gray, *Vice-President for Class III.*
Edwin H. Hall, *Corresponding Secretary.*
William Watson, *Recording Secretary.*
Francis Blake, *Treasurer.*
A. Lawrence Rotch, *Librarian.*

Councillors for Three Years.

Elihu Thomson, of Class I.
William T. Sedgwick, of Class II.
John H. Wright, of Class III.

Member of Committee of Finance.

Eliot C. Clarke.

Rumford Committee.

Erasmus D. Leavitt,
Edward C. Pickering,
Charles R. Cross,
Amos E. Dolbear,
Arthur G. Webster,
Theodore W. Richards,
Elihu Thomson.

C. M. Warren Committee.

Leonard P. Kinnicutt,
Samuel Cabot,
Arthur M. Comey,
Robert H. Richards,
Henry P. Talbot.
Arthur Michael,
Charles R. Sanger.

The following standing committees were appointed by the Chair: —

Committee of Publication.

SETH C. CHANDLER, of Class I., EDWARD L. MARK, of Class II.,
CRAWFORD H. TOY, of Class III.

Committee on the Library.

HARRY M. GOODWIN, of Class I., SAMUEL HENSHAW, of Class II.,
HENRY W. HAYNES, of Class III.

Auditing Committee.

HENRY G. DENNY, FREDERICK J. STIMSON.

On the recommendation of the Rumford Committee, it was

Voted, To award the Rumford Premium to Ernest Fox Nichols of Columbia University.

On the motion of the Recording Secretary, it was

Voted, That the thanks of the Academy be tendered to William M. Davis for his faithful and efficient services as Corresponding Secretary.

David G. Lyon gave an account, illustrated with lantern projections, of "The Recently Discovered Code of Laws, Promulgated by Hammurabi, King of Babylon, Twenty-third Century B.C."

The following papers were presented by title: —

"Studies on Transformation of Saturnia Moths, with Notes on the Life-History and Affinities of Brahmaea Japonica." By A. S. Packard.

"Heredity of Coat Color in Mice," by G. M. Allen. Presented by E. L. Mark.

"Sexual Reproduction in the Mucorineae," by A. F. Blakeslee. Presented by R. Thaxter.

"On Mandragora as an Anæsthetic," by C. R. Randolph. Presented by M. H. Morgan.

"A Problem in Statics and its Relation to Certain Algebraic Invariants," by Maxime Bôcher.

Contribution No. 2 from the Research Laboratory of Physical Chemistry, Massachusetts Institute of Technology, "The Electrical Conductivity of very Dilute Solutions of Hydrochloric and Nitric Acids," by H. M. Goodwin and Raymond Haskell. Presented by H. M. Goodwin.

AMERICAN ACADEMY OF ARTS AND SCIENCES.

Report of the Council. — Presented May 11, 1904.

BIOGRAPHICAL NOTICES.

Horace Gray	Francis C. Lowell.
William Sumner Appleton	Charles C. Smith.
James Elliot Cabot	T. W. Higginson.
Alfred Perkins Rockwell	R. H. Richards.
Horatio Hollis Hunnewell	C. S. Sargent.

REPORT OF THE COUNCIL.

The Academy has lost eleven members by death since the last report of the Council at the annual meeting of May 13, 1903: two Resident Fellows, Alfred P. Rockwell, Charles S. Storrow; three Associate Fellows, H. E. von Holst, J. P. Lesley, and George S. Morison; six Foreign Honorary Members, L. Cremona, K. Gegenbaur, W. E. H. Lecky, T. Mommsen, Sir L. Stephen, K. A. von Zittel. One Resident Fellow has resigned; one Associate Fellow has been transferred to Resident Fellowship.

New members have been elected as follows: — Resident Fellows (including two elected May 13, 1903), 4; Associate Fellows, 5; Foreign Honorary Members, 6. The roll of the Academy now includes 199 Resident Fellows, 100 Associate Fellows, and 73 Foreign Honorary Members.[1]

HORACE GRAY.

Horace Gray was born in Boston, March 24, 1828, the son of Horace Gray and Harriet Upham. His mother, the daughter of Jabez Upham, a distinguished lawyer, died while he was a little child, and when he was nine years old his father married Sarah Russell Gardner, the mother of his half-brothers, John Chipman and Russell. His paternal grandfather was William Gray, the famous merchant and the owner of a great fleet of ships. Horace Gray graduated at Harvard College in 1845, being in his eighteenth year, very young even for those days. As a youth he knew animals, birds, and flowers. He was famed among boys for the accuracy with which he threw a stone, sometimes hitting a bird on the wing. After graduation he went to Europe for general travel, but was recalled to this country by the failure of his father in business. He entered the Harvard Law School in 1847, two years after his graduation, younger, nevertheless, than most of his fellow-students. He took the degree of LL.B. in 1849, studied in the office of Sohier and Welch, and was admitted to the Bar in 1851. Soon afterward he was employed by Mr. Cushing, reporter of decisions, to go on circuit with the full bench. As has been said by Senator Hoar, "He had already acquired a great stock of learning for a man of his age. Even then his wonderful capacity for research, the instinct which, when some interesting question of law was up, would direct Gray's thumb and finger to some obscure volume of English Reports of Law or Equity, was almost like the scent of a wild animal or a bird of prey. When any one of the great lawyers of Massachusetts was arguing or was waiting to argue a great case, the young reporter would often appear to him with a case which the counsel had not discovered, and was pat to the question. So, although he was hardly out of his boyhood, they all got to like him as a companion, and to respect him as a lawyer. When Cushing died, most of these lawyers joined in a recommendation for the appointment of Gray, who was then but twenty-six years old." He was reporter for six years, being also engaged in practice during the time. For about two years he was a partner of Ebenezer Rockwood Hoar, afterwards his associate in the Supreme Court of Massachusetts and Attorney-General of the United States. Later he was in partnership with Wilder Dwight and Charles F. Blake. Mr. Gray's practice at the bar was considerable, though not very large; the cases which he argued were more important than numerous. He was appointed by Governor Andrew associate justice of the Supreme Judicial Court of Massachusetts, August 23, 1864; chief justice of the same court by Governor Washburn, September 5, 1873; and finally was appointed associate justice of the Supreme Court of the United States by President Arthur, December 20, 1881. He resigned that office August, 1902, and died at Nahant on the fifteenth day of September, in that year.

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[1] Note: In the Proceedings of last year, Vol. XXXVIII., p. 730, the number of Associate Fellows at the time of the Annual Report of the Council should have been given as 99 (not 98), and of Foreign Honorary Members as 73 (not 72).

Law is more than justice. Its administration seeks something more than justice between man and man in the particular case before the court. It is order as well as justice, established that the citizen may know, surely and beforehand, what will be the consequence of his acts, so that he may direct them accordingly. In not a few matters, it is better that the law should be settled than that it should be right. If given words have an interpretation absolutely fixed, for example, they can be used with safety, however absurd may have been the interpretation originally reached; but if they are interpreted now in one sense and now in another, no man is safe.

Horace Gray had a strong sense of order. He had a great memory, and its order was even more remarkable than its extent and its accuracy. His learning was ready for use, so arranged that everything which related to the subject he was considering could be produced without admixture of the irrelevant. His mental index was so perfect that nothing was found under the wrong heading. The orderly administration of courts he delighted in. It is literally true, I believe, that, without notice, he could have discharged in any American or English court the duties of any officer from crier to chief justice, so that his example would have profited the regular incumbent. He had been a model reporter of the opinions of others, and he aided greatly those who reported his own opinions. The ordinary judge, wisely enough, leaves clerical details to his clerk, and refuses to burden his mind with them. Judge Gray thoroughly respected a good clerk, and was outspoken in his commendation when he found one; but he always directed his clerk in nice technicalities, and a good clerk knew the value of his direction. His standard of decorum was high. He perceived that faults of deportment, insignificant in themselves, when often repeated may bring a court into real disrespect. As head of the judiciary of Massachusetts, he had to maintain by example the dignity of lesser men as well as his own. If his masterfulness and a certain quickness of temper made him not always tactful in his rebukes, it must be remembered that the leaders of the bar were the sufferers, and that a beginner was often helped by his exceptional kindness and gentleness. The rule by which he ordered his own actions was severer than that which he applied to the conduct of others. "While any capital case was on trial before him, in the days when such trials were had in the Supreme Judicial Court, he never took part in any social function or entertainment. He was once heard to say that he believed that, when a man's life was at stake, the judge presiding at the trial should give himself entirely to the cause before him, and that during the trial, while not in court, he should hold himself aloof from the ordinary distractions of the world, and keep his mind in contemplation only of the responsible and serious duty which he was performing." *

These matters of detail, important as he deemed them, were not allowed to encumber his administration of the law. His knowledge of the elaborate system of equity was used to give relief to a deserving suitor, not to delay or deny justice. In his administration equitable remedies, such as injunctions, were used to make good the defects of the common law, not sold, like common-law writs, for a fee to the clerk. On the other hand, he developed and extended those remedies when a new need called for a novel exercise of the jurisdiction of the chancellor. He was an historical rather than a philosophical lawyer. As he himself put it in one of his earliest opinions, "This subject illustrates the wisdom

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* Memorial of the Boston Bar Association, p. 13.

of the common law in taking for its guides judicial opinions, given after argument, under the responsibility of determining the rights of parties in actual controversies, rather than the theories of scholars and commentators, however learned or acute." * He liked historical research, and, until he knew everything that history could tell him, he was unwilling to decide a case. He had both the thoroughness and the accuracy of the scholar. No relevant detail was overlooked. Having learned the development of the doctrine which governed the case before him, he had the scholar's proper pride in setting out that development so fully that no other man need repeat the investigation. In his longer opinions, the historical statement often made the largest part, but this could hardly be otherwise. He did not value precedents for their age or peculiarity. He turned to the Year Books not very often, and always referred to the case last decided in the authoritative courts of England and the United States. His antiquarian learning did not cost him his sense of proportion.

Once a judge, he gave to his judicial duties his entire service. With his talents and habits of mind, that service would have been of great value had it begun only in middle life. Appointed to the Supreme Court of Massachusetts at thirty-six, he had gathered before middle life a fund of judicial experience. For a year he sat on the bench of Massachusetts with Theron Metcalf, born five years before the adoption of the federal constitution. As circuit justice of the first circuit, he had for four years as a subordinate colleague a district judge seventy-one years Judge Metcalf's junior, born only six years before the outbreak of the rebellion. His opinions are contained in forty-three volumes of Massachusetts reports and in seventy-nine volumes of reports of the Supreme Court of the United States. He learned to be a judge while his habits of mind were flexible. In matters other than judicial, his opinions were vigorously expressed without much hesitation, and some of his prejudices were considerable. In deciding a case, his consideration was not less patient than was his research exhaustive. As his secretary during his last twenty months in Massachusetts, I observed that he examined literally every case cited in every brief, unless the case cited was already familiar to him. Lawyers know almost as well as judges how recklessly citations are sometimes shovelled into a brief in the hope that the court may find in them something relevant which counsel have not had time to discover, and they will appreciate Judge Gray's unalterable patience. One of his latest secretaries has told me

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* Kimball *v.* Aetna Ins. Co., 9 All. 540, 551.

that his patience was the same when he was seventy years old. Again, he found in me, as I doubt not he found in some of the other young men who afterwards served him, a youth fresh from the law school, ready to argue with a good deal of confidence any legal point whatsoever, and not inclined to drop the argument when he had before him so lively and open-minded a disputant as the chief justice. Yet I remember that he never checked my persistence but once, and then most gently. He was in no hurry to make up his mind, glad even to the last moment to hear anything which might properly affect his decision. After leaving his brother judges, he began his further consideration of a case by discussion with his secretary, and examined and considered the authorities by himself or with the help of his secretary in bringing the books. He used no stenographer. In Massachusetts his dictation was taken down in long hand; in Washington he wrote the first rough draft with a pencil, and this was copied out by his secretary and corrected by the judge in printed proof. His handwriting — rather handsome when given room to extend itself — when confined was not always easily legible. Proud was the moment when a secretary was able to read an inscription which the judge himself could not decipher. I have said that he kept his mind always open to reconsideration for sufficient reason, but he had none of that paralysis of doubt which leads some otherwise excellent judges to postpone a decision until the decision has ceased to be of much use. He knew that a decision upon matters of fact can hardly be made too soon. "He at once made it a rule," Mr. Samuel Hoar has told us, "always to decide every question of fact submitted to him for determination before he went to bed that night, and to make at the time a minute of the decision; the questions of law could safely wait for a more deliberate examination."

He excelled at Nisi Prius, and at equity sittings, but his lasting reputation, like that of other great judges, is based upon his written opinions. His style had not much of that felicity and distinction which sometimes — though rarely — make literature of a law report, but his meaning was clear. This was his object, and almost his only one. He did not deem himself to be the sole author of the opinion which he composed, but rather the mouthpiece of a body of which he was only one member. Hence this style was impersonal. He spent no time searching for a neatly turned phrase, though he would change his words over and over again if their meaning seemed obscure. Rhetorical effort in the expression of matters not purely literary, even when successful, often savors of affectation. The opinions of some judges manifest this affectation; his never. Nature and experience enabled him to write easily.

His skill in framing rules, decrees, and such formal matters, helped by his sense of order, was wonderful. In the correction of all that he wrote he spared no pains.

As has been said, his historical study made not a few of his opinions longer than they would have been if written by another judge. Doubtless he gave himself to historical research in cases where another judge would have neglected it, satisfied that research could not affect the decision. I do not find that his historical exposition was ever irrelevant. That the experience of the past has important bearing upon such matters as the lawfulness of acts done under color of military authority, the precise definition of the term soldier,* an abutter's ownership of the fee in a street,† the legality of a lease of land in a rebel state,‡ to take a few of his earliest important opinions at random, must be plain even to one who is not a lawyer. And, as he said himself, "In construing any act of legislation, whether a statute enacted by the legislature, or a constitution established by the people as the supreme law of the land, regard is to be had, not only to all parts of the act itself, and of any former act of the same law-making power, of which the act in question is an amendment; but also to the condition, and to the history, of the law as previously existing, and in the light of which the new act must be read and interpreted." § So also of cases concerned with the conflict of laws and with international law.‖ In the Arlington Heights case, the wealth of learning contained in his dissenting opinion has seemed to many persons, as it seemed to the majority of the court, beside the question involved; but it must be remembered that three of his colleagues agreed with him, among them Judge Bradley, after Gray himself the most learned member of the court.

"He is often called a great case lawyer," said Mr. E. R. Thayer, one of his secretaries in Washington, "as if his greatest gifts were in the direction of recording the thoughts of others rather than in original thought of his own. I believe this to be a misconception, and one for which the style of his opinions is largely responsible. I have often regretted some features of this style, as failing to do justice to himself; but these features were due primarily to a conception of the judge's duty to which he held with the strength that characterized all his views on questions of principle and obligation. In the matter of giving credit to others he held himself up to the strictest standard applicable to forms of literature in which originality is a more essential feature than in opinion writing, and would say nothing as his own which another judge had said before him. This, with a knowledge of the past which seemed like a miracle, and a habit of condensing his own statements as severe as his quotations from others were generous, made it appear as if the larger part of his work was that of marshalling cases. But any of his secretaries knows that it was not so; and I suppose there is not one of them who has not many times urged him, as I have so often done, to incorporate in his opinion the forcible and brilliant original reasoning which, as we listened to it, seemed to bring light into the darkest corners of his subject. But he would only answer, 'You will find it all there' (indicating a passage where another judge had said something like it, but generally, as it seemed to us, not half so well), or else he would point to the passage which can always be found somewhere in his opinions, usually near the end, in which his own matter is stated with the condensation almost of a head-note, and would say, 'Why does n't that cover it, after all?'"

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* Tyler *v.* Pomeroy, 8 All. 480.

† Boston *v.* Richardson, 8 All. 146.

‡ Kershaw *v.* Kelsey, 100 Mass. 561.

§ U. S. *v.* Wong Kim Ark, 169 U. S. 649, 653, 654.

‖ Ross *v.* Ross, 129 Mass. 243; The Paquete Habana, 175 U. S. 677.

His exposition was not the collocation of unrelated facts. Fair in his statement, he did not put his opinion together until he saw the result which his study and exposition would establish, and the bearing upon that result of every precedent cited. This is conspicuous in Jackson *v.* Phillips, 14 All. 539, where he established the extent of the great doctrine of charitable trusts, which has done so much for Massachusetts. Here his language, without rhetorical exaggeration, has an unusual stateliness. So firmly is the opinion put together, that it can be appreciated fully only by reading it as a whole, but some of its paragraphs may be taken as illustrations of his style. The court had to determine the validity of a bequest made in 1861 to "create a public sentiment that will put an end to negro slavery in this country."

"The bequest itself manifests its immediate purpose to be to educate the whole people upon the sin of a man's holding his fellowman in bondage; and its ultimate object, to put an end to negro slavery in the United States; in either respect, a lawful charity.

"It is universally admitted that trusts for the promotion of religion and education are charities. Gifts for the instruction of the public in the cure of the diseases of quadrupeds or birds useful to man, or for the prevention of cruelty to animals (either by publishing newspapers on the subject, or by providing establishments where killing them for the market might be attended with as little suffering as possible), have been held charitable in England. London University *v.* Yarrow, 23 Beav.

159; s. c. 1 De Gex & Jones, 72; Marsh *v.* Means, 3 Jur. (N. S.) 790; Tatham *v.* Drummond, 11 Law Times (N. S.) 325. To deliver men from a bondage which the law regards as contrary to natural right, humanity, justice, and sound policy, is surely not less charitable than to lessen the sufferings of animals. . . . This bequest directly tends to carry out the principles thus declared in the fundamental law of the Commonwealth. And certainly no kind of education could better accord with the religion of Him who came to preach deliverance to the captives, and taught that you should love your neighbor as yourself and do unto others as you would that they should do unto you.

"The authorities already cited show that the peaceable redemption or manumission of slaves in any manner not prohibited by law is a charitable object. It falls indeed within the spirit, and almost within the letter, of many clauses in the Statute of Elizabeth. It would be an anomaly in a system of law, which recognized as charitable uses the relief of the poor, the education and preferment of orphans, marriages of poor maids, the assistance of young tradesmen, handicraftsmen, and persons decayed, the relief of prisoners and the redemption of captives, to exclude the deliverance of an indefinite number of human beings from a condition in which they were so poor as not even to own themselves, in which their children could not be educated, in which marriages had no sanction of law or security of duration, in which all their earnings belonged to another, and they were subject, against the law of nature, and without any crime of their own, to such an arbitrary dominion as the modern usages of nations will not countenance over captives taken from the most barbarous enemy.

"The next question arises upon the bequest in trust for the benefit of fugitive slaves who might from time to time escape from the slaveholding States of the Union.

"The validity of this bequest must be determined according to the law as it stood at the time when the testator died and from which his will took effect. It is no part of the duty of this court to maintain the constitutionality, the justice, or the policy of the fugitive slave acts, now happily repealed. But the Constitution of the United States, at the time of the testator's death, declared that no person held to service or labor in one State should be discharged therefrom by escaping into another. It may safely be assumed that, under such a constitution, a bequest to assist fugitive slaves to escape from those to whom their service was thus recognized to be due, could not have been withheld and enforced as a lawful charity. The epithets with which the testator accompanied this bequest show that he set his own ideas of moral duty above his allegiance to his State or his country; and warrant the conjecture that he would have been well pleased to have the fund applied in a manner inconsistent with the Constitution and laws of the United States. But he has used no words to limit its use to illegal methods, and has left his trustees untrammelled as to the mode of its application. . . .

"A bequest for the benefit of fugitive slaves is not necessarily unlawful. The words 'relief or redemption of prisoners and captives' have always been held in England to include those in *prison* under condemnation for crime, as well as persons confined for debt; and to support gifts for distributing bread and meat among them annually, or for enabling poor imprisoned debtors to compound with their creditors. Duke, 131, 156; Attorney General *v.* Ironmongers' Co., C. P. Coop. Pract. Cas. 285, 290; Attorney General *v.* Painterstainers' Co., 2 Cox Ch. 51; Attorney General *v.* Drapers' Co., Tudor, 591, 592; s. c. 4 Beav. 67; 36th Report of Charity Commissioners to Parliament, pt. vi. 856-868. It would be hardly consistent with charity or justice to favor the relief of those undergoing punishment for crimes of their own committing, or imprisonment for not paying debts of their own contracting; and yet prohibit a like relief to those who were in equal need, because they had withdrawn themselves from a service imposed upon them by local laws without their fault or consent. . . .

"To supply sick or destitute fugitive slaves with food and clothing, medicine and shelter, or to extinguish by purchase the claims of those asserting a right to their service and labor, would in no wise have tended to impair the claim of the latter or the operation of the Constitution and laws of the United States; and would clearly have been within the terms of this bequest. If, for example, the trustees named in the will had received this fund from the executor without question, and had seen fit to apply it for the benefit of fugitive slaves in such a manner, they could not have been held liable as for a breach of trust.

"This bequest, therefore, as well as the previous one, being capable of being applied according to its terms in a lawful manner at the time of the testator's death, must, upon the settled principles of construction, be held a valid charity."

A like subordination of great historical learning and research to the questions raised in a particular case is exemplified in United States *v.* Wong Kim Ark, 169 U. S. 649, decided thirty years after Jackson *v.* Phillips. The question there concerned the citizenship of a child of Chinese parents living in the United States at the time of his birth. After elaborate historical exposition covering some forty octavo pages, the matter is thus summed up.

"The foregoing considerations and authorities irresistibly lead us to these conclusions: The Fourteenth Amendment affirms the ancient and fundamental rule of citizenship by birth within the territory, in the allegiance and under the protection of the country, including all children here born of resident aliens, with the exceptions or qualifications (as old as the rule itself) of children of foreign sovereigns or their ministers, or born on foreign public ships, or of enemies within and during a hostile occupation of part of our territory, and with the single additional exception of children of members of the Indian tribes owing direct allegiance to their several tribes. The Amendment, in clear words and in manifest intent, includes the children born within the territory of the United States, of all other persons, of whatever race or color, domiciled within the United States. Every citizen or subject of another country, while domiciled here, is within the allegiance and the protection, and consequently subject to the jurisdiction, of the United States. His allegiance to the United States is direct and immediate, and, although but local and temporary, continuing only so long as he remains within our territory, is yet, in the words of Lord Coke, in Calvin's Case, 7 Rep. 6*a*, 'strong enough to make a natural subject, for if he hath issue here, that issue is a natural-born subject;' and his child, as said by Mr. Binney in his essay before quoted, 'if born in the country, is as much a citizen as the natural-born child of a citizen, and by operation of the same principle.' It can hardly be denied that an alien is completely subject to the political jurisdiction of the country in which he resides — seeing that, as said by Mr. Webster, when Secretary of State, in his Report to the President on Thrasher's Case in 1851, and since repeated by this court, 'independently of a residence with intention to continue such residence; independently of any domiciliation; independently of the taking of any oath of allegiance or of renouncing any former allegiance, it is well known that, by the public law, an alien, or a stranger born, for so long a time as he continues within the dominions of a foreign government, owes obedience to the laws of that government, and may be punished for treason, or other crimes, as a native-born subject might be, unless his case is varied by some treaty stipulation.'" 169 U. S. 693, 694.

Even in his longest opinions I do not find diffuseness of expression. The majority in number of his opinions are both short and terse, not a few of these dealing with important questions of law. A good example of compressed statement and reasoning is Osborne *v.* Morgan, 130 Mass. 102, a case concerned with liability for negligence causing damage to a fellow servant, covering four pages and a half.

While he was a judge, without thought of other ambition, yet he was no narrow specialist. He worked hard and fast, without haste, and took substantial vacations. He said that a man could do more work in five days of the week than in six, in eleven months of the year than in twelve. To speak as a New Englander, he loafed not infrequently when his work was over, but he never dawdled. He loved to be out of doors, went often into the woods fishing and shooting; notwithstanding his great size he rode on horseback until middle life, and later took up the game of golf. He read many books of many sorts with marvellous quickness. His literary taste was simple and healthy. He admired the English of the Bible, and wished that he might be a delegate to any convention of the Episcopal Church before which a proposal to substitute the revised for the authorized version should come up. His contribution to the discussion would have been both learned and vigorous. He was always interested in public affairs, and at considerable sacrifice retained his citizenship in Boston during the last twenty years of his life. He could seldom vote here, but he took care that his name was kept on the voting list. He liked good company. With habits of mind fresh and almost boyish to the end of his life, he liked especially the society of the young. Young people in Washington spoke of his liveliness and the interest they found in his conversation.

He was married on June 4, 1889, to Jane, daughter of his late colleague, the Honorable Stanley Matthews, and the marriage was serenely happy. His parents were Congregationalists, and he was brought up in that communion. In middle life he attended the services at Trinity Church, Boston, whose rector, Phillips Brooks, was his friend. He was confirmed in the Protestant Episcopal Church at Washington, in 1889, and was thereafter a regular communicant. The strength of his religious feelings was manifested throughout his life in private and on every fitting public occasion; it may be judged by his commemoration of his colleagues in the Supreme Court of Massachusetts, Chief Justice Chapman and Justices Metcalf, Wells, and Colt, and by his oration on Chief Justice Marshall.

He was not a pioneer like Mansfield and Marshall, breaking out a way through regions of law hitherto untravelled; a pioneer comes hardly once in a century of our legal history: but for thirty-eight years he labored with success to make the crooked ways of the law straight and its rough places plain.

FRANCIS C. LOWELL.

WILLIAM SUMNER APPLETON.

William Sumner Appleton was born in Boston January 11, 1840, and died in his native city April 28, 1903. He was the elder son of the Hon. Nathan Appleton, for nineteen years a Fellow of the Academy, and for three sessions a Representative in Congress from the Boston district, by his second wife, Harriot Coffin, daughter of Jesse Sumner, of Boston. On both his father's side and his mother's he was descended from ancestors who were among the first settlers of Massachusetts, Samuel Appleton at Ipswich and William Sumner at Dorchester. Of his father it was truly said, not long after his death, that he was "a merchant of large enterprise and unsullied integrity; a member of many learned societies; a writer of many able essays on commerce and currency; a wise and prudent counsellor in all private and public affairs; who had served with marked distinction in the legislative halls both of the State and of the nation, and who had enjoyed through life the esteem, respect, and confidence of the community in which he lived."* And in the equally just words of another: "Emphatically a merchant, his mind was not narrowed nor his heart contracted by the influence of a special calling. His inquiry and his information were extensive. Acquainted with political history and the principles of civil government, with all questions of national finance, with some branches of physical science, with Christian theology and biblical criticism, so far as was needful for the stability or the defence of his own faith, he was ready to communicate the fruits of his research for the benefit of others."† Such was the example which was set before young Appleton as he was growing up, and such the sturdy and sterling qualities which he afterward exhibited in his own life and character.

As a small boy he was sent to a boarding-school at Jamaica

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* Memoir by Hon. Robert C. Winthrop in Proceedings of the Massachusetts Historical Society, vol. v. p. 250.

† Sermon preached July 21, 1861, the Sunday after the Funeral of the late Hon. Nathan Appleton, by Ezra S. Gannett, D.D., p. 17.

Plain, then kept by Mr. Cornelius M. Vinson, which had enjoyed a wide reputation under the management of Mr. Charles W. Greene, its first principal. He was fitted for college in the private Latin School of Mr. Epes S. Dixwell, who had recently resigned the head mastership of the Boston Latin School, and who was the most eminent classical teacher in this community. The thoroughness of his preparation was seen in his high rank in the first three years of his college course. He graduated in 1860 in the first half of his class, being entitled to a part at Commencement, a disquisition on "A Uniform Coinage." This assignment shows, as does his election to membership in the New England Historic-Genealogical Society, in February, 1859, while he was a Junior in college, at how early a period those tastes were developed which he afterward cultivated so assiduously and so successfully; and it is little matter for surprise that he did not gain the high distinction in college which might have been anticipated for him if his attention had not been diverted by other, and to him more fascinating, studies. In the year in which he graduated he was one of the founders of the Boston Numismatic Society, and he was its secretary from that time until his death. His interest in coins and medals dates back to his boyhood, and is mentioned in some of his earliest letters. This study never lost its attractiveness for him, and was pursued to the end of his life. At his death his collection, which was remarkable both for the beauty and the rarity of its specimens, included about twelve thousand coins and three thousand medals, the greater part of which he had himself catalogued with great minuteness. Besides being one of the original members of the Numismatic Society, he was one of the Publishing Committee of the American Journal of Numismatics from 1870 to 1891, and made many short communications to it. Closely akin to his interest in numismatics, and growing out of it, was his interest in heraldry, which he had thoroughly studied and well mastered. Probably no person on this side of the Atlantic had a more minute and more comprehensive knowledge of the subject; and his opinion on any question connected with it was wellnigh conclusive. From 1865 to 1868 he was one of the Publishing Committee of the short-lived Heraldic Journal, and wrote much for it. In 1867, at the early age of twenty-seven, he was honored with an appointment as one of the commission to attend the annual assay of the United States Mint. It was a well-merited compliment, and reflected scarcely less honor on the administration which conferred it than it did on the recipient. From 1864 to 1872 he was one of the Publishing Committee of the New England Historical and Genealogical Register.

Immediately after graduating Mr. Appleton spent two months in travel in Canada and the Western States; and in April, 1862, he went abroad for the first of six visits to Europe, passing most of the summer and autumn in Great Britain, and reaching home at the end of November. Inheriting from his father, who died in 1861, and his mother, who died in 1867, an ample fortune, he had no need to pursue any gainful profession; but he nevertheless entered the Cambridge Law School, and graduated in July, 1865, with the degree of LL.B. Two weeks later he again sailed for Europe. He remained abroad a little more than a year, and travelled in France, Belgium, Holland, Germany, Switzerland, Austria, Turkey, Palestine, Egypt, Greece, and Italy. During a large part of this trip he had Phillips Brooks, afterward Bishop Brooks, as his travelling companion, and for four weeks in the Holy Land they rode side by side during the day, and slept every night in the same tent. In less than two years after his return, in June, 1868, he started for a journey around the world. Besides countries already mentioned he visited Ireland, Denmark, Sweden, Russia, Poland, Dalmatia, Montenegro, and Spain; and in January, 1869, he sailed from Marseilles for Bombay and the Far East. After travelling in Hindostan and visiting most of the accessible places of interest in the Philippines, China, and Japan, he reached home by the way of San Francisco in the autumn of 1869. While absent he was elected, in May, 1869, a member of the Massachusetts Historical Society; and for nearly thirty-four years, whether at home or abroad, he took a deep and active interest in the work of the Society. In a letter to the writer of this memoir, from Pyrmont in June, 1887, he expressed a strong wish to have the serial parts of the Proceedings sent to him, and added: "It is a pleasure to keep up with the work of the Society, and there might chance occasionally to be a suggestion of something which one could do on this side the water."

In February and March, 1871, he visited the Southern States and Cuba. But this seems only to have increased his fondness for more distant travel; and in the following June he went again to Europe, remaining abroad rather more than a year. On this visit, August 12, 1871, he was married, at the United States Legation in Berne, Switzerland, to Edith Stuart, daughter of his cousin William Stuart Appleton, of Baltimore, Maryland. His eldest child, a daughter, was born in Paris, before his return to Boston.

The next four years were passed at home; and at the annual meeting of the Historical Society in April, 1873, he was elected a member of the Standing Committee. At the end of a year's service he was elected Cabinet-Keeper, which office he filled for six years. On the creation of the Record Commission of Boston, in 1875, he was appointed one of the two Commissioners, and he held this office, to which no salary was attached, until the Commission was abolished in July, 1892. As a Commissioner he edited the Ninth Report, "Boston Births, Baptisms, Marriages, and Deaths, 1630–1699"; the Twenty-first Report, "Dorchester Births, Deaths, and Marriages"; and the Twenty-fourth Report, "Boston Births, 1700–1800."

In November, 1876, he went to Europe for the fifth time, returning in June, 1877. During this visit he extended his travels to the islands of Sicily and Malta. After his return, in November, 1878, he was elected a Fellow of the American Academy of Arts and Sciences in Class III., Section II., Philology and Archæology. He never, however, contributed anything to the printed Transactions of the Academy.

After remaining at home for nearly nine years, actively engaged in his favorite pursuits, he sailed, in May, 1886, for his last and longest visit to Europe, returning in June, 1889. Most of the time was passed in Germany, France, England, Belgium, Holland, and Italy; and for two successive summers he resided in the little town of Pyrmont, once a noted watering-place, in Germany, not far from Hanover, where he spent one winter. A little more than two years after his return, January 20, 1892, his wife died. The remainder of his life was spent quietly, a part of each year in a new house which he had built on Beacon Street, near Hereford Street, and a part on his large and beautiful estate near Oak Hill, Newton Centre. For a country life he acquired a genuine taste, and often spoke of his reluctance to come into town. He found a congenial occupation in cutting his own trees and overseeing the routine of a farmer's life.

In 1898 Mr. Appleton drew up a short paper which was found after his death in an envelope marked "Autobiographical Statement," and which begins as follows: "A short memoir of my life must be written for several Societies to which I belong, the most important perhaps being that for the Massachusetts Historical Society. I hope that in that one at least something may be printed to the following effect, being all that I care to leave in the form of an autobiographical statement." This paper is so characteristic of the writer that a biographer would find small excuse for not complying with so plain an injunction. With one or two unimportant omissions it is here given just as it was written: "It has been my fortune, or misfortune, to be generally in the minority. I have always belonged to a religious body which is small in acknowledged numbers, even if it has been, as I think, of far greater influence than mere figures would warrant; and in this body I seem at this date to be in a small minority, one of those who adhere to old-fashioned, conservative, Christian Unitarianism. I was an independent in politics long before the mugwump was imagined, and almost never voted a straight party ticket, but declined to allow my name to be brought before nominating conventions, republican and democratic. . . . I have alone voted *no* in a meeting of my class at Harvard, and did not regret it. I was one of those who voted against the amendment which allows a pauper to be chosen Governor of the Commonwealth. I have always regretted the death of the Election Sermon and the murder of Fast Day. I am one of those, apparently few in this country, who think that things are not necessarily bad because old or old-fashioned, and therefore I deeply regret that Harvard's catalogue of her graduates is no longer printed in Latin. I am one of those who object to the naked boys over the door of Boston's Public Library, as poor in art, worse as part of a seal, and worst of all in their offensive vulgarity."

To this he added: "I claim the credit of the discovery of the two 'Candler' manuscripts in the Bodleian Library at Oxford, and the recognition of their value to American genealogists. I also claim the credit, at least equally with any other person, of first calling attention to the importance for American genealogists and families of the wills at Doctors' Commons, now Her Majesty's Principal Registry of Probate. . . . In 1885 I established the arms and seal of Harvard University, for which I received the formal thanks of the Corporation."

Filed with this statement was a copy of a clause in a will drawn up by him in June, 1868, before his marriage, "which circumstances happily kept from realization," and of which he very much wished mention should be made in any record of his life. By it he gave to fifteen trustees, most of whom are no longer living, "the sum of two hundred and fifty thousand dollars, together with my real estate lying on Commonwealth Avenue and Clarendon Street in said Boston, being two lots,— to be held by them in trust, and for such uses and purposes as are understood by the words 'Gallery of Art and School of Design,' and I desire that they should become and be a corporation of fifteen members, with power to fill any vacancies in their number, and that the consent and assent of nine should be necessary to any act in order to be binding; and that a suitable building be erected on the land in their hands at an expense not to exceed one hundred and twenty-five thousand dollars. I also give them all works of art, such as paintings, which are in my possession, and which are not family memorials, and my wish and hope is, that such action should be taken by them as that the collection of works of art belonging to the Boston Athenæum should be permanently placed in the building to be erected by them. And my object in this bequest is to contribute as far as possible to the improvement and development of the Fine Arts in this country, to which end I earnestly desire the above-named trustees to labor in the exercise of this trust which I give to them, confiding in their ability and readiness to perform the same so that it may be a means of great benefit." * Before leaving this subject it may be proper to add that by his last will, dated April 25, 1902, Mr. Appleton gave in a certain remote contingency a very valuable and important part of his collection of medals and coins to the Historical Society; and in another more remote contingency he directed the trustees under his will to pay over specified sums to numerous literary, scientific, and charitable societies or institutions, among which were the Massachusetts Historical Society, the American Academy of Arts and Sciences, and the New England Historic-Genealogical Society.

Mr. Appleton's investigations in numismatics, heraldry, and genealogy, the studies which earliest and longest engaged his attention, were thorough, exact, and methodical. He was never satisfied until he had reached the final result; and in the case of the Sumner Genealogy he printed ten supplements. He never allowed himself to be deceived by pretentious claims to family antiquity and high descent based on no sufficient foundations; and he remorselessly pricked many bubble reputations. Besides his numerous contributions to the various periodicals of which he was either an editor or contributor, Mr. Appleton printed separately at least ten genealogies and other works. The earliest of these was an account, printed

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* The will from which this is an extract was drawn up before the first movement for establishing the Museum of Fine Arts. That institution was not incorporated until February, 1870.

in 1867, of the "Ancestry of Mary Oliver," wife of Samuel Appleton, Jr., of Ipswich, son of the first immigrant, with an Appendix. His next publication was an elaborate account, printed in the following year, of the "Cranes of Chilton," from whom also he was descended. In the Introduction to it he makes a very characteristic statement, which shows how thorough and accurate was his work: "In preparing this record of a line of my own ancestry, I am glad to have an opportunity of showing the richness of genealogical material in England, and also the great difficulty of forming a sure and true pedigree. The Cranes of Chilton, though belonging to the gentry of the county of Suffolk, were by no means an important family in English history; yet I have been able to examine at least twenty manuscripts, giving a genealogy of this race, no one of which is free from mistake." This was followed two years afterward by a third contribution to his family genealogy, in a still larger volume on the "Ancestry of Priscilla Baker," wife of Isaac Appleton, of Ipswich. "The chief reason for printing this volume," he says in his Introduction, "is the opportunity which it gives of preparing better and fuller genealogies of the families of Symonds and Reade than can be found in any book which I have ever seen." It is a perfect storehouse of wills and other documents relating to the families mentioned. Next we have, in 1873 and 1874, two editions of the "Genealogy of the Appleton Family." Of these only a very small number of copies were printed, as a rough sketch of a larger Genealogy of the family in its various branches which it was his intention and hope to prepare, but which was left unfinished at his death. His largest and most important work in this field was the "Record of the Descendants of William Sumner, of Dorchester, Mass., 1636," published in 1879, and continued by supplementary leaflets almost to the last year of his life. After a considerable interval he printed, in 1893, an important collection of "Early Wills illustrating the Ancestry of Harriot Coffin, with Genealogical and Biographical Notes"; and this was followed in 1896 by "Gatherings toward a Genealogy of the Coffin Family." In 1899 he prepared and printed a small pamphlet entitled "The Family of Armistead of Virginia." This embodied little original research, but was mainly an abridgment and rearrangement of a larger genealogy prepared by Lyon G. Tyler, President of William and Mary College, and printed in the Quarterly of that institution. "The reason of my own interest in the family," says Mr. Appleton, "appears in the sixth generation. I have therefore arranged in simple genealogical shape so much of the matter in the Quarterly as was necessary for the purpose," which was to trace his wife's ancestry. His last genealogical publication was issued a little more than a year before his death under the title of "Family Letters from the Bodleian Library, with Notes." The collection comprises thirty-six letters relating to the family of Appleton of Little Waldingfield, the immediate relatives of the immigrant to New England. An examination of these independent publications shows how persistent was the energy which he put into his genealogical researches, and fully justifies his reputation as a genealogist.

His wide travels had stored his mind with a fund of useful knowledge, and on many subjects he was an authority of the first rank. His interest in the broader aspects of history, it must be said, was subsidiary to his interest in his earlier studies, and his very numerous communications in the Proceedings of the Historical Society, which were usually quite short, were mainly illustrative of them. These communications need not be enumerated here; and it will be sufficient to mention two or three of the more important ones. The first of these, and one of the longest, was made in April, 1870, at a social meeting held at the house in Beacon Street, which had been the residence of his father and mother. He then exhibited a rare collection of coins and medals relating to America, and communicated a minute description of them which is printed in the eleventh volume of the Proceedings of the Society.

In March, 1895, he communicated a complete roll of the United States Senate for the first century of its existence, with notes. This was a work of great labor and research, involving a vast amount of correspondence; and in the end he was able to ascertain the date and place of death of all the members, eight hundred and forty-eight in number, with only two exceptions. A separate edition was afterward published for sale. Two years later he read a short paper on the character and personal relations of the Whigs of Massachusetts during the period from 1840 to 1850. In February, 1898, after the sale of the Tremont Street estate, the stated meeting was held at his new house in Beacon Street, and the use of his parlors was offered to the Council for any other meetings which it might be convenient to hold there before the completion of the Society's new building. His last important contribution was in June, 1901, when he read a keen and searching paper on Heraldry in America.

Even a cursory examination of his contributions to the Proceedings of the Society indicates how great an interest he took in carrying on the work which its founders had in view, and what special fields of inquiry most attracted him. He never served on any Publishing Committee, and with the exception of one year on the Executive Committee and six years as Cabinet-Keeper he had no direct and avowed part in determining the policy of the Society; but few of the so-called working members did more or better work, in a quiet and unobtrusive way. His written style was clear, direct, and forceful, but it lacked that flexibility of expression which so often comes from constant practice, and in which specialists are so apt to be deficient. An ineffective delivery and an indistinctness of speech, especially in his later years, detracted much from the interest with which his papers and remarks were listened to; but their value was at once seen when written out in his clear and beautiful handwriting or transferred to the printed page.

Like his father, who printed late in life a correspondence with a clergyman of the Church of England on "The Doctrines of Original Sin and the Trinity," Mr. Appleton was a steadfast Unitarian of the older school — the school of Channing and Gannett — and looked with strong aversion on the latitudinarian tendencies which were creeping into the denomination. In 1896 he printed for private distribution a pamphlet entitled "Views of Unitarian Belief held by a Layman of Boston. Written for the Unitarian Club of Boston, but never read before it." It was a vigorous and outspoken statement of his theological opinions and of his total dissent from the then recent action of the National Conference of Unitarian Churches at Saratoga. He felt, as did many others, that the denomination had taken a further step backward from the high ground assumed at the formation of the Conference, thirty years before, in the city of New York. His earliest associations were with the "Federal Street meeting-house," where his father regularly worshipped; but after the removal of the congregation to Arlington Street, and later in life, he was an attendant at King's Chapel.

In his early years he was fond of dancing and social life. From 1864 to 1871 he was a member of the Independent Corps of Cadets, and for the last four years was in command of a company, with the rank of first lieutenant. After his wife's death he did not go much into society. In college he belonged to few if any of the undergraduate fraternities; but he was afterward a member of the Thursday Evening Club, which he frequently entertained at his house, and of the Union Club. He was also one of the founders of the Bostonian Society, an original member of the Eastern Yacht Club, and a member of the Boston Art Club, the Boston Society of Natural History, the Prince Society, the American Historical Association, and various other societies.

He had five children, a son and four daughters, all of whom were living at his death. To them he left an unsullied reputation, as did his father and a long line of honored ancestors on both sides of the Atlantic. To his associates in the societies in which he was an active worker, and especially to those who were brought into closest relation with him, he left a bright example of large and faithful service.

CHARLES C. SMITH.

JAMES ELLIOT CABOT.

Our late associate, Elliot Cabot, of whom I have been appointed to write a sketch, was to me from my college days an object of peculiar interest, on a variety of grounds. He was distantly related to me, in more than one way, through the endless intermarriages of the old Essex county families. Though two years and a half older, he was but one year in advance of me in Harvard College. He and his chum, Henry Bryant, who had been my schoolmate, were among the early founders of the Harvard Natural History Society, then lately established, of which I was an ardent member; and I have never had such a sensation of earthly glory as when I succeeded Bryant in the responsible function of Curator of Entomology in that august body. I used sometimes in summer to encounter Cabot in the Fresh Pond marshes, then undrained, which he afterwards described so delightfully in the "Atlantic Monthly" in his paper entitled "Sedge Birds" (XXIII. 384). On these occasions he bore his gun, and I only the humbler weapon of a butterfly net. After we had left college, I looked upon him with envy as one of the early and successful aspirants to that German post-collegiate education which was already earnestly desired, but rarely attained, by the more studious among Harvard graduates. After his return, I was brought more or less in contact with him, at the close of the "Dial" period, and in the following years of Transcendentalism; and, later still, I was actively associated with him for a time in that group of men who have always dreamed of accomplishing something through the Harvard Visiting Committee, and have retired from it with hopes unaccomplished. Apart from his labors as Emerson's scribe and editor, he seemed to withdraw himself more and more from active life as time went on, and to accept gracefully the attitude which many men find so hard, — that of being, in a manner, superseded by the rising generation. This he could do more easily, since he left a family of sons to represent in various forms the tastes and gifts that were combined in him; and he also left a manuscript autobiography, terse, simple, and modest, like himself, to represent what was in its way a quite unique career. Of this sketch I have been allowed to avail myself through the courtesy of his sons.

James Elliot Cabot was born in Boston, June 18, 1821, his birthplace being in Quincy Place, upon the slope of Fort Hill, in a house which had belonged to his grandfather, Samuel Cabot, brother of George Cabot, the well-known leader of the Federalists in his day. These brothers belonged to a family originating in the island of Jersey and coming early to Salem, Mass. Elliot Cabot's father was also named Samuel, while his mother was the eldest child of Thomas Handasyd Perkins and Sarah Elliot; the former being best known as Colonel Perkins, who gave his house and grounds on Pearl Street toward the foundation of the Blind Asylum bearing his name, and also gave profuse gifts to other Boston institutions; deriving meanwhile his military title from having held command of the Boston Cadets. Elliot Cabot was, therefore, born and bred in the most influential circle of the little city of that date, and he dwelt in what was then the most attractive part of Boston, though long since transformed into a business centre.

His summers were commonly spent at Nahant, then a simple and somewhat primitive seaside spot, and his childhood was also largely passed in the house in Brookline built by Colonel Perkins for his daughter. Elliot Cabot went to school in Boston under the well-known teachers of that day, — Thayer, Ingraham, and Leverett. When twelve years old, during the absence of his parents in Europe, he was sent to a boarding-school in Brookline, but spent Saturday and Sunday with numerous cousins at the house of Colonel Perkins, their common grandfather, who lived in a large and hospitable manner, maintaining an ampler establishment than is to be found in the more crowded Boston of to-day. Personally he was a man of marked individuality, and I remember hearing from another of his grandchildren an amusing account of the scene which occurred, on one of these Sunday evenings, after the delivery of a total abstinence sermon by the Rev. Dr. Channing, of whom Colonel Perkins was one of the leading parishioners. The whole theory of total abstinence was then an absolute innovation, and its proclamation, which came rather suddenly from Dr. Channing, impressed Colonel Perkins much as it might have moved one of Thackeray's English squires; in so much that he had a double allowance of wine served out that evening to each of his numerous grandsons in place of their accustomed wine-glass of diluted beverage, and this to their visible disadvantage as the evening went on.

Elliot Cabot entered Harvard College in 1836 as Freshman, and though he passed his entrance examinations well, took no prominent rank in his class, but read all sorts of out-of-the-way books and studied natural history. He was also an early reader of Carlyle's "Sartor Resartus," then just published; and was, in general, quite disposed to pursue his own course in mental culture. He belonged to the Hasty Pudding Club and to the Porcellian Club, but spent much time with his classmates Henry Bryant and William Sohier in shooting excursions, which had then the charm of being strictly prohibited by the college. The young men were obliged to carry their guns slung for concealment in two parts, the barrels separated from the stock, under their cloaks, which were then worn largely in the place of overcoats. This taste was strengthened by the love of Cabot's elder brother, afterwards Dr. Samuel Cabot, for ornithology; and as the latter was then studying medicine in Paris, the young men used to send him great numbers of specimens for purposes of exchange. Dr. Henry Bryant is well remembered in Boston for the great collection of birds given by him to the Boston Natural History Society.

Soon after his graduation, in 1840, Elliot Cabot went abroad with the object of joining his elder brother in Switzerland, visiting Italy, wintering in Paris, and returning home in the spring; but this ended in his going for the winter to Heidelberg instead, a place then made fascinating to all young Americans through the glowing accounts of it in Longfellow's "Hyperion." They were also joined by two other classmates,—Edward Holker Welch, now well known in the Roman Catholic priesthood, and John Fenwick Heath of Virginia, well remembered by the readers of Lowell's letters. All of these four were aiming at the profession of the law, although not one of them, I believe, finally devoted himself to its practice. Migrating afterwards to Berlin, after the fashion of German students, they were admitted to the University on their Harvard degrees by Ranke, the great historian, who said as he inspected their parchments, "Ah! the High School at Boston!" which they thought showed little respect for President Quincy's parchment, until they found that "Hoch Schule" was the German equivalent for University. There they heard the lectures of Schelling, then famous, whom they found to be a little man of ordinary appearance, old, infirm, and taking snuff constantly, as if to keep himself awake. Later they again removed, this time to Göttingen, where Cabot busied himself with the study of Kant, and also attended courses in Rudolph Wagner's laboratory. Here he shared more of the social life of his companions, frequented their Liederkränze, learned to fence and to dance, and spent many evenings at students' festivals.

Cabot sums up his whole European reminiscences as follows: "As I look back over my residence in Europe, what strikes me is the waste of time and energy from having had no settled purpose to keep my head steady. I seem to have been always well employed and happy, but I had been indulging a disposition to mental sauntering, and the picking up of scraps, very unfavorable to my education. I was, I think, naturally inclined to hover somewhat above the solid earth of practical life, and thus to miss its most useful lessons. The result I think was to confirm me in the vices of my mental constitution and to cut off what chance there was of my accomplishing something worth while."

In March, 1843, he finally left Göttingen for home by way of Belgium and England, and entered the Harvard Law School in the autumn, taking his degree there two years later, in 1845. Renewing acquaintance with him during this period, I found him to be, as always, modest and reticent in manner, bearing unconsciously a certain European prestige upon him which so commanded the respect of a circle of young men that we gave him the sobriquet of "Jarno," after the well-known philosophic leader in Goethe's "Wilhelm Meister." Whatever he may say of himself, I cannot help still retaining somewhat of my old feeling about the mental training of the man, who, while in the Law School, could write a paper so admirable as Cabot's essay entitled "Immanuel Kant" (Dial, IV. 409), an essay which seems to me now, as it then seemed, altogether the simplest and most effective statement I have ever encountered of the essential principles of that great thinker's philosophy. I remember that when I told Cabot that I had been trying to read Kant's "Critique of Pure Reason" in an English translation, but could not understand it, he placidly replied that he had read it twice in German and had thought he comprehended it, but that Meiklejohn's translation was beyond making out, so that I need not be discouraged.

After graduating from the Law School, he went for a year into a law office in Boston, acting as senior partner to my classmate, Francis Edward Parker, who, being a born lawyer, as Cabot was not, found it for his own profit to sever the partnership at the end of a year, while Cabot retired from the profession forever. His German training had meanwhile made him well known to the leaders of a new literary enterprise, originating with Theodore Parker and based upon a meeting at Mr. Emerson's house in 1849, the object being the organization of a new magazine which should be, in Theodore Parker's phrase, "the Dial with a beard." Liberals and reformers were present at the meeting, including men so essentially diverse as Sumner and Thoreau. Parker was, of course, to be the leading editor, and became such. Emerson also consented, "rather weakly," as Cabot says in his memoranda, to appear, and contributed only the introductory address, while Cabot himself agreed to act as corresponding secretary and business manager. The "Massachusetts Quarterly Review" sustained itself with difficulty for three years, — showing more of studious and systematic work than its predecessor the "Dial," but far less of freshness and originality, — and then went under.

A more successful enterprise in which he was meanwhile enlisted was a trip to Lake Superior with Agassiz, in 1850, when Cabot acted as secretary and wrote and illustrated the published volume of the expedition, — a book which was then full of fresh novelties, and which is still very readable. Soon after his return, he went into his brother Edward's architect office in Boston to put his accounts in order, and ultimately became a partner in the business, erecting various buildings.

He was married on Sept. 28, 1857, to Elizabeth Dwight, daughter of Edmund Dwight, Esq., a woman of rare qualities and great public usefulness, who singularly carried on the tradition of those Essex County women of an earlier generation, who were such strong helpmates to their husbands. Of Mrs. Cabot it might almost have been said what was said by John Lowell in 1826 of his cousin Elizabeth Higginson, wife of her double-first cousin, George Cabot: "She had none of the advantages of early education afforded so bountifully to the young ladies of the present age; but she surpassed *all* of them in the acuteness of her observation, in the knowledge of human nature, and in her power of expressing and defending the opinions which she had formed." * Thus Elliot Cabot writes of his wife: "From the time when the care of her children ceased to occupy the most of her time, she gradually became one of the most valuable of the town officials, as well as the unofficial counsellor of many who needed the unfailing succor of her inexhaustible sympathy and practical helpfulness."

Cabot visited Europe anew after his marriage, and after his return served for nine years as a school committee-man in Brookline, where he resided. He afterwards did faithful duty for six years as chairman of the examining committee of Harvard Overseers. He gave for a single year a series of lectures on Kant at Harvard University, and for a time acted as Instructor in Logic there, which included a supervision of the forensics or written discussions then in vogue. The Civil War aroused his sympathies strongly, especially when his brother Edward and his personal friend Francis L. Lee became respectively Colonel and Lieutenant-colonel of the 44th Massachusetts Volunteer Infantry. Elliot Cabot himself enlisted in a drill club, and did some work for the Sanitary Commission. He also assisted greatly in organizing the Museum of Fine Arts and in the administration of the Boston Athenaeum.

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* Lodge's Cabot, p. 12, note.

Though a life-long student, he wrote little for the press, — a fact which recalls Theodore Parker's remark about him that he "could make a good law argument, but could not address it to the jury." He rendered, however, a great and permanent service, far outweighing that performed by most American authors of his time, as volunteer secretary to Ralph Waldo Emerson, a task which constituted his main occupation for five or six years. After Emerson's death, Cabot also wrote his memoirs, by the wish of the family, — a book which will always remain the primary authority on the subject with which it deals, although it was justly criticised by others for a certain restricted tone which made it seem to be, as it really was, the work of one shy and reticent man telling the story of another. In describing Emerson the biographer often unconsciously described himself also; and the later publications of Mr. Emerson's only son show clearly that there was room for a more ample and varied treatment in order to complete the work.

Under these circumstances Cabot's home life, while of even tenor, was a singularly happy one. One of his strongest and life-long traits was his love of children, — a trait which he also eminently shared with Emerson. The group formed by him with two grandchildren in his lap, to whom he was reading John Gilpin or Hans Andersen, is one which those who knew him at home would never forget. It was characteristic also that in his German copy of Kant's "Critique of Pure Reason," already mentioned, there were found some papers covered with drawings of horses and carts which had been made to amuse some eager child. Akin to this was his strong love of flowers, united with a rare skill in making beautiful shrubs grow here and there in such places as would bring out the lines and curves of his estate at Beverly. Even during the last summer of his life he was cutting new little vistas on the Beverly hills. His sketches of landscape in water-color were also very characteristic both of his delicate and poetic appreciation of nature and of his skill and interest in drawing. In 1885, while in Italy, he used to draw objects seen from the car window as he travelled; and often in the morning, when his family came down to breakfast at hotels, they found that he had already made an exquisite sketch in pencil of some tower or arch.

His outward life, on the whole, seemed much akin to the lives led by that considerable class of English gentlemen who adopt no profession, dwelling mainly on their paternal estates, yet are neither politicians nor fox hunters; pursuing their own favorite studies, taking part from time to time in the pursuits of science, art, or literature, even holding minor public functions, but winning no widespread fame. He showed, on the other hand, the freedom from prejudice, the progressive tendency, and the ideal proclivities which belong more commonly to Americans. He seemed to himself to have accomplished nothing; and yet he had indirectly aided a great many men by the elevation of his tone and the breadth of his intellectual sympathy. If he did not greatly help to stimulate the thought of his time, he helped distinctly to enlarge and ennoble it. His death occurred at Brookline on Jan. 16, 1903. He died as he had lived, a high-minded, stainless, and in some respects unique type of American citizen.

ALFRED PERKINS ROCKWELL.

THE death of Alfred Perkins Rockwell removes one more of the few remaining men who, just entering upon a scientific career, dropped all personal aims at the first call of his country. Fortunately General Rockwell was able to re-enter his chosen profession as professor of mining at the Sheffield Scientific School, where he had been a student after his graduation from Yale University in 1855. In 1868 he was called to the same chair at the Massachusetts Institute of Technology, where he contributed not a little to the success of that department.

Instruction in mining in this country was at that time largely descriptive, by lectures, since laboratory methods were not then developed.

General Rockwell's administrative ability was soon more fully utilized as chief of the fire department of the city of Boston after the great fire of 1872, and subsequently as president of the Eastern Railroad Company. To these civic duties he brought the same high standard of duty, the same loyalty to the right, the same power of organization, which brought him during three years of the civil war from a volunteer second lieutenant to brevet brigadier general. Good judgment in placing his forces and courage in leading them to success characterized his services in these positions. While only a captain he was commended by three generals on three occasions within a year.

The Massachusetts Commandery of the Military Order of the Loyal Legion showed its regard for the man and its appreciation of his services by electing him to its council in 1870 and 1871, as its senior vice-commander in 1877, and as its commander in 1878.

His paper on the "Operations against Charleston, 1860," contributed to the Military Historical Society of Massachusetts, and his careful and scholarly editing of Volumes III and IV of its Memoirs, bear witness to the use he made of the greater leisure of later years.

General Rockwell was a member not only of this society, but of the Geological Society of France, the Boston Natural History Society, the American Association for the Advancement of Science, and of various social clubs in Boston.

His relations with his fellow men were marked by a dignity of manner, a self-respecting modesty, and his cheerful disposition made him a welcome member of the various societies to which he belonged.

Born in Norwich, Conn., Oct. 15, 1834, he died suddenly at New Haven on Dec. 24, 1903. The funeral services were held at Trinity Church, Boston, December 28.

HORATIO HOLLIS HUNNEWELL.

Horatio Hollis Hunnewell was elected a resident Fellow of the Academy, May 28, 1872, in class 2, section 2 (botany). He died on May 20, 1902. He was born in Watertown, Massachusetts, on July 27, 1810, and was the son of Walter Hunnewell, of the fifth generation of the New England Hunnewells, a graduate of the Harvard class of 1787, and a physician of standing highly respected in the community, and of Susanna Cooke.

Adeline Fowl, a niece of Mrs. Walter Hunnewell, still remembered for her beauty and charm of manner, had married Samuel Welles, the head of the Paris banking-house of Welles & Company. At the suggestion of Mrs. Welles young Hunnewell was sent to Paris at the age of fifteen to learn the banking business in her husband's house. He applied himself to this with such assiduity that at the end of ten years he was admitted a partner in the bank and married a daughter of John Welles of Boston, a cousin of Samuel Welles. Mr. Hunnewell's career now seemed assured, and he looked forward with complacency to passing his life in Paris in the active pursuit of business, but the financial crisis of 1837 destroyed credits and compelled the winding up of the affairs of John Welles & Company. During the Revolution of 1830 Mr. Hunnewell, as a member of the *Garde Nationale*, carried a musket and helped protect his employer's bank. Nearly seventy years later he was gratified by receiving from the Government of France a medal commemorating his services as a soldier.

Mr. Hunnewell returned to Boston disappointed, and, as he acknowledged later, nearly broken-hearted. His optimism, the strong feature of his character, soon asserted itself, however, and he began at once to make a position for himself in the business community. His industry, broad-mindedness, and astuteness, and his confidence in the future of the country, soon brought him success and the attention of the group of Boston capitalists who during the last half of the nineteenth century made this city an important factor in the development of the West. By these men Mr. Hunnewell's advice was sought and respected, and with them he acquired in building railroads and in other enterprises great wealth and filled many positions of responsibility.

Soon after his return from France, Mr. Hunnewell became actively interested in horticulture, and for the remainder of his life he devoted a large part of his time to the improvement of the estate in Natick which had belonged to his wife's family and which has given the name of Wellesley to a town later separated from Natick. This estate Mr. Hunnewell lived to see one of the most famous and interesting in America. He seems to have inherited his love of flowers from his father, whose garden in Watertown was the most attractive in the town: for the love of gardening, which was the passion of his later years, showed itself early, and this man of great affairs boasted only of his first financial transaction, — the successful sale of a number of young cherry-trees which he had grafted as a boy just before he went to France. Gardening taught Mr. Hunnewell the value of experiment and the importance of science. It led him to test trees and other plants not previously introduced into New England, and our knowledge of the value in this climate of many plants, especially coniferous trees, is based largely on his experiments at Wellesley. As a horticulturist Mr. Hunnewell's most useful work was the formation of his collection of coniferous trees, which now contains the best specimens of several species to be found in this country and which is one of the largest and the most interesting in America, and his experiments in the cultivation of rhododendrons, which have now become common garden plants in this part of the country through knowledge obtained first at Wellesley.

His appreciation of the importance of science as a basis for good gardening was shown in our associate's long-sustained support of the botanical departments of Harvard University. For twenty years he was an active member of the Visiting Committee of the Botanic Garden and a generous contributor to the expenses of the Garden and the

Botanical Museum at Cambridge. His interest in the Arnold Arboretum began with the inception of that undertaking and ceased only with his life. He was one of the largest benefactors of the Arboretum, where the Hunnewell Building is a witness of his interest in increasing the knowledge of trees. One of the last acts of his life was to endow the Botanical Department of Wellesley College.

For forty-five years Mr. Hunnewell was an active member of the Massachusetts Horticultural Society, rendering it valuable service for thirty-four years as a member of its Finance Committee, during nineteen of which he was Chairman of that important Committee, and as Vice-President for eleven years. Much of the success of this Society is due to his wise management of its finances and his unfailing interest in its exhibitions. For twenty-six years Mr. Hunnewell served the Massachusetts Humane Society as a Trustee, and for five years as its President. His liberality was not confined to the advancement of botany and horticulture. No appeal in behalf of a worthy object was made to him in vain. In Wellesley a public library, a town hall, and a public park are monuments to his public spirit. The Public Library in his native town and the hospitals and the Museum of Fine Arts in Boston shared in his success.

Simple in tastes, unassuming, modest and retiring, hospitable without ostentation, of broad sympathies, interested to the end in all that interested three generations of his descendants, happy with his trees and with his flowers, which never failed to delight him, happiest in making others happy, loved and respected by old and young, our associate lived wisely, and in his life gave an example of public spirit, enterprise, generosity, and kindliness of high value to the community. To those who knew him the recollection of this friend is a delight and an inspiration.

C. S. SARGENT.

American Academy of Arts and Sciences.

OFFICERS AND COMMITTEES FOR 1904–05.

PRESIDENT.

William W. Goodwin.

VICE-PRESIDENT.

Class I.	Class II.	Class III.
John Trowbridge,	Henry P. Walcott,	John C. Gray.

CORRESPONDING SECRETARY.

Edwin H. Hall.

RECORDING SECRETARY.

William Watson.

TREASURER.

Francis Blake.

LIBRARIAN.

A. Lawrence Rotch.

COUNCILLORS.

Class I.	Class II.	Class III.
Arthur G. Webster,	Edward L. Mark,	Arlo Bates,
	Terms expire 1905.	
Lewis J. Johnson,	Edward H. Bradford,	Morris H. Morgan,
	Terms expire 1906.	
Elihu Thomson,	William T. Sedgwick,	John H. Wright.
	Terms expire 1907.	

COMMITTEE OF FINANCE.

William W. Goodwin, Francis Blake, Eliot C. Clarke.

RUMFORD COMMITTEE.

Charles R. Cross, *Chairman*,
Erasmus D. Leavitt, Edward C. Pickering, Amos E. Dolbear,
Arthur G. Webster, Theodore W. Richards, Elihu Thomson.

C. M. WARREN COMMITTEE.

Leonard P. Kinnicutt, *Chairman*,
Robert H. Richards, Samuel Cabot, Arthur M. Comey,
Arthur Michael. Henry P. Talbot, Charles R. Sanger.

COMMITTEE OF PUBLICATION.

Edward L. Mark, of Class II., *Chairman*,
Seth C. Chandler, of Class I., Crawford H. Toy, of Class III.

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A. Lawrence Rotch, *Chairman*,
Harry M. Goodwin, of Class I., Samuel Henshaw, of Class II.,
Henry W. Haynes, of Class III.

AUDITING COMMITTEE.

Henry G. Denny, Frederick J. Stimson.

LIST

OF THE

FELLOWS AND FOREIGN-HONORARY MEMBERS.

(Corrected to June 30, 1904.)

RESIDENT FELLOWS. — 199.

(Number limited to two hundred.)

Class I. — *Mathematical and Physical Sciences.* — 79.

Section I. — 18.

Mathematics and Astronomy.

Solon I. Bailey,	Cambridge.
Maxime Bôcher,	Cambridge.
William E. Byerly,	Cambridge.
Seth C. Chandler,	Cambridge.
Gustavus Hay,	Boston.
Percival Lowell,	Boston.
James Mills Peirce,	Cambridge.
Edward C. Pickering,	Cambridge.
William H. Pickering,	Cambridge.
Henry S. Pritchett,	Boston.
John Ritchie, Jr.,	Roxbury.
Edwin F. Sawyer,	Brighton.
Arthur Searle,	Cambridge.
William E. Story,	Worcester.
Henry Taber,	Worcester.
Harry W. Tyler,	Boston.
O. C. Wendell,	Cambridge.
P. S. Yendell,	Dorchester.

Section II. — 24.

Physics.

A. Graham Bell,	Washington, D.C.
Clarence J. Blake,	Boston.
Francis Blake,	Weston.
George A. Campbell,	Boston.
Harry E. Clifford,	Newton.
Charles R. Cross,	Brookline.
Amos E. Dolbear,	Somerville.
A. W. Duff,	Worcester.
H. M. Goodwin,	Roxbury.
Edwin H. Hall,	Cambridge.
Hammond V. Hayes,	Cambridge.
William L. Hooper,	Somerville.
William W. Jacques,	Newton.
Frank A. Laws,	Boston.
Henry Lefavour,	Boston.
Theodore Lyman,	Brookline.
Benjamin O. Peirce,	Cambridge.
A. Lawrence Rotch,	Boston.
Wallace C. Sabine,	Boston.
John S. Stone,	Boston.
Elihu Thomson,	Swampscott.
John Trowbridge,	Cambridge.
A. G. Webster,	Worcester.
Robert W. Willson,	Cambridge.

Section III. — 21.

Chemistry.

Samuel Cabot,	Boston.

Arthur M. Comey,	Cambridge.
James M. Crafts,	Boston.
Charles W. Eliot,	Cambridge.
Charles L. Jackson,	Cambridge.
Walter L. Jennings,	Worcester.
Leonard P. Kinnicutt,	Worcester.
Charles F. Mabery,	Cleveland, O.
Arthur Michael,	Boston.
George D. Moore,	Worcester.
Charles E. Munroe,	Wash'gton, D.C.
John U. Nef,	Chicago, Ill.
Arthur A. Noyes,	Boston.
Robert H. Richards,	Jamaica Plain.
Theodore W. Richards,	Cambridge.
Charles R. Sanger,	Cambridge.
Stephen P. Sharples,	Cambridge.
Francis H. Storer,	Boston.
Henry P. Talbot,	Newton.
Charles H. Wing,	Ledger, N. C.
Edward S. Wood,	Boston.

SECTION IV. — 16.

Technology and Engineering.

Alfred E. Burton,	Boston.
Eliot C. Clarke,	Boston.
Heinrich O. Hofman,	Jamaica Plain.
Ira N. Hollis,	Cambridge.
L. J. Johnson,	Cambridge.
Gaetano Lanza,	Boston.
E. D. Leavitt,	Cambridge.
William R. Livermore,	New York.
Hiram F. Mills,	Lowell.
Cecil H. Peabody,	Brookline.
Andrew H. Russell,	Manila.
Albert Sauveur,	Cambridge.
Peter Schwamb,	Arlington.
H. L. Smyth,	Cambridge.
George F. Swain,	Boston.
William Watson,	Boston.

CLASS II. — *Natural and Physiological Sciences.* — 68.

SECTION I. — 15.

Geology, Mineralogy, and Physics of the Globe.

H. H. Clayton,	Milton.
Algernon Coolidge,	Boston.
William O. Crosby,	Jamaica Plain.
William M. Davis,	Cambridge.
Benj. K. Emerson,	Amherst.
O. W. Huntington,	Newport, R. I.
Robert T. Jackson,	Cambridge.
T. A. Jaggar, Jr.,	Cambridge.
William H. Niles,	Cambridge.
Charles Palache,	Cambridge.
John E. Pillsbury,	Boston.
Nathaniel S. Shaler,	Cambridge.
Robert DeC. Ward,	Cambridge.
John E. Wolff,	Cambridge.
J. B. Woodworth,	Cambridge.

SECTION II. — 12.

Botany.

F. S. Collins,	Malden.
Geo. E. Davenport,	Medford.
William G. Farlow,	Cambridge.
Charles E. Faxon,	Jamaica Plain.
Merritt L. Fernald,	Cambridge.
William F. Ganong,	Northampton.
George L. Goodale,	Cambridge.
John G. Jack,	Jamaica Plain.
B. L. Robinson,	Cambridge.
Charles S. Sargent,	Brookline.
Arthur B. Seymour,	Cambridge.
Roland Thaxter,	Cambridge.

SECTION III. — 25.

Zoölogy and Physiology.

Alexander Agassiz,	Cambridge.
Robert Amory,	Boston.

James M. Barnard,	Milton.
Henry P. Bowditch,	Jamaica Plain.
William Brewster,	Cambridge.
Louis Cabot,	Brookline.
William E. Castle,	Cambridge.
Samuel F. Clarke,	Williamstown.
W. T. Councilman,	Boston.
Charles B. Davenport,	Chicago, Ill.
Harold C. Ernst,	Jamaica Plain.
Edward G. Gardiner,	Boston.
Samuel Henshaw,	Cambridge.
Theodore Hough,	Boston.
John S. Kingsley,	Somerville.
Edward L. Mark,	Cambridge.
Charles S. Minot,	Milton.
Edward S. Morse,	Salem.
George H. Parker,	Cambridge.
William T. Porter,	Boston.
James J. Putnam,	Boston.
Samuel H. Scudder,	Cambridge.
William T. Sedgwick,	Boston.
James C. White,	Boston.
William M. Woodworth,	Cambridge.

SECTION IV. — 16.

Medicine and Surgery.

Samuel L. Abbot,	Boston.
Edward H. Bradford,	Boston.
Arthur T. Cabot,	Boston.
David W. Cheever,	Boston.
Frank W. Draper,	Boston.
Thomas Dwight,	Nahant.
Reginald H. Fitz,	Boston.
Charles F. Folsom,	Boston.
Frederick I. Knight,	Boston.
Samuel J. Mixter,	Boston.
W. L. Richardson,	Boston.
Theobald Smith,	Jamaica Plain.
O. F. Wadsworth,	Boston.
Henry P. Walcott,	Cambridge.
John C. Warren,	Boston.
Francis H. Williams,	Boston.

CLASS III. — *Moral and Political Sciences.* — 52.

SECTION I. — 10.

Philosophy and Jurisprudence.

James B. Ames,	Cambridge.
John C. Gray,	Boston.
G. Stanley Hall,	Worcester.
Geo. F. Hoar,	Worcester.
Francis C. Lowell,	Boston.
Hugo Münsterberg,	Cambridge.
Josiah Royce,	Cambridge.
Jeremiah Smith,	Cambridge.
Frederic J. Stimson,	Dedham.
Edward H. Strobel,	Cambridge.

SECTION II. — 22.

Philology and Archæology.

Charles P. Bowditch,	Jamaica Plain.
Lucien Carr,	Cambridge.
Franklin Carter,	Williamstown.
Joseph T. Clarke,	Boston.
Henry G. Denny,	Roxbury.
William Everett,	Quincy.
J. W. Fewkes,	Washington, D.C.
William W. Goodwin,	Cambridge.
Henry W. Haynes,	Boston.
Charles R. Lanman,	Cambridge.
David G. Lyon,	Cambridge.
Morris H. Morgan,	Cambridge.
Bennett H. Nash,	Boston.
Frederick W. Putnam,	Cambridge.
Edward Robinson,	Boston.
Edward S. Sheldon,	Cambridge.
Herbert Weir Smyth,	Cambridge.
F. B. Stephenson,	Boston.
Crawford H. Toy,	Cambridge.
John W. White,	Cambridge.
John H. Wright,	Cambridge.
Edward J. Young,	Waltham.

SECTION III. — 10.

Political Economy and History.

Charles F. Adams,	Lincoln.
Edward Atkinson,	Brookline.
Andrew McF. Davis,	Cambridge.
Ephraim Emerton,	Cambridge.
A. C. Goodell,	Salem.
Henry C. Lodge,	Nahant.
A. Lawrence Lowell,	Boston.
James F. Rhodes,	Boston.
Charles C. Smith,	Boston.
F. W. Taussig,	Cambridge.

SECTION IV. — 10.

Literature and the Fine Arts.

Francis Bartlett,	Boston.
John Bartlett,	Cambridge.
Arlo Bates,	Boston.
George S. Boutwell,	Groton.
T. W. Higginson,	Cambridge.
George L. Kittredge,	Cambridge.
Charles Eliot Norton,	Cambridge.
Denman W. Ross,	Cambridge.
William R. Ware,	Milton.
Barrett Wendell,	Boston.

ASSOCIATE FELLOWS. — 100.

(Number limited to one hundred. Elected as vacancies occur.)

CLASS I. — *Mathematical and Physical Sciences.* — 37.

SECTION I. — 14.

Mathematics and Astronomy.

Edward E. Barnard,	Williams Bay, Wis.
S. W. Burnham,	Chicago.
George Davidson,	San Francisco.
Fabian Franklin,	Baltimore.
Asaph Hall,	Goshen, Conn.
George W. Hill,	W. Nyack, N.Y.
E. S. Holden,	New York.
Emory McClintock,	Morristown, N.J.
E. H. Moore,	Chicago.
Simon Newcomb,	Washington.
Charles L. Poor,	New York.
George M. Searle,	Washington.
J. N. Stockwell,	Cleveland, O.
Chas. A. Young,	Princeton, N. J.

SECTION II. — 6.

Physics.

Carl Barus,	Providence, R.I.
G. E. Hale,	Williams Bay, Wis.
S. P. Langley,	Washington.
T. C. Mendenhall,	
A. A. Michelson,	Chicago.
E. L. Nichols,	Ithaca, N. Y.

SECTION III. — 9.

Chemistry.

T. M. Drown,	So. Bethlehem, Pa.
Wolcott Gibbs,	Newport, R. I.
Frank A. Gooch,	New Haven.
Eugene W. Hilgard,	Berkeley, Cal.
S. W. Johnson,	New Haven.
J. W. Mallet,	Charlottesville, Va.
E. W. Morley,	Cleveland, O.
J. M. Ordway,	New Orleans.
Ira Remsen,	Baltimore.

SECTION IV. — 8.

Technology and Engineering.

Henry L. Abbot,	Cambridge.
Cyrus B. Comstock,	New York.
W. P. Craighill,	Charlestown, W. Va.
John Fritz,	Bethlehem, Pa.
James D. Hague,	New York.
F. R. Hutton,	New York.
William Sellers,	Edge Moor, Del.
Robt. S. Woodward,	New York.

CLASS II. — *Natural and Physiological Sciences.* — 34.

SECTION I. — 11.

Geology, Mineralogy, and Physics of the Globe.

Cleveland Abbe,	Washington.
George J. Brush,	New Haven.
T. C. Chamberlin,	Chicago.
Edward S. Dana,	New Haven.
Walter G. Davis,	Cordova, Arg.
Samuel F. Emmons,	Washington.
G. K. Gilbert,	Washington.
S. L. Penfield,	New Haven.
R. Pumpelly,	Newport, R.I.
Israel C. Russell,	Ann Arbor.
Charles D. Walcott,	Washington.

SECTION II. — 6.

Botany.

L. H Bailey,	Ithaca, N. Y.
D. H. Campbell,	Palo Alto, Cal.
J. M. Coulter,	Chicago.
C. G. Pringle,	Charlotte, Vt.
John D. Smith,	Baltimore.
W. Trelease,	St. Louis.

SECTION III. — 9.

Zoölogy and Physiology.

Joel A. Allen,	New York.
W. K. Brooks,	Lake Roland, Md
F. P. Mall,	Baltimore.
S. Weir Mitchell,	Philadelphia.
H. F. Osborn,	New York.
A. S. Packard,	Providence, R.I.
A. E. Verrill,	New Haven.
C. O. Whitman,	Chicago.
E. B. Wilson,	New York.

SECTION IV. — 8.

Medicine and Surgery.

John S. Billings,	New York.
W. S. Halsted,	Baltimore.
Abraham Jacobi,	New York.
W. W. Keen,	Philadelphia
William Osler,	Baltimore.
T. Mitchell Prudden,	New York.
Wm. H. Welch,	Baltimore.
H. C. Wood,	Philadelphia.

CLASS III. — *Moral and Political Sciences.* — 29.

SECTION I. — 7.

Philosophy and Jurisprudence.

James C. Carter,	New York.
Joseph H. Choate,	New York.
Melville W. Fuller,	Washington.
William W. Howe,	New Orleans.
Charles S. Peirce,	Milford, Pa.
G. W. Pepper,	Philadelphia.
T. R. Pynchon,	Hartford, Conn.

SECTION II. — 7.

Philology and Archæology.

Timothy Dwight,	New Haven.
B. L. Gildersleeve,	Baltimore.
D. C. Gilman,	Baltimore.
T. R. Lounsbury,	New Haven.
Rufus B. Richardson,	Athens.
Thomas D. Seymour,	New Haven.
A. D. White,	Ithaca, N.Y.

SECTION III. — 7.

Political Economy and History.

Henry Adams,	Washington.
G. P. Fisher,	New Haven.
Arthur T. Hadley,	New Haven.
Henry C. Lea,	Philadelphia.
Alfred T. Mahan,	New York.
H. Morse Stephens,	Ithaca.
W. G. Sumner,	New Haven.

SECTION IV. — 8.

Literature and the Fine Arts.

James B. Angell,	Ann Arbor, Mich.
L. P. di Cesnola,	New York.
H. H. Furness,	Wallingford, Pa.
R. S. Greenough,	Florence.
Herbert Putnam,	Washington.
Augustus St. Gaudens,	Windsor, Vt.
John S. Sargent,	London.
E. C. Stedman,	Bronxville, N. Y.

FOREIGN HONORARY MEMBERS.—73.

(Number limited to seventy-five. Elected as vacancies occur.)

CLASS I.—*Mathematical and Physical Sciences.*—25.

SECTION I.—8.

Mathematics and Astronomy.

Arthur Auwers,	Berlin.
George H. Darwin,	Cambridge.
Sir William Huggins,	London.
Felix Klein,	Göttingen.
Émile Picard,	Paris.
H. Poincaré,	Paris.
Otto Struve,	Karlsruhe.
H. C. Vogel,	Potsdam.

SECTION II.—6.

Physics.

Ludwig Boltzmann,	Leipsic.
Oliver Heaviside,	Newton Abbot.
F. Kohlrausch,	Berlin.
Joseph Larmor,	Cambridge.
Lord Rayleigh,	Witham.
Joseph J. Thomson,	Cambridge.

SECTION III.—6.

Chemistry.

Adolf Ritter von Baeyer,	Munich
Marcellin Berthelot,	Paris.
J. H. van't Hoff,	Berlin.
D. Mendeleeff,	St. Petersburg.
Sir H. E. Roscoe,	London.
Julius Thomsen,	Copenhagen.

SECTION IV.—5.

Technology and Engineering.

Sir Benjamin Baker,	London.
Lord Kelvin,	Largs.
Maurice Lévy,	Paris.
H. Müller-Breslau,	Berlin.
W. Cawthorne Unwin,	London.

CLASS II.—*Natural and Physiological Sciences.*—25.

SECTION I.—6.

Geology, Mineralogy, and Physics of the Globe.

Sir Archibald Geikie,	London.
Julius Hann,	Vienna.
Albert Heim,	Zurich.
Sir John Murray,	Edinburgh.
Freih. v. Richthofen,	Berlin.
Henry C. Sorby,	Sheffield.

SECTION II.—6.

Botany.

E. Bornet,	Paris.
A. Engler,	Berlin.
Sir Joseph D. Hooker,	Sunningdale.
W. Pfeffer,	Leipsic.
H. Graf zu Solms-Laubach,	Strassburg.
Eduard Strasburger,	Bonn.

SECTION III. — 6.

Zoölogy and Physiology.

Sir Michael Foster,	Cambridge.
Ludimar Hermann,	Königsberg.
A. von Kölliker,	Würzburg.
H. Kronecker,	Bern.
E. Ray Lankester,	London.
Elias Metschnikoff,	Paris.

SECTION IV. — 7.

Medicine and Surgery.

Emil von Behring,	Marburg.
Sir T. L. Brunton,	London.
A. Celli,	Rome.
Sir V. A. H. Horsley,	London.
R. Koch,	Berlin.
Lord Lister,	London.
F. v. Recklinghausen,	Strassburg.

CLASS III. — *Moral and Political Sciences.* — 23.

SECTION I. — 5.

Philosophy and Jurisprudence.

A. J. Balfour,	Prestonkirk.
Heinrich Brunner,	Berlin.
A. V. Dicey,	Oxford.
F. W. Maitland,	Cambridge.
Sir Frederick Pollock, Bart.,	London.

SECTION II. — 7.

Philology and Archæology.

Ingram Bywater,	Oxford.
F. Delitzsch,	Berlin.
W. Dörpfeld,	Athens.
Sir John Evans,	Hemel Hempstead.
H. Jackson,	Cambridge.
J. W. A. Kirchhoff,	Berlin.
G. C. C. Maspero,	Paris.

SECTION III. — 5.

Political Economy and History.

James Bryce,	London.
Adolf Harnack,	Berlin.
Sir G. O. Trevelyan, Bart.,	London.
John Morley,	London.
Pasquale Villari,	Florence.

SECTION IV. — 6.

Literature and the Fine Arts.

E. de Amicis,	Florence.
Gaston Boissier,	Paris.
Georg Brandes,	Copenhagen.
F. Brunetière,	Paris.
Jean Léon Gérôme,	Paris.
Rudyard Kipling,	Burwash.

STATUTES AND STANDING VOTES.

◆

STATUTES.

Adopted May 30, 1854: *amended September* 8, 1857, *November* 12, 1862, *May* 24, 1864, *November* 9, 1870, *May* 27, 1873, *January* 26, 1876, *June* 16, 1886, *October* 8, 1890, *January* 11 *and May* 10, 1893, *May* 9 *and October* 10, 1894, *March* 13, April 10 *and May* 8, 1895, *May* 8, 1901, *and January* 8, 1902.

CHAPTER I.

OF FELLOWS AND FOREIGN HONORARY MEMBERS.

1. The Academy consists of Resident Fellows, Associate Fellows and Foreign Honorary Members. They are arranged in three Classes, according to the Arts and Sciences in which they are severally proficient, viz.: Class I. The Mathematical and Physical Sciences; — Class II. The Natural and Physiological Sciences; — Class III. The Moral and Political Sciences. Each Class is divided into four Sections, viz.: Class I., Section 1. Mathematics and Astronomy: — Section 2. Physics: — Section 3. Chemistry; — Section 4. Technology and Engineering. Class II., Section 1. Geology, Mineralogy, and Physics of the Globe; — Section 2. Botany; — Section 3. Zoölogy and Physiology; — Section 4. Medicine and Surgery. Class III., Section 1. Philosophy and Jurisprudence: — Section 2. Philology and Archæology; — Section 3. Political Economy and History; — Section 4. Literature and the Fine Arts.

2. The number of Resident Fellows shall not exceed two hundred. Only residents in the Commonwealth of Massachusetts shall be eligible to election as Resident Fellows, but resident fellowship may be retained after removal from the Commonwealth. Each Resident Fellow shall pay an admission fee of ten dollars and such annual assessment, not exceeding ten dollars, as shall be voted by the Academy at each annual

meeting. Resident Fellows only may vote at the meetings of the Academy.

3. The number of Associate Fellows shall not exceed one hundred, of whom there shall not be more than forty in either of the three classes of the Academy. Associate Fellows shall be chosen from persons residing outside of the Commonwealth of Massachusetts. They shall not be liable to the payment of any fees or annual dues, but on removing within the Commonwealth they may be transferred by the Council to resident fellowship as vacancies there occur.

4. The number of Foreign Honorary Members shall not exceed seventy-five; and they shall be chosen from among persons most eminent in foreign countries for their discoveries and attainments in either of the three departments of knowledge above enumerated. There shall not be more than thirty Foreign Members in either of these departments.

CHAPTER II.

OF OFFICERS.

1. There shall be a President, three Vice-Presidents, one for each Class, a Corresponding Secretary, a Recording Secretary, a Treasurer, and a Librarian, which officers shall be annually elected, by ballot, at the Annual Meeting, on the second Wednesday in May.

2. At the Annual Meeting of 1901, nine Councillors shall be elected by ballot, one from each Class of the Academy to serve for one year, one from each Class for two years, and one from each Class for three years; and at annual meetings thereafter three Councillors shall be elected in the same manner, one from each Class, to serve for three years; but the same Fellow shall not be eligible for two successive terms. The nine Councillors, with the President, the three Vice-Presidents, the two Secretaries, the Treasurer, and the Librarian, shall constitute the Council. Five members shall constitute a quorum. It shall be the duty of this Council to exercise a discreet supervision over all nominations and elections. With the consent of the Fellow interested, they shall have power to make transfers between the several Sections of the same Class, reporting their action to the Academy.

3. If any office shall become vacant during the year, the vacancy shall be filled by a new election, and at the next stated meeting, or at a meeting called for this purpose.

CHAPTER III.

OF NOMINATIONS OF OFFICERS.

1. At the stated meeting in March, the President shall appoint from the next retiring Councillors a Nominating Committee of three Fellows, one for each class.

2. It shall be the duty of this Nominating Committee to prepare a list of candidates for the offices of President, Vice-Presidents, Corresponding Secretary, Recording Secretary, Treasurer, Librarian, Councillors, and the Standing Committees which are chosen by ballot; and to cause this list to be sent by mail to all the Resident Fellows of the Academy not later than four weeks before the Annual Meeting.

3. Independent nominations for any office, signed by at least five Resident Fellows and received by the Recording Secretary not less than ten days before the Annual Meeting, shall be inserted in the call for the Annual Meeting, which shall then be issued not later than one week before that meeting.

4. The Recording Secretary shall prepare for use, in voting at the Annual Meeting, a ballot containing the names of all persons nominated for office under the conditions given above.

5. When an office is to be filled at any other time than at the Annual Meeting, the President shall appoint a Nominating Committee in accordance with the provisions of Section 1, which shall announce its nomination in the manner prescribed in Section 2 at least two weeks before the time of election. Independent nominations, signed by at least five Resident Fellows and received by the Recording Secretary not later than one week before the meeting for election, shall be inserted in the call for that meeting.

CHAPTER IV.

OF THE PRESIDENT.

1. It shall be the duty of the President, and, in his absence, of the senior Vice-President present, or next officer in order as above enumerated, to preside at the meetings of the Academy; to summon extraordinary meetings, upon any urgent occasion; and to execute or see to the execution of the Statutes of the Academy. Length of continuous membership in the Academy shall determine the seniority of the Vice-Presidents.

2. The President, or, in his absence, the next officer as above enumerated, is empowered to draw upon the Treasurer for such sums of money as the Academy shall direct. Bills presented on account of the Library, or the Publications of the Academy, must be previously approved by the respective committees on these departments.

3. The President, or, in his absence, the next officer as above enumerated, shall nominate members to serve on the different committees of the Academy which are not chosen by ballot.

4. Any deed or writing to which the common seal is to be affixed shall be signed and sealed by the President, when thereto authorized by the Academy.

CHAPTER V.

OF STANDING COMMITTEES.

1. At the Annual Meeting there shall be chosen the following Standing Committees, to serve for the year ensuing, viz.: —

2. The Committee of Finance, to consist of the President, Treasurer, and one Fellow chosen by ballot, who shall have full control and management of the funds and trusts of the Academy, with the power of investing or changing the investment of the same at their discretion. The general appropriations for the expenditures of the Academy shall be moved by this Committee at the Annual Meeting, and all special appropriations from the general and publication funds shall be referred to or proposed by this Committee.

3. The Rumford Committee, of seven Fellows, to be chosen by ballot, who shall consider and report on all applications and claims for the Rumford Premium, also on all appropriations from the income of the Rumford Fund, and generally see to the due and proper execution of this trust.

4. The C. M. Warren Committee, of seven Fellows, to be chosen by ballot, who shall consider and report on all applications for appropriations from the income of the C. M. Warren Fund, and generally see to the due and proper execution of this trust.

5. The Committee of Publication, of three Fellows, one from each Class, to whom all communications submitted to the Academy for publication shall be referred, and to whom the printing of the Memoirs and the Proceedings shall be intrusted.

6. The Committee on the Library, of the Librarian ex officio and three other Fellows, one from each class, who shall examine the Library, and make an annual report on its condition and management.

7. An Auditing Committee of two Fellows, for auditing the accounts of the Treasurer.

CHAPTER VI.

OF THE SECRETARIES.

1. The Corresponding Secretary shall conduct the correspondence of the Academy, recording or making an entry of all letters written in its name, and preserving on file all letters which are received; and at each meeting he shall present the letters which have been addressed to the Academy since the last meeting. Under the direction of the Council for Nomination, he shall keep a list of the Resident Fellows, Associate Fellows, and Foreign Honorary Members, arranged in their Classes and in Sections in respect to the special sciences in which they are severally proficient; and he shall act as secretary to the Council.

2. The Recording Secretary shall have charge of the Charter and Statute-book, journals, and all literary papers belonging to the Academy. He shall record the proceedings of the Academy at its meetings; and after each meeting is duly opened, he shall read the record of the preceding meeting. He shall notify the meetings of the Academy, apprise officers and committees of their election or appointment, and inform the Treasurer of appropriations of money voted by the Academy. He shall post up in the Hall a list of the persons nominated for election into the Academy; and when any individual is chosen, he shall insert in the record the names of the Fellows by whom he was nominated.

3. The two Secretaries, with the Chairman of the Committee of Publication, shall have authority to publish such of the records of the meetings of the Academy as may seem to them calculated to promote its interests.

CHAPTER VII.

OF THE TREASURER.

1. The Treasurer shall give such security for the trust reposed in him as the Academy shall require.

2. He shall receive officially all moneys due or payable, and all bequests or donations made to the Academy, and shall pay such sums as the Academy may direct. He shall keep an account of all receipts and expenditures; shall submit his accounts to the Auditing Committee; and shall report the same at the expiration of his term of office.

3. The Treasurer shall keep separate accounts of the income and appropriation of the Rumford Fund and of other special funds, and report the same annually.

4. All moneys which there shall not be present occasion to expend shall be invested by the Treasurer, under the direction of the Finance Committee.

CHAPTER VIII.

Of the Librarian and Library.

1. It shall be the duty of the Librarian to take charge of the books, to keep a correct catalogue of them, to provide for the delivery of books from the Library, and to appoint such agents for these purposes as he may think necessary. He shall make an annual report on the condition of the Library.

2. The Librarian, in conjunction with the Committee on the Library, shall have authority to expend such sums as may be appropriated, either from the General, Rumford or other special Funds of the Academy, for the purchase of books, and for defraying other necessary expenses connected with the Library.

3. To all books in the Library procured from the income of the Rumford Fund, or other special funds, the Librarian shall cause a stamp or label to be affixed, expressing the fact that they were so procured.

4. Every person who takes a book from the Library shall give a receipt for the same to the Librarian or his assistant.

5. Every book shall be returned in good order, regard being had to the necessary wear of the book with good usage. If any book shall be lost or injured, the person to whom it stands charged shall replace it by a new volume or set, if it belongs to a set, or pay the current price of the volume or set to the Librarian; and thereupon the remainder of the set, if the volume belonged to a set, shall be delivered to the person so paying for the same.

6. All books shall be returned to the Library for examination at least one week before the Annual Meeting.

7. The Librarian shall have custody of the Publications of the Academy and shall distribute copies among the Associate Fellows and Foreign Honorary Members, at their request. With the advice and consent of the President, he may effect exchanges with other associations.

CHAPTER IX.

OF MEETINGS.

1. There shall be annually four stated meetings of the Academy; namely, on the second Wednesday in May (the Annual Meeting), on the second Wednesday in October, on the second Wednesday in January, and on the second Wednesday in March. At these meetings only, or at meetings adjourned from these and regularly notified, shall appropriations of money be made, or alterations of the statutes or standing votes of the Academy be effected.

2. Fifteen Fellows shall constitute a quorum for the transaction of business at a stated meeting. Seven Fellows shall be sufficient to constitute a meeting for scientific communications and discussions.

3. The Recording Secretary shall notify the meetings of the Academy to each Fellow residing in Boston and the vicinity; and he may cause the meetings to be advertised, whenever he deems such further notice to be needful.

CHAPTER X.

OF THE ELECTION OF FELLOWS AND HONORARY MEMBERS.

1. Elections shall be made by ballot, and only at stated meetings.

2. Candidates for election as Resident Fellows must be proposed by two Resident Fellows of the section to which the proposal is made, in a recommendation signed by them, and this recommendation shall be transmitted to the Corresponding Secretary, and by him referred to the Council for nomination. No person recommended shall be reported by the Council as a candidate for election, unless he shall have received a written approval, signed at a meeting of the Council by at least five of its members. All nominations thus approved shall be read to the Academy at a stated meeting, and shall then stand on the nomination list during the interval between two stated meetings, and until the balloting. No person shall be elected a Resident Fellow, unless he shall have been resident in this Commonwealth one year next preceding his election. If any person elected a Resident Fellow shall neglect for one year to pay his admission fee, his election shall be void; and if any Resident Fellow shall neglect to pay his annual assessments

for two years, provided that his attention shall have been called to this article, he shall be deemed to have abandoned his Fellowship; but it shall be in the power of the Treasurer, with the consent of the Council, to dispense (*sub silentio*) with the payment both of the admission fee and of the assessments, whenever in any special instance he shall think it advisable so to do.

3. The nomination of Associate Fellows may take place in the manner prescribed in reference to Resident Fellows. The Council may in like manner originate nominations of Associate Fellows, which must be read at a stated meeting previous to the election, and be exposed on the nomination list during the interval.

4. Foreign Honorary Members shall be chosen only after a nomination made at a meeting of the Council, signed at the time by at least seven of its members, and read at a stated meeting previous to that on which the balloting takes place.

5. Three fourths of the ballots cast must be affirmative, and the number of affirmative ballots must amount to eleven to effect an election of Fellows or Foreign Honorary Members.

6. A majority of any section of the Academy is empowered to present lists of persons deemed best qualified to fill vacancies occurring in the number of Foreign Honorary Members or Associate Fellows allotted to it; and such lists, after being read at a stated meeting, shall be referred to the Council for Nomination.

7. If, in the opinion of a majority of the entire Council, any Fellow — Resident or Associate — shall have rendered himself unworthy of a place in the Academy, the Council shall recommend to the Academy the termination of his Fellowship; and provided that a majority of two thirds of the Fellows at a stated meeting, consisting of not less than fifty Fellows, shall adopt this recommendation, his name shall be stricken off the roll of Fellows.

CHAPTER XI.

OF AMENDMENTS OF THE STATUTES.

1. All proposed alterations of the Statutes or additions to them, shall be referred to a committee, and, on their report at a subsequent meeting, shall require for enactment a majority of two thirds of the members present, and at least eighteen affirmative votes.

2. Standing votes may be passed, amended, or rescinded, at any

stated meeting, by a majority of two thirds of the members present. They may be suspended by a unanimous vote.

CHAPTER XII.

OF LITERARY PERFORMANCES.

1. The Academy will not express its judgment on literary or scientific memoirs or performances submitted to it, or included in its publications.

STANDING VOTES.

1. Communications of which notice had been given to the Secretary shall take precedence of those not so notified.

2. Resident Fellows who have paid all fees and dues chargeable to them are entitled to receive one copy of each volume or article printed by the Academy, on application to the Librarian personally or by written order, within two years from the date of publication. And the current issues of the Proceedings shall be supplied, when ready for publication, free of charge, to all the Fellows and members of the Academy who desire to receive them.

3. The Committee of Publication shall fix from time to time the price at which the publications of the Academy may be sold. But members may be supplied at half this price with volumes which they are not entitled to receive free, and which are needed to complete their sets.

4. Two hundred extra copies of each paper accepted for publication in the Memoirs or Proceedings of the Academy shall be placed at the disposal of the author, free of charge.

5. Resident Fellows may borrow and have out from the Library six volumes at any one time, and may retain the same for three months, and no longer.

6. Upon special application, and for adequate reasons assigned, the Librarian may permit a larger number of volumes, not exceeding twelve, to be drawn from the Library for a limited period.

7. Works published in numbers, when unbound, shall not be taken from the Hall of the Academy, except by special leave of the Librarian.

8. Books, publications, or apparatus shall be procured from the income of the Rumford Fund only on the certificate of the Rumford Committee that they, in their opinion, will best facilitate and encourage the making of discoveries and improvements which may merit the Rumford Premium.

9. A meeting for receiving and discussing scientific communications may be held on the second Wednesday of each month not appointed for stated meetings, excepting July, August, and September.

RUMFORD PREMIUM.

In conformity with the terms of the gift of Benjamin, Count Rumford, granting a certain fund to the American Academy of Arts and Sciences, and with a decree of the Supreme Judicial Court for carrying into effect the general charitable intent and purpose of Count Rumford, as expressed in his letter of gift, the Academy is empowered to make from the income of said fund, as it now exists, at any Annual Meeting, an award of a gold and a silver medal, being together of the intrinsic value of three hundred dollars, as a premium to the author of any important discovery or useful improvement in light or in heat, which shall have been made and published by printing, or in any way made known to the public, in any part of the continent of America, or any of the American islands; preference being always given to such discoveries as shall, in the opinion of the Academy, tend most to promote the good of mankind; and to add to such medals, as a further premium for such discovery and improvement, if the Academy see fit so to do, a sum of money not exceeding three hundred dollars.

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